PROBLEM SOLUTIONS

CHAPTER 1. *PRELIMINARY CONCEPTS*

1-1 A 1.4-cm diameter sphere placed in a freestream at 18 m/s at 20°C and 1 atm. Compute the diameter Reynolds number for 3 cases:

(a) Air: Table A-2 - at 20°C, $\rho = 1.205$ kg/m^3, $\mu = 1.81$ E-5 Pa-s. Then

$$\mathrm{Re_D} = \rho \mathrm{VD}/\mu = \frac{(1.205)(18)(0.014)}{1.81\text{E-}5} = \mathbf{16,800} \qquad \text{(Ans.)}$$

(b) Water: Table A-1 - at 20°C, $\rho = 998$ kg/m^3, $\mu = 1.002$ mPa-s:

$$\mathrm{Re_D} = (998)(18)(0.014)/(0.001002) = \mathbf{251,000} \qquad \text{(Ans.)}$$

(c) Hydrogen: Table A-3, $\mathrm{M} = 2.016$, then $\mathrm{R} = 8313/\mathrm{M} = 4124$ m^2/s^2-°K. Thus estimate $\rho = \mathrm{p}/\mathrm{RT} = (101350)/(4124)(293) = 0.0838$ kg/m^3. From Table 1-2 for hydrogen,

$$\mu \approx \mu_o (\mathrm{T}/\mathrm{T_o})^{\mathrm{n}} = (8.411\text{E-}6)(293/273)^{.068} = 8.83 \text{ E-6 Pa-s}$$

Then $\mathrm{Re_D} = (0.0838)(18)(0.014)/(8.83\text{ E-}6) = \mathbf{2,400}$ (Ans.)

1-2 At what wind velocity will an 8-mm-diameter wire "sing" at middle C (256 Hz)?

For air at 20°C, assume $\mathrm{v} \approx 1.5\text{E-}5$ m^2/s. From Fig. 1-8 guess a vortex-shedding Strouhal number of 0.2 [check the Reynolds number afterward]. Then

$\mathrm{fD}/\mathrm{U} \approx 0.2 = (256)(0.008)/\mathrm{U}$, or $\mathrm{U} \approx 10.24$ m/s. At this speed the Reynolds number is $\mathrm{Re_D} = \mathrm{UD}/\mathrm{v} = (10.24)(0.008)/1.5\text{E-}5 = 5400$. This is nicely in the range where $\mathrm{fD}/\mathrm{U} = 0.2$. Perhaps we could iterate just a little more closely to obtain

$$\mathrm{fD}/\mathrm{U} \approx 0.205, \ \mathrm{Re} = \mathrm{UD}/\mathrm{v} \approx 5300, \text{ or } \mathrm{U} = \mathbf{10.0\ m/s} \qquad \text{(Ans.)}$$

1-3 If $\mathrm{U} = 12$ m/s in Prob. 1-2 above, what is the wire drag in N/m?

For air assume $\rho = 1.205$ kg/m^3 and $\mathrm{v} = 1.5\text{E-}5$ m^2/s. The Reynolds number is

$$\mathrm{Re_D} = \mathrm{UD}/\mathrm{v} = (12)(0.008)/(1.5\text{E-}5) = 6400$$

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From Fig. 1-9 at this Reynolds number, estimate a drag coefficient of 1.1. Then

$$F_{drag} = C_D\left(\frac{1}{2}\right)\rho V^2\,(DL) = 1.1(0.5)(1.205)(12)^2\,(0.008)(1.0) = \mathbf{0.76\ N/m} \qquad \text{(Ans.)}$$

1-4 Given, without proof, the Poiseuille-paraboloid laminar-pipe-flow formula from Chap. 3, $u = (C/\mu)(R^2 - r^2)$, find the wall shear stress if $u_{max} = 30$ m/s, $D = 1$ cm, and $\mu = 0.3$ kg/(m-s). [The exact analysis will be given in Sect. 3-3.1.]

