Chapter 6

6-1 Eq. (2-36): $S_{ut} = 3.4H_B = 3.4(300) = 1020$ MPa

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(1020) = 510$ MPa

Table 6-2: $a = 1.38,\ b = -0.067$

Eq. (6-18): $k_a = aS_{ut}^b = 1.38(1020)^{-0.067} = 0.868$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$

Eq. (6-17): $S_e = k_a k_b S'_e = (0.868)(0.969)(510) = 429$ MPa *Ans.*

6-2 **(a)** Table A-20: S_{ut} = 80 kpsi

Eq. (6-10): $S'_e = 0.5(80) = 40$ kpsi *Ans.*

(b) Table A-20: S_{ut} = 90 kpsi

Eq. (6-10): $S'_e = 0.5(90) = 45$ kpsi *Ans.*

(c) Aluminum has no endurance limit. *Ans.*

(d) Eq. (6-10): S_{ut} > 200 kpsi, $S'_e = 100$ kpsi *Ans.*

6-3 $S_{ut} = 120$ kpsi, $\sigma_{ar} = 70$ kpsi

Fig. 6-23: $f = 0.82$

Eq. (6-10): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.82(120)\right]^2}{60} = 161.4 \text{ kpsi}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{60}\right) = -0.0716$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 117\,000 \text{ cycles}$$ *Ans.*

6-4 $S_{ut} = 1600$ MPa, $\sigma_{ar} = 900$ MPa

Fig. 6-23: S_{ut} = 1600 MPa. Off the graph, so estimate f = 0.77.

Eq. (6-10): S_{ut} > 1400 MPa, so S_e = 700 MPa

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(1600)\right]^2}{700} = 2168.3 \text{ MPa}$$

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$

Eq. (6-15): $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400$ cycles *Ans.*

6-5 $S_{ut} = 230$ kpsi, $N = 150\,000$ cycles

Fig. 6-23, point is off the graph, so estimate: $f = 0.77$

Eq. (6-10): $S_{ut} > 200$ kpsi, so $S_e' = S_e = 100$ kpsi

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(230)\right]^2}{100} = 313.6$ kpsi

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$

Eq. (6-12): $S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0$ kpsi *Ans.*

6-6 $S_{ut} = 1100$ MPa = 160 kpsi

Fig. 6-23: $f = 0.79$

Eq. (6-10): $S_e' = S_e = 0.5(1100) = 550$ MPa

Eq. (6-13): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.79(1100)\right]^2}{550} = 1373$ MPa

Eq. (6-14): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.79(1100)}{550}\right) = -0.06622$

Eq. (6-12): $S_f = aN^b = 1373(150\,000)^{-0.06622} = 624$ MPa *Ans.*

6-7 $S_{ut} = 150$ kpsi, $S_{yt} = 135$ kpsi, $N = 500$ cycles

Fig. 6-23: $f = 0.80$

From Fig. 6-21, we note that below 10^3 cycles on the *S-N* diagram constitutes the low-cycle region. The stress-life approach is not very reliable in this region, but for a rough response to this question, we can write an equation in log-log scale for the line between $(10^0, S_{ut})$ and $(10^3, fS_{ut})$ as

$$S_f = S_{ut} N^{(\log f)/3} = 150(500)^{[\log(0.80)]/3} = 123 \text{ kpsi} \qquad \textit{Ans.}$$

The testing should be done at a completely reversed stress of 123 kpsi, which is below the yield strength, so it is possible. *Ans.*

6-8 d = 1.5 in, S_{ut} = 110 kpsi

Eq. (6-10): $S'_e = 0.5(110) = 55$ kpsi

Table 6-2: a = 2.00, b = – 0.217

Eq. (6-18): $k_a = aS_{ut}{}^{b} = 2.00(110)^{-0.217} = 0.721$

Eq. (6-19): $k_b = 0.879\, d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$

Eq. (6-17): $S_e = k_a k_b S'_e = 0.721(0.842)(55) = 33.4$ kpsi *Ans.*

6-9 For AISI 4340 as-forged steel,

Eq. (6-10): S_e = 100 kpsi

Table 6-2: a = 12.7, b = – 0.758

Eq. (6-18): $k_a = 12.7(260)^{-0.758} = 0.188$

Eq. (6-19): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$S_e = 0.188(0.907)(100) = 17.1$ kpsi *Ans.*

For AISI 1040:

$S'_e = 0.5(113) = 56.5$ kpsi

$k_a = 12.7(113)^{-0.758} = 0.353$

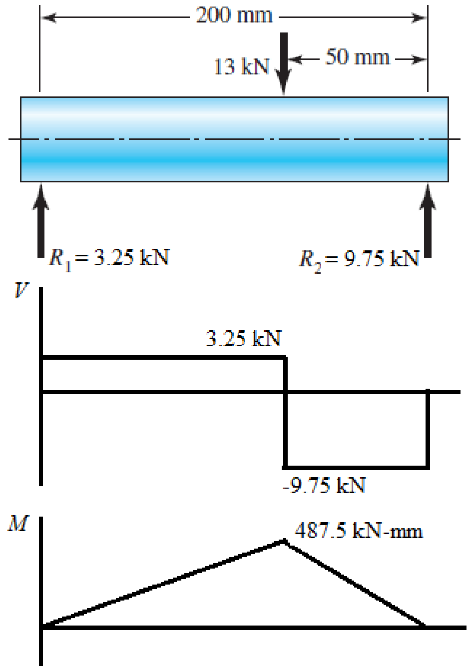
$k_b = 0.907$ (same as 4340)

Each of the other modifying factors is unity

$S_e = 0.353(0.907)(56.5) = 18.1$ kpsi *Ans.*

Not only is AISI 1040 steel a contender, it has a superior endurance strength. *Ans.*

6-10 From Table A-20, $S_{ut} = 570$ MPa, $S_y = 310$ MPa. From a free-body diagram analysis, the bearing reaction forces are found to be $R_1 = 3.25$ kN and $R_2 = 9.75$ kN. The shear-force and bending-moment diagrams are shown. The critical location is at the section where the bending moment is maximum, on the outer surface where the bending stress is maximum. With a rotating shaft, the bending stress will be completely reversed.



$$\sigma_{\max} = \sigma_{ar} = \frac{Mc}{I} = \frac{487\,500(25/2)}{(\pi/64)(25)^4} = 317.8 \text{ MPa}$$

(a) $$n_y = \frac{S_y}{\sigma_{\max}} = \frac{310}{317.8} = 0.98 \qquad \textit{Ans.}$$

Yielding is predicted, on the outer surface. For some applications, this might not prevent the part from being used, so we will continue checking for fatigue.

(b) Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(570) = 285$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(570)^{-0.217} = 0.767$

Eq. (6-19): $k_b = 1.24d^{-0.107} = 1.24(25)^{-0.107} = 0.879$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): $f = 0.87$

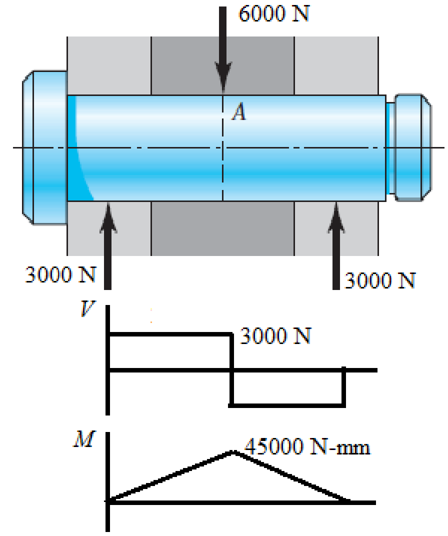
Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.87(570)]^2}{192} = 1280.8$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(570)}{192}\right) = -0.1374$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$$

$N = 25000$ cycles *Ans.*

6-11 From Table A-20, S_{ut} = 400 MPa, S_y = 220 MPa. Free-body, shear-force, and bending-moment diagrams are shown.



$$\sigma_{\max} = \frac{32M}{\pi d^3} = \frac{32(45000)}{\pi d^3} = 458366 / d^3$$

The load is repeatedly applied and released, so from Eqs. (6-8) and (6-9),

$$\sigma_m = \sigma_a = \sigma_{\max} / 2 = 229183 / d^3$$

Be sure to confirm that the units are legitimate for stress in MPa and d in mm.

For yielding,

$$n_y = 1.5 = \frac{S_y}{\sigma_{\max}} = \frac{220}{458366 / d^3}$$

$$d = 14.62 \text{ mm}$$

Now check fatigue, opting for the linear Goodman criterion for simplicity of solving for the diameter. First, determine the adjusted endurance limit.

Eq. (6-10): $S_e' = 0.5 S_{ut} = 0.5(400) = 200$ MPa

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(400)^{-0.217} = 0.828$

Estimate the size factor from the diameter determined for yielding. It can be adjusted later.