Examining the formula, we see that the maximum velocity occurs on the centerline:

$$u_{mx} = u(r = 0) = CR^2/\mu = 30 \text{ m/s} = C(0.005)^2/(0.3)\text{, or: } C = 3.6E5\ \text{N}/\left(\text{m}^2\text{-s}^2\right)$$

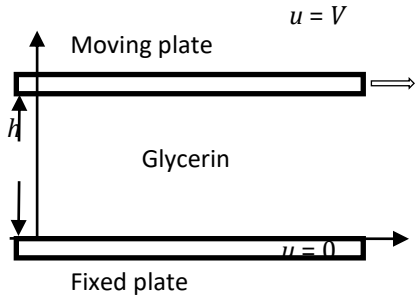
With C thus known for this data, we may evaluate wall shear stress by differentiation:

$$\tau_{wall} = \mu\left|\frac{\partial u}{\partial r}\right|_{r=0} = \mu\left(\frac{2RC}{\mu}\right) = 2RC = 2(0.005)(3.6E5) = \mathbf{3600\ Pa} \qquad \text{(Ans.)}$$

We should check the Reynolds number Re_D but we don't know the density. But "oil" is usually in the range $\rho \approx 900$ kg/m^3. Then $Re_D = \rho u_{max} D/\mu = (900)(30)(0.01)/(0.3) \approx 900$, which is well within the laminar-flow range.

1-5 Glycerin at $20^\circ C$ is confined between two large parallel plates. One plate is fixed and the other moves parallel at 17 mm/s. The distance between the plates is 3 mm. Assuming no-slip, estimate the shear stress in the glycerin, in Pa.

Solution: Glycerin at $20^\circ C$ is confined between two large parallel plates. One plate is fixed and the other moves parallel at $V = 17$ mm/s. The distance *h* between the plates is 3 mm.



For glycerin at $20^\circ C$, the viscosity $\mu = 1.5$ kg/m·s.

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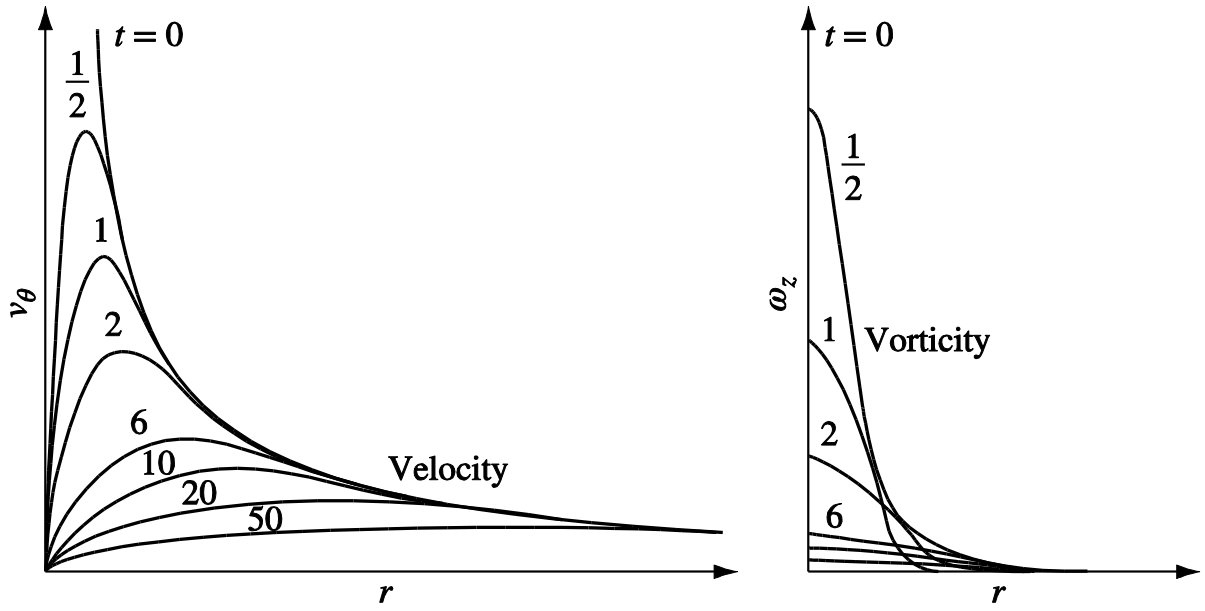
Assuming no-slip, the shear stress $\tau = \dfrac{\mu V}{h} = \dfrac{(1.5\ \text{kg/m}\cdot\text{s})(17\times10^{-3}\ \text{m/s})}{(3\times10^{-3}\ \text{m/s})} = 8.5\ \text{Pa}$. (Ans.)

1-6 Given a plane unsteady viscous flow in polar coordinates:

$$v_r = 0;\ v_\theta = \frac{C}{r}\left[1-\exp\left(-\frac{r^2}{4vt}\right)\right]$$



Compute the vorticity and sketch some profiles of vorticity and velocity.

From Appendix B, the vorticity is

$$\omega_z = \frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) = \frac{C}{2vt}\exp\left(-\frac{r^2}{4vt}\right)$$

The instantaneous velocity and vorticity profiles are plotted at top. At $t = 0$, the flow is a "line" vortex, irrotational everywhere except at the origin $(\omega = \infty)$.

1-7 Given the two-dimensional unsteady flow $u = x/(1+t)$, $v = y/(1+2t)$, find the equation for the streamlines which pass through the point (x_0, y_0) at time $(t = 0)$. From the geometric requirement for two-dimensional streamlines at any instant,

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$$\left.\frac{dy}{dx}\right|_{streamline} = \frac{v}{u} = \frac{y(1+t)}{x(1+2t)}$$

Holding time t constant, we may separate the variables and integrate to obtain $y = C\,x^n$, where $n = (1+t)/(1+2t)$. To satisfy the initial condition, we must have $C = y_0/(x_0)^n$. The final result for the (unsteady) streamlines is

$$\frac{y}{y_0} = \left(\frac{x}{x_0}\right)^n, \quad n = \frac{1+t}{1+2t} \qquad \text{(Ans.)}$$

1-8 For the inviscid streamline approaching the forward stagnation point of the cylinder in Fig. 1-5, evaluate the strain rates and the time to go from $(2R, \pi)$ to (R, π)

From Eqs. (1-2), $v_r = U_\infty\left(1 - R^2/r^2\right)\cos\theta, \quad v_\theta = -U_\infty\left(1 + R^2/r^2\right)\sin\theta$

Then, from Appendix B, evaluate the normal and shear-strain rates along the line $\theta = \pi$:

$$\varepsilon_{rr} = \frac{\partial v_r}{\partial r} = \frac{2U_\infty R^2\cos\theta}{r^3}\Big|_{\theta=\pi} = -\frac{2U_\infty R^2}{r^3}$$

$$\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r} = -\frac{2U_\infty R^2\cos\theta}{r^3}\Big|_{\theta=\pi} = +\frac{2U_\infty R^2}{r^3}$$

$$\varepsilon_{r\theta} = \frac{1}{r}\frac{\partial v_r}{\partial\theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} = \frac{4U_\infty R^2\sin\theta}{r^3}\Big|_{\theta=\pi} = 0 \qquad \text{(Ans. } \mathbf{a}\text{)}$$

The particle moving toward the stagnation point gets shorter in the "r" direction and fatter by the same amount in the "θ" direction, thus maintaining constant volume for this incompressible flow. The shear strain rate is *zero* because we are on a line of symmetry.

For part (b), by definition, the radial velocity along the stagnation line $(\theta = \pi)$ is

$$v_r = \frac{dr}{dt} = -U_\infty\left(1 - R^2/r^2\right)$$

We may separate the variables and integrate to find the time of travel between $(2R)$ and (R):

$$-U_\infty t = \int_{2R}^{R}\frac{r^2 dr}{r^2 - R^2} = \left[r + \frac{R}{2}\ln\left(\frac{r-R}{r+R}\right)\right]_{2R}^{R} = -\infty \qquad \text{(Ans. } \mathbf{b}\text{)}$$

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It takes infinite time to actually *reach* the stagnation point, where $V = 0$.

1-9 Use the approximate equation of state for water,

$$p/p_o \approx (A+1)(\rho/\rho_o)^n - A \quad \text{with } A \approx 3000,\ n = 7$$

to compute the following quantities for water, $p_o = 1$ atm, $\rho_o = 998$ kg/m^3:

(a) the pressure required to double the density of water:

$$p/p_o = (3000+1)(2.0)^7 - 3000 = 381128, \text{ or: } p = \mathbf{381,000\ atm} \qquad \text{(Ans. } \mathbf{a}\text{)}$$

(b) the bulk modulus K of water at 1 atm. By definition,

$$K = \rho \frac{dp}{d\rho}\Big|_T = \rho p_o (A+1) \frac{n\rho^{n-1}}{\rho_o^n} = n p_o (A+1) = 21007\ p_o \text{ at 1 atm.}$$