Eq. (6-19): $k_b = 1.24 d^{-0.107} = 1.24(15)^{-0.107} = 0.93$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.767)(0.879)(1)(285) = 192$ MPa *Ans.*

(c) For completely reversed stress, the fatigue factor of safety can be assessed as the ratio of the endurance limit to the completely reversed stress.

$$n_f = \frac{S_e}{\sigma_{ar}} = \frac{192}{317.8} = 0.60 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

(d) Fig. 6-23, or Eq. (6-11): f = 0.87

Eq. (6-13): $$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{[0.87(570)]^2}{192} = 1280.8$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(570)}{192}\right) = -0.1374$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{317.8}{1280.8}\right)^{\frac{1}{-0.1374}} = 25444$$

N = 25000 cycles *Ans.*

6-12 D = 1 in, d = 0.8 in, T = 1800 lbf·in, and from Table A-20 for AISI 1020 CD, S_{ut} = 68 kpsi, and S_y = 57 kpsi.

(a) Fig. A-15-15: $\frac{r}{d}=\frac{0.1}{0.8}=0.125,\ \frac{D}{d}=\frac{1}{0.8}=1.25,\ K_{ts}=1.40$

Get the notch sensitivity either from Fig. 6-27, or from the curve-fit Eqs. (6-33) and (6-36). Using the equations,

$$\sqrt{a}=0.190-2.51\left(10^{-3}\right)\left(68\right)+1.35\left(10^{-5}\right)\left(68\right)^2-2.67\left(10^{-8}\right)\left(68^3\right)=0.07335$$

$$q_s=\frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}}=\frac{1}{1+\frac{0.07335}{\sqrt{0.1}}}=0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of T = 1800 lbf·in,

$$\tau_a=K_{fs}\frac{Tr}{J}=\frac{K_{fs}16T}{\pi d^3}=\frac{1.32(16)(1800)}{\pi(0.8)^3}=23\,635\text{ psi}=23.6\text{ kpsi}$$

Eq. (6-10): $S_e'=0.5(68)=34\text{ kpsi}$

Eq. (6-18): $k_a = 2.00(68)^{-0.217} = 0.80$
Eq. (6-19): $k_b = 0.879(0.8)^{-0.107} = 0.90$
Eq. (6-25): $k_c = 0.59$
Eq. (6-17) (labeling for shear): $S_{se} = 0.80(0.90)(0.59)(34) = 14.4$ kpsi

For purely reversing torsion, use Eq. (6-58) for the ultimate strength in shear.
Eq. (6-58): $S_{su} = 0.67\ S_{ut} = 0.67(68) = 45.6$ kpsi
Fig. 6-23: $f = 0.9$

Adjusting the fatigue strength equations for shear,

Eq. (6-13): $$a=\frac{\left(f\,S_{su}\right)^2}{S_{se}}=\frac{\left[0.9(45.6)\right]^2}{14.4}=117.0\text{ kpsi}$$

Eq. (6-14): $$b=-\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right)=-\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.4}\right)=-0.151\,61$$

Eq. (6-15): $$N=\left(\frac{\tau_a}{a}\right)^{\frac{1}{b}}=\left(\frac{23.6}{117.0}\right)^{\frac{1}{-0.151\,61}}=38.5\left(10^3\right)\text{ cycles}\qquad \textit{Ans.}$$

(b) Estimate the ultimate strength at the operating temperature.

Eq. (6-26): $(S_T/S_{RT})_{750^\circ}=0.98+3.5(10^{-4})(750)-6.3(10^{-7})750^2=0.89$

Thus, $\left(S_{ut}\right)_{750^\circ}=\left(S_T/S_{RT}\right)_{750^\circ}\left(S_{ut}\right)_{70^\circ}=0.89(68)=60.5\text{ kpsi}$

Eq. (6-10): $(S'_e)_{750^\circ} = 0.5(S_{ut})_{750^\circ} = 0.5(60.5) = 30.3$ kpsi
Eq. (6-17): $S_{se} = 0.80(0.90)(0.59)(30.3) = 12.9$ kpsi

Note that we use $k_d = 1$ since the ultimate strength has been adjusted for the operating temperature.
Eq. (6-58): $S_{su} = 0.67(S_{ut})_{750^\circ} = 0.67(60.5) = 40.5$ kpsi

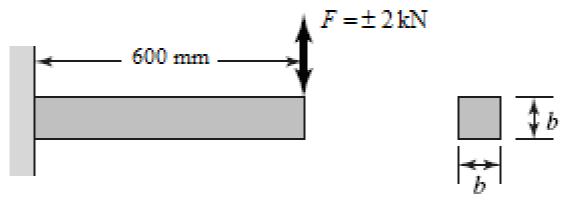
$$a = \frac{(f\,S_{su})^2}{S_{se}} = \frac{[0.9(40.5)]^2}{12.9} = 103.0 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(40.5)}{12.9}\right) = -0.150\,37$$

$$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.6}{103.0}\right)^{\frac{1}{-0.150\,37}} = 18.0(10^3) \text{ cycles} \qquad \textit{Ans.}$$

6-13 $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)
First evaluate the fatigue strength.