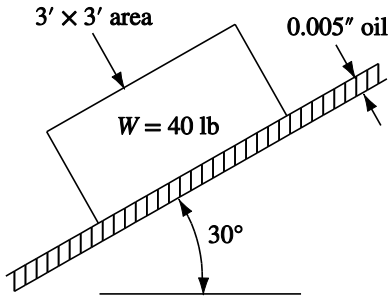
Thus the bulk modulus is $K \approx 21{,}007$ atm $\approx \mathbf{2.13\ E9\ Pa}$ (Ans. **b**)

(c) the speed of sound at 1 atm:

$$a\big|_{1\ atm} = (K/\rho_o)^{1/2} = \left(\frac{2.13\ \text{E9 Pa}}{998\ \text{kg/m}^3}\right)^{1/2} = \mathbf{1460\ m/s} \qquad \text{(Ans. } \mathbf{c}\text{)}$$

These are accurate estimates of the measured compressibility and sound speed of water.

1-10 As shown, a plate slides down an incline on a film of oil of viscosity $\mu = 5\text{E-}4$ slug/ft-s



(a) Estimate the terminal sliding velocity V*:

Acceleration is zero, so

$$W \sin\theta = \tau A = \mu \frac{\Delta V}{\Delta y} A$$

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where A is the plate area touching the oil film.

Assuming a linear velocity profile, $\Delta V = V^*$ and $\Delta y =$ the film thickness, hence

$$W\sin\theta = 40\sin(30°) = \mu\frac{\Delta V}{\Delta y}A = (0.0005)\left(\frac{V^*}{0.005/12}\right)(9)$$

in English units. Hence solve for $V^*(\text{terminal}) = \mathbf{1.85\,ft/s}$ (Ans. **a**)

(b) Estimate the time for the plate to accelerate from rest to 99% terminal velocity: If x is down the incline, then a dynamic force balance gives

$$\sum F_x = W\sin\theta - \mu\frac{V}{\Delta y}A = \frac{W}{g}\frac{dV}{dt},$$

or: $$\frac{dV}{dt} + \left(\frac{g\mu A}{W\Delta y}\right)V = g\sin\theta$$

The solution to this first-order linear ordinary differential equation is

$$V = V^*\left[1 - \exp\left(-\frac{g\mu A}{W\Delta y}t\right)\right] = 0.99V^* \text{ if } t^* = \frac{4.605W\Delta y}{g\mu A}$$

For our data, then, the time to reach 99% of terminal velocity is

$$t^* = \frac{4.605(40)(0.005/12)}{(32.2)(0.0005)(9.0)} = \mathbf{0.53\,sec}$$ (Ans. **b**)

1-11 Estimate the viscosity of nitrogen at 86 MPa and 49°C. From Appendix A-3, for N_2, read $T_c = 226°R = 126°K$, $p_c = 33.5$ atm, $\mu_c = 18.0$ E-6 Pa-s. At this high pressure, we cannot use "low density" formulas but rather must use Fig. 1-17. Compute ratios:

$$\frac{T}{T_c} = \frac{49+273}{126} = 2.55; \quad \frac{p}{p_c} = \frac{86E6}{33.5(101350)} = 25.3; \text{ Read } \frac{\mu}{\mu_c} \approx 2.5 \pm 0.1$$

Then our estimate is $\mu = 2.5\,\mu_c = 2.5(18.0) = \mathbf{45 \pm 2\ \mu Pa\text{-}s}$ (Ans.)

The agreement with the measured value (also 45 μPa-s) is excellent.

1-12 Estimate the thermal conductivity of helium at 420°C and 1 atm. This is truly "low-density", since $p < p_c$ and $T >> T_c$. A power-law estimate would be based on 0°C:

$$k \approx k_o(T/T_o)^n \approx (0.142\text{W/m-K})\left(\frac{420+273}{273}\right)^{072} \approx \mathbf{0.278\ W/m\text{-}K}$$

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Alternately, we could use the kinetic theory formula, Eq. (1-41). From Appendix A-5 for helium, $\sigma = 2.551$ Å, $T_\varepsilon = 10.22°K,$ and $M = 4.003.$ First use Eq. (1-34) to compute

$$\Omega_v \approx 1.147\left(\frac{420+273}{10.22}\right)^{-0.145} + \left(\frac{420+273}{10.22}+0.5\right)^{.20} = 0.6226 \qquad \text{[check with Table 1-1]}$$

Our estimate from (1-41) then is

$$k = \frac{0.0833\sqrt{T}}{\sigma^2\Omega_v\sqrt{M}} = \frac{0.0833\sqrt{(693)}}{(2.551)^2(0.6226)\sqrt{4.003}} = \mathbf{0.27\ W/m\text{-}K}$$

The agreement with the experimental value of 0.28 W/m-K is good for both estimates.