$$S'_e = 0.5(770) = 385 \text{ MPa}$$
$$k_a = 38.6(770)^{-0.650} = 0.51$$



Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

Eq. (6-17): $S_e = 0.51(0.85)(385) = 167$ MPa
Fig. 6-23: $f = 0.83$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{167} = 2446 \text{ MPa}$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{167}\right) = -0.1943$$

Eq. (6-12): $$S_f = aN^b = 2446(10^4)^{-0.1943} = 409 \text{ MPa}$$

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b.

$$n = \frac{S_f}{\sigma_a} = \frac{409(10^6)}{7200/b^3} = 1.5$$
$$b = 0.0298 \text{ m} \quad \text{Select } b = 30 \text{ mm.}$$

Since the size factor was guessed, go back and check it now.

Eq. (6-24): $d_e = 0.808(hb)^{1/2} = 0.808b = 0.808(30) = 24.2 \text{ mm}$

Eq. (6-19): $k_b = \left(\frac{24.2}{7.62}\right)^{-0.107} = 0.88$

Our guess of 0.85 was slightly conservative, so we will accept the result of

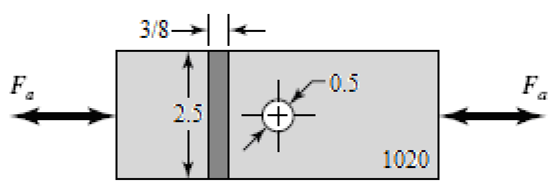
b = 30 mm. *Ans.*

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3}(10^{-6}) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

6-14 Given: w =2.5 in, t = 3/8 in, d = 0.5 in, n_d = 2. From Table A-20, for AISI 1020 CD, S_{ut} = 68 kpsi and S_y = 57 kpsi.



Eq. (6-10): $S'_e = 0.5(68) = 34$ kpsi

Table 6-2: $k_a = 2.00(68)^{-0.217} = 0.80$

Eq. (6-20): k_b = 1 (axial loading)

Eq. (6-25): k_c = 0.85

Eq. (6-17): S_e = 0.80(1)(0.85)(34) = 23.1 kpsi

Table A-15-1: $d / w = 0.5/2.5 = 0.2,\ K_t = 2.5$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). The relatively large radius is off the graph of Fig. 6-26, so we will assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(68) + 1.51(10^{-5})(68)^2 - 2.67(10^{-8})(68^3) = 0.09799$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.09799}{\sqrt{0.25}}} = 0.836$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5 - 1) = 2.25$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5-0.5)} = 3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{23.1}{3F_a} = 2$$

$$F_a = 3.85 \text{ kips} \quad \textit{Ans.}$$

6-15 Given: $D = 2$ in, $d = 1.8$ in, $r = 0.1$ in, $M_{\max} = 25\,000$ lbf · in, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, S_{ut} = 120 kpsi and S_y = 66 kpsi.

Eq. (6-10): $S'_e = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(120)^{-0.217} = 0.71$

Eq. (6-23): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S'_e = (0.71)(0.92)(1)(60) = 39.2$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \Rightarrow K_t = 2.1$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1 - 1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eqs. (6-8) and (6-9): $\sigma_m = K_f \dfrac{\sigma_{\max}+\sigma_{\min}}{2} = (1.96)\dfrac{(43.7+0)}{2} = 42.8 \text{ kpsi}$

$$\sigma_a = K_f \left|\frac{\sigma_{\max}-\sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7-0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-41): $n_f = \left(\dfrac{\sigma_a}{S_e}+\dfrac{\sigma_m}{S_{ut}}\right)^{-1} = \left(\dfrac{42.8}{39.2}+\dfrac{42.8}{120}\right)^{-1}$

$$n_f = 0.69 \quad \textit{Ans.}$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \quad \textit{Ans.}$$

6-16 From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is M = 2.1(200) = 420 kN·mm. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{420\,(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: r/d = 3/35 = 0.086, D/d = 50/35 = 1.43, K_t =1.7

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations, with S_{ut} = 470 MPa and r = 3 mm,

$$\sqrt{a} = 1.24 - 2.25\left(10^{-3}\right)(470) + 1.60\left(10^{-6}\right)(470)^2 - 4.11\left(10^{-10}\right)(470)^3 = 0.4933$$

$$q = \frac{1}{1+\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\dfrac{0.4933}{\sqrt{3}}} = 0.78$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$

Eq. (6-10): $S_e' = 0.5 S_{ut} = 0.5(470) = 235$ MPa

Eq. (6-18): $k_a = a S_{ut}^b = 3.04(470)^{-0.217} = 0.80$

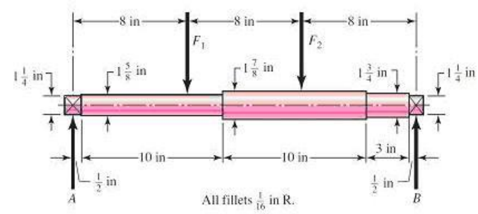
Eq. (6-19): $k_b = 1.24 d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

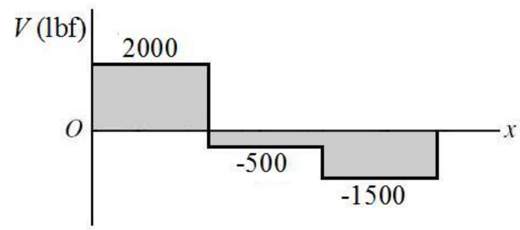
Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.80)(0.85)(1)(235) = 160$ MPa

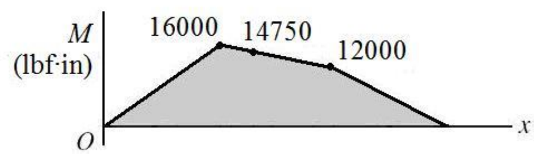
$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{160}{1.55(99.8)} = 1.03 \quad \text{Infinite life is predicted.} \qquad \textit{Ans.}$$

6-17 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 2000 lbf and R_B = 1500 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.





M = 16 000 – 500 (2.5) = 14 750 lbf · in



With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: r/d = 0.0625/1.625 = 0.04, D/d = 1.875/1.625 = 1.15, K_t =1.95

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{ar}}} = \frac{27.0}{1.72(35.0)} = 0.45 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

Fig. 6-23: $f = 0.87$

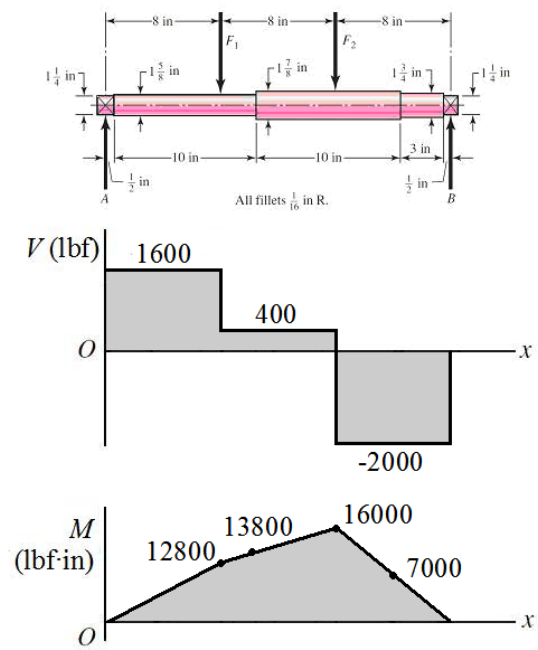
Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(85)]^2}{27.0} = 202.5$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(85)}{27.0}\right) = -0.1459$

Eq. (6-15): $N = \left(\dfrac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\dfrac{(1.72)(35.0)}{202.5}\right)^{\frac{1}{-0.1459}} = 4082$ cycles

$N = 4100$ cycles *Ans.*

6-18 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 1600 lbf and R_B = 2000 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.



$M = 12\,800 + 400\,(2.5) = 13\,800$ lbf · in

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{ar} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-26, or from the curve-fit Eqs. (6-33) and (6-35). Using the equations,

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.07690}{\sqrt{0.0625}}} = 0.76$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-10): $S_e' = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 2.00(85)^{-0.217} = 0.76$

Eq. (6-19): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-25): $k_c = 1$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(0.835)(1)(42.5) = 27.0$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{ar}} = \frac{27.0}{1.72(32.8)} = 0.48 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S*-*N* diagram to estimate the life.
Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{\left(f\,S_{ut}\right)^2}{S_e} = \frac{\left[0.87(85)\right]^2}{27.0} = 202.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(85)}{27.0}\right) = -0.1459$$

Eq. (6-15): $$N = \left(\frac{K_f \sigma_{ar}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{202.5}\right)^{\frac{1}{-0.1459}} = 6370 \text{ cycles}$$

$N = 6400$ cycles *Ans.*

6-19 Table A-20: $S_{ut} = 120$ kpsi, $S_y = 66$ kpsi

N = (950 rev/min)(10 hr)(60 min/hr) = 570 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We’ll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we will evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are R_1 = 2 kips and R_2 = 6 kips. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2\text{ kip}(10\text{ in}) = 20\text{ kip}\cdot\text{in}$$

$$\sigma_{ar} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3}\text{ kpsi}$$

Now we will get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, let us estimate a value of q = 0.85 from observation of Fig. 6-26, and check it later.