1-13 According to Table C-5 and Fig. 1-15, at what pressure is the viscosity of CO_2 equal to approximately 30×10^{-5} Pa·s when the temperature is 800°R ?

Solution: $T = 800°R = 444.4444$ K, $T_c = 548°R = 304.4444$ K, $T_r = \dfrac{T}{T_c} = \dfrac{444.4444\text{ K}}{304.4444\text{ K}} = 1.46$

$\mu = 30\times10^{-5}$ Pa·s, $\mu_c = 3.43\times10^{-5}$ Pa·s, $\mu_r = \dfrac{\mu}{\mu_c} = \dfrac{30\times10^{-5}\text{ Pa}\cdot\text{s}}{34.3\times10^{-6}\text{ Pa}\cdot\text{s}} = 8.75$

Thus, $p_r = 25$ (from Fig. 1-15).

Then, pressure $p = p_c p_r = (72.9\text{ atm})\times 25 = 1822.5\text{ atm}$. (Ans.)

1-14 Fit the given viscosity-vs-temperature data for ammonia gas to power-law and Sutherland-law formulas.

(a) The power-law is an excellent fit to this data. Taking $T_o = 300°K,$ we obtain, by least-squares to a $\log(\mu)$ vs. $\log(T)$ plot,

$$\frac{\mu}{\mu_o} \approx \left(\frac{T}{T_o}\right)^{\mathbf{1.051}} \pm 0.3\% \text{ for } T_o = 300°K \qquad \text{(Ans. } \mathbf{a}\text{)}$$

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(b) The Sutherland-law is not an especially good fit to the data, which has μ rising with T at an increasing rate. It may be fit to least squares by minimizing the functional

$$\sum_{i=1}^{6}\left[\mu_i^* - \frac{\left(T_i^*\right)^{3/2}\left(1+S^*\right)}{T_i^*+S^*}\right]^2 \quad \text{where } \mu^* = \frac{\mu}{\mu_o}, T^* = \frac{T}{T_o}, S^* = \frac{S}{T_o}$$

for the given six data points. The minimum is found by differentiating the functional with respect to S^*, v with the result $S^* = 1.91$, or:

$$S_{\text{best fit}} \approx \mathbf{573°K} \qquad \text{(Ans. b)}$$

The error is $\pm 2.4\%$, or eight rimes more than the power-law fit.

1-15 Experimental data for the viscosity of helium at low pressure are as follows:

T, °C	0	100	200	300	400	500
μ, Pa·s	1.87×10^{-5}	2.32×10^{-5}	2.73×10^{-5}	3.12×10^{-5}	3.48×10^{-5}	3.48×10^{-5}

Fit these values to a suitable formula.

Solution: Experimental data for the viscosity of helium in Kelvin scale at low pressure are as follows:

T, K	273	373	473	573	673	773
μ, Pa·s	1.87×10^{-5}	2.32×10^{-5}	2.73×10^{-5}	3.12×10^{-5}	3.48×10^{-5}	3.48×10^{-5}

Using power law curve, $\dfrac{\mu}{\mu_0} = \left(\dfrac{T}{T_0}\right)^n$.

For $\mu_0 = 1.87 \times 10^{-5}$ Pa·s and $T_0 = 273$ K

T	373 K	473 K	573 K	673 K	773 K
n	0.691	0.688	0.69	0.688	0.597

Therefore, the mean value $n = \dfrac{\sum_{i=1}^{5} n_i}{5} = 0.671$ ($\pm 4\,\%$ accuracy for $250\ \text{K} \le T \le 1000\ \text{K}$). (Ans.)

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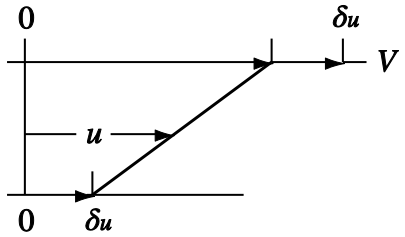
1-16 Analyze newtonian flow between parallel plates (Fig. 1-15) with a finite *slip* velocity $\delta u = l\left(du/dy\right)$ at both walls.