Fig. A-15-9: $D/d = 1.4d/d = 1.4, \quad r/d = 0.1d/d = 0.1, \quad K_t = 1.65$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now, evaluate the fatigue strength.

$$S_e^{'} = 0.5(120) = 60\text{ kpsi}$$

$$k_a = 2.00(120)^{-0.217} = 0.71$$

Since the diameter is not yet known, assume a typical value of k_b = 0.85 and check later. All other modifiers are equal to one.

$$S_e = (0.71)(0.85)(60) = 36.2\text{ kpsi}$$

Determine the desired fatigue strength from the *S-N* diagram.

Fig. 6-23: $f = 0.82$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.82(120)]^2}{36.2} = 267.5$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.82(120)}{36.2}\right) = -0.1448$

Eq. (6-12): $S_f = aN^b = 267.5(570\,000)^{-0.1448} = 39.3\text{ kpsi}$

Compare strength to stress and solve for the necessary *d.*

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{39.3}{(1.55)(203.7/d^3)} = 1.6$$

d = 2.34 in

Since the size factor and notch sensitivity were guessed, go back and check them now.

Eq. (6-19): $k_b = 0.91d^{-0.157} = 0.91(2.34)^{-0.157} = 0.80$

This is a little lower than our initial guess.

From Fig. 6-26 with $r = d/10$ = 0.234 in, we are off the graph, but it appears our guess for q of 0.85 is low. Assuming the trend of the graph continues, we'll choose q = 0.91 and iterate the problem with the new values of k_b and q.

Intermediate results are S_e = 34.1 kpsi, S_f = 37.2 kpsi, and K_f = 1.59. This gives

$$n_f = \frac{S_f}{K_f \sigma_{ar}} = \frac{37.2}{(1.59)(203.7/d^3)} = 1.6$$

d = 2.41 in *Ans.*

A quick check of k_b and q show that our estimates are still reasonable for this diameter.

6-20 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[25^2 + 3(0)^2\right]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[25^2 + 3(15^2)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{25.00}{40}\right)\left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)}\right)^2}\right] = 1.31 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(25.00/40) + (25.98/130)} = 1.21$ *Ans.*

6-21 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[10^2 + 3(0)^2\right]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(20)^2\right]^{1/2} = 34.64 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[10^2 + 3\left(20^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{34.64}\right)^2\left(\frac{10.00}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)}\right)^2}\right\} = 1.74 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(10.00/40) + (34.64/130)} = 1.94$ *Ans.*

6-22 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[12^2 + 3(10)^2\right]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{max} = \left(\sigma^2_{max} + 3\tau^2_{max}\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[\left(12 + 0\right)^2 + 3\left(10 + 15\right)^2\right]^{1/2} = 44.93 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{max}} = \frac{60}{44.93} = 1.34 \quad \textit{Ans.}$$

(a) Goodman, Equation (6-41)

$$n_f = \frac{1}{(21.07/40) + (25.98/80)} = 1.17 \quad \textit{Ans.}$$

(b) Gerber, Equation (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2\left(\frac{21.07}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(21.07)}\right)^2}\right\} = 1.47 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $n_f = \dfrac{1}{(21.07/40) + (25.98/130)} = 1.38 \quad \textit{Ans.}$

6-23 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma^2_a + 3\tau^2_a\right)^{1/2} = \left[0^2 + 3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma^2_m + 3\tau^2_m\right)^{1/2} = 0 \text{ kpsi}$$

$$\sigma'_{max} = \left(\sigma^2_{max} + 3\tau^2_{max}\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[3\left(30\right)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{max}} = \frac{60}{51.96} = 1.15 \quad \textit{Ans.}$$

(a) through (c)

With a mean stress of zero, the Goodman, Gerber, and Morrow criteria all simplify to the same simple comparison of the alternating stress to the endurance limit,

$$n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the *S*-*N* diagram is applicable for σ'_a.

Fig. 6-23: $f = 0.875$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.875(80)]^2}{40} = 122.5$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.875(80)}{40}\right) = -0.08101$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{51.96}{122.5}\right)^{\frac{1}{-0.08101}} = 39\,600 \text{ cycles} \quad \textit{Ans.}$$

6-24 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$

Obtain von Mises stresses for the alternating, mean, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[15^2 + 3(0)^2\right]^{1/2} = 15.00 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(\sigma_a + \sigma_m)^2 + 3(\tau_a + \tau_m)^2\right]^{1/2}$$

$$= \left[(15)^2 + 3(15)^2\right]^{1/2} = 30.00 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \quad \textit{Ans.}$$

(a) Goodman, Eq. (6-41)

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \quad \textit{Ans.}$$

(b) Gerber, Eq. (6-48)

$$n_f = \frac{1}{2}\left(\frac{80}{15.00}\right)^2\left(\frac{25.98}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)}\right)^2}\right\} = 1.43 \quad \textit{Ans.}$$

(c) Morrow

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 50 = 80 + 50 = 130$ kpsi

Eq. (6-46): $$n_f = \frac{1}{(25.98/40) + (15.00/130)} = 1.31 \quad \textit{Ans.}$$

6-25 Given: $F_{\max} = 28 \text{ kN}, F_{\min} = -28 \text{ kN}$. From Table A-20, for AISI 1040 CD, $S_{ut} = 590 \text{ MPa}, S_y = 490 \text{ MPa}$,

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-10): $S_e' = 0.5(590) = 295 \text{ MPa}$

Eq. (6-18): $k_a = aS_{ut}^b = 3.04(590)^{-0.217} = 0.76$

Eq. (6-20): $k_b = 1 \quad \text{(axial)}$

Eq. (6-25): $k_c = 0.85$

Eq. (6-17): $S_e = k_a k_b k_c S_e' = (0.76)(1)(0.85)(295) = 190.6 \text{ MPa}$

Fig. 6-26: $q = 0.83$

Fig. A-15-1: $d/w = 0.24, K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

Note, since $\sigma_m = 0$, the stress is completely reversing, and

$$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{324.2} = 0.59 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate the life from the *S-N* diagram. With $\sigma_m = 0$, the stress state is completely reversed, and the *S-N* diagram is applicable for σ_a.

Fig. 6-23: $f = 0.87$

Eq. (6-13): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(590)]^2}{190.6} = 1382$

Eq. (6-14): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.87(590)}{190.6}\right) = -0.1434$

Eq. (6-15): $N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{324.2}{1382}\right)^{\frac{1}{-0.1434}} = 24\,613 \text{ cycles}$

N = 25 000 cycles *Ans.*

6-26 $S_{ut} = 590$ MPa, $S_y = 490$ MPa, $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life.
From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (12\,000)}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Goodman criteria, Equation (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590} \right)^{-1}$$

$$n_f = 1.14 \quad \textit{Ans.}$$

Gerber criteria, Equation (6-48):

$$n_f = \frac{1}{2} \left(\frac{S_{ut}}{\sigma_m} \right)^2 \frac{\sigma_a}{S_e} \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut} \sigma_a} \right)^2} \right]$$

$$= \frac{1}{2} \left(\frac{590}{231.6} \right)^2 \frac{92.63}{190.6} \left[-1 + \sqrt{1 + \left(\frac{2(231.6)(190.6)}{590(92.63)} \right)^2} \right]$$

$$n_f = 1.42 \quad \textit{Ans.}$$

Morrow criteria:

Estimate the fatigue strength coefficient.

Eq. (6-44): $\sigma'_f = S_{ut} + 345 = 590 + 345 = 935$ MPa

Eq. (6-46): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma'_f} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{935} \right)^{-1}$$

$$n_f = 1.36 \quad \textit{Ans.}$$

The results are consistent with Fig. 6-36, where for a mean stress that is about half of the yield strength, the Goodman line should predict failure significantly before the other two.

6-27 $S_{ut} = 590$ MPa, $S_y = 490$ MPa

From Prob. 6-25: $S_e = 190.6$ MPa, $K_f = 2.2$

(a) $F_{\max} = 28$ kN, $F_{\min} = 0$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \qquad \textit{Ans.}$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 0}{2(10)(25-6)} \right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 0}{2(10)(25-6)} \right] = 162.1 \text{ MPa}$$

For the Goodman criteria, Eq. (6-41):

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{162.1}{190.6} + \frac{162.1}{590} \right)^{-1} = 0.89 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{162.1}{1 - (162.1/590)} = 223.5 \text{ MPa}$$

Fig. 6-23: $f = 0.87$

Eq. (6-13): $$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{\left[0.87(590)\right]^2}{190.6} = 1382$$

Eq. (6-14): $$b = -\frac{1}{3} \log\left(\frac{f\, S_{ut}}{S_e} \right) = -\frac{1}{3} \log\left(\frac{0.87(590)}{190.6} \right) = -0.1434$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b} = \left(\frac{223.5}{1382} \right)^{\frac{1}{-0.1434}} = 329\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \quad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 12\,000}{2(10)(25-6)} \right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 12\,000}{2(10)(25-6)} \right] = 231.6 \text{ MPa}$$

Eq. (6-41): $$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} = \left(\frac{92.63}{190.6} + \frac{231.6}{590} \right)^{-1} = 1.14 \quad \textit{Ans.}$$

(c) $F_{\max} = 12$ kN, $F_{\min} = -28$ kN

The compressive load is the largest, so check it for yielding.