The velocity profile is still linear, but with slip at both walls the slope is less, as shown in the sketch:

$$du/dy = \left(V - 2\delta u\right)/h$$



Introducing δu from the slip relation, we obtain

$$\frac{du}{dy} = \frac{V}{h + 2l} \text{ or: } \tau_w = \frac{\mu V}{h + 2l} \text{ at both walls} \qquad \text{(Ans.)}$$

1-17 Derive Eq. (1-106) from a balance of forces on the differential surface-area alement shown in the problem.

Since the sliver of area is negligibly thin, it has no weight. The pressures act on a projected surface area $dS_x dS_y$. The surface tension forces are slanted slightly upward, at angles $(d\theta_x/2)$ and $(d\theta_y/2)$, respectively. The force balance is

$$2T\ dS_y \sin\left(d\theta_x/2\right) + 2T\ dS_x \sin\left(d\theta_y/2\right) + \left(p - p_a\right) dS_x dS_y = 0$$

For differentially small angles, $\sin\left(d\theta\right) = d\theta$. Clean up this equation and rearrange:

$$p = p_a - T\left(\frac{d\theta_x}{dS_x} + \frac{d\theta_y}{dS_y}\right) = p_a - T\left(\frac{1}{R_x} + \frac{1}{R_y}\right) \qquad \text{(Ans.)}$$

since, by definition, $d\theta/dS = 1/R$, where R is the radius of curvature.

1-18 Two bubbles of radii R_1 and R_2 coalesce isothermally into a single bubble R_3. Find the radius of the new (single) bubble.

Because of surface tension, the pressure inside a bubble (which has two surfaces) is higher than ambient, $p = p_o + 4T/R$. Assuming that no interior-bubble air mass escapes during the coalescence, $m_1 + m_2 = m_3$, or, for an ideal isothermal gas of temperature T,

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$$\frac{p_o + 4\mathrm{l}/R_1}{\Re T}\frac{4\pi}{3}R_1^3 + \frac{p_o + 4\mathrm{l}/R_2}{\Re T}\frac{4\pi}{3}R_2^3 = \frac{p_o + 4\mathrm{l}/R_3}{\Re T}\frac{4\pi}{3}R_3^3$$

where $\Re$ is the gas constant. Canceling common terms and cleaning up, we have

$$p_o R_3^3 + 4\mathrm{l}\,R_3^2 = p_o\left(R_1^3 + R_2^3\right) + 4\mathrm{l}\,\left(R_1^2 + R_2^2\right) \qquad (Ans.)$$

This must be solved numerically or algebraically for the new radius R_3.

1-19 In Prob. 1-1, if the temperature, sphere size, and velocity remain the same for air flow, at what air pressure will the Reynolds number $\mathrm{Re_D}$ be equal to 10,000?

Solution: From Prob. 1–1, $T = 293$ K, $D = 0.014$ m, $V = 18$ m/s, and $\mu = 1.81$E-5 Pa-s. Use the specified Reynolds number to compute the required air density:

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{\rho(18\ m/s)(0.014\ m)}{1.81E\text{-}5\ kg/m\text{-}s} = 10,000 \text{ Solve } \rho = 0.718\ kg/m^3$$

Ideal gas: $\rho = 0.718 = \dfrac{p}{RT} = \dfrac{p}{(287)(293)}$, Solve for $p = 60400\ Pa \approx \mathbf{60}\ kPa$ (*Ans.*)

1-20 A solid cylinder of mass *m*, radius *R*, and length *L* falls concentrically through a vertical tube of radius $R + \Delta R$, where $\Delta R << R$. The tube is filled with gas of viscosity μ and mean free path ℓ. Neglect fluid forces on the front and back faces of the cylinder and consider only shear stress in the annular region, assuming a linear velocity profile. Find an analytic expression for the terminal velocity of fall, *V*, of the cylinder (*a*) for no-slip; (*b*) with slip, Eq. (1-91).

Solution: (*a*) For no-slip, the shear stress in the thin annular region between cylinders is

$$\tau = \mu\frac{\delta u}{\delta y} = \mu\frac{V}{\Delta R}, \text{ then } W = mg = F_{shear} = \tau A_{wall} = \left(\mu\frac{V}{\Delta R}\right)(2\pi RL)$$

$$\text{Solve for } V_{no\text{-}slip} = \frac{mg\Delta R}{2\pi RL} \qquad Ans.\ (a)$$

(*b*) For slip, modify the shear stress (see Prob. 1.16 for another example):

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