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{-28\,000}{10(25-6)} = -147.4 \text{ MPa}$$

$$n_y = \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \quad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{12\,000 - (-28\,000)}{2(10)(25-6)} \right| = 231.6 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{12\,000 + (-28\,000)}{2(10)(25-6)} \right] = -92.63 \text{ MPa}$$

For $\sigma_m < 0$, Eq. (6-42): $$n_f = \frac{S_e}{\sigma_a} = \frac{190.6}{231.6} = 0.82 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress, consistent with the horizontal fatigue line in Fig. 6-34. Get *a* and *b* from part (a).

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b} = \left(\frac{231.6}{1382} \right)^{\frac{1}{-0.1434}} = 257\,000 \text{ cycles} \qquad \textit{Ans.}$$

6-28 Eq. (2-36): $S_{ut} = 0.5(400) = 200$ kpsi

Eq. (6-10): $S_e' = 0.5(200) = 100$ kpsi

Eq. (6-18): $k_a = aS_{ut}^b = 11.0(200)^{-0.650} = 0.35$

Eq. (6-24): $d_e = 0.37d = 0.37(0.375) = 0.1388$ in

Eq. (6-19): $k_b = 0.879d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

Eq. (6-17): $S_e = (0.35)(1)(100) = 35.0$ kpsi

$$F_a = \frac{40-20}{2} = 10 \text{ lb} \qquad F_m = \frac{40+20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

(a) Goodman criterion, Eq. (6-41):

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{35.0} + \frac{69.54}{200}$$

$$n_f = 0.99 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress. Using the Goodman criterion,

Eq. (6-58): $$\sigma_{ar} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{23.18}{1-(69.54/200)} = 35.54 \text{ kpsi}$$

Fig. 6-23: $f = 0.78$

Eq. (6-13): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.78(200)\right]^2}{35} = 695.3$$

Eq. (6-14): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.78(200)}{35.0}\right) = -0.2164$$

Eq. (6-15): $$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{35.54}{695.3}\right)^{\frac{1}{-0.2164}} = 929\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) Gerber criterion, Eq. (6-48):

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{200}{69.54}\right)^2 \frac{23.18}{35.0}\left[-1+\sqrt{1+\left(\frac{2(69.54)(35.0)}{200(23.18)}\right)^2}\right]$$

$= 1.23$ Infinite life is predicted *Ans.*

6-29 $E = 207.0$ GPa

(a) $I = \frac{1}{12}(20)(4^3) = 106.7\ \text{mm}^4$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3\ \text{N} \quad \textit{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9\ \text{N} \quad \textit{Ans.}$$

(b) Get the fatigue strength information.

Eq. (2-36): S_{ut} = =3.4H_B = 3.4(490) = 1666 MPa

From problem statement: $S_y = 0.9S_{ut}$ = 0.9(1666) = 1499 MPa

Eq. (6-10): $S'_e = 700$ MPa

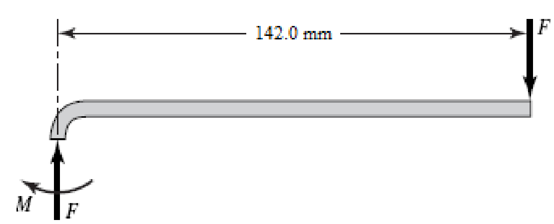
Eq. (6-18): $k_a = 1.38(1666)^{-0.067} = 0.84$

Eq. (6-24): $d_e = 0.808[20(4)]^{1/2} = 7.23$ mm

Eq. (6-19): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-17): S_e = 0.84(1)(700) = 588 MPa

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$M = 142F$ N·mm, $A = 4(20) = 80\ \text{mm}^2$,
$h = 4$ mm, $r_i = 4$ mm, $r_o = r_i + h = 8$ mm,
$r_c = r_i + h/2 = 6$ mm

Table 3-4: $$r_n = \frac{h}{\ln(r_o/r_i)} = \frac{4}{\ln(8/4)} = 5.7708\ \text{mm}$$

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$
$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$
$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-76) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$
$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$
$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$
$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$
$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$
$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$
$$(\sigma_i)_a = \left|\frac{-166.2 - (-498.6)}{2}\right| = 166.2 \text{ MPa}$$
$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$
$$(\sigma_o)_a = \left|\frac{310.8 - 103.6}{2}\right| = 103.6 \text{ MPa}$$
$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is –498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa, so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.

Inner radius:

Since $\sigma_m < 0$, Eq. (6-42): $n_f = \dfrac{S_e}{\sigma_a} = \dfrac{588}{166.2} = 3.54$

Outer radius:

Since $\sigma_m > 0$, using the Goodman line, Eq. (6-41),