
CHAPTER 4

LONG-TERM FINANCIAL PLANNING AND GROWTH

Answers to Concepts Review and Critical Thinking Questions

1. The reason is that, ultimately, sales are the driving force behind a business. A firm's assets, employees, and, in fact, just about every aspect of its operations and financing exist to directly or indirectly support sales. Put differently, a firm's future need for things like capital assets, employees, inventory, and financing are determined by its future sales level.
2. Two assumptions of the sustainable growth formula are that the company does not want to sell new equity, and that financial policy is fixed. If the company raises outside equity, or increases its debt-equity ratio, it can grow at a higher rate than the sustainable growth rate. Of course, the company could also grow faster than the rate at which its profit margin increases, if it changes its dividend policy by increasing the retention ratio, or its total asset turnover increases.
3. The internal growth rate is greater than 15 percent, because at a 15 percent growth rate the negative EFN indicates that there is excess internal financing. If the internal growth rate is greater than 15 percent, then the sustainable growth rate is certainly greater than 15 percent, because there is additional debt financing used in that case (assuming the firm is not 100 percent equity-financed). As the retention ratio is increased, the firm has more internal sources of funding, so the EFN will decline. Conversely, as the retention ratio is decreased, the EFN will rise. If the firm pays out all its earnings in the form of dividends, then the firm has no internal sources of funding (ignoring the effects of accounts payable); the internal growth rate is zero in this case and the EFN will rise to the change in total assets.
4. The sustainable growth rate is greater than 20 percent, because at a 20 percent growth rate the negative EFN indicates that there is excess financing still available. If the firm is 100 percent equity financed, then the sustainable and internal growth rates are equal and the internal growth rate would be greater than 20 percent. However, when the firm has some debt, the internal growth rate is always less than the sustainable growth rate, so it is ambiguous whether the internal growth rate would be greater than or less than 20 percent. If the retention ratio is increased, the firm will have more internal funding sources available, and it will have to take on more debt to keep the debt-equity ratio constant, so the EFN will decline. Conversely, if the retention ratio is decreased, the EFN will rise. If the retention rate is zero, both the internal and sustainable growth rates are zero, and the EFN will rise to the change in total assets.
5. Presumably not, but, of course, if the product had been *much* less popular, then a similar fate would have awaited due to lack of sales.
6. Since customers did not pay until shipment, receivables rose. The firm's NWC, but not its cash, increased. At the same time, costs were rising faster than cash revenues, so operating cash flow declined. The firm's capital spending was also rising. Thus, all three components of cash flow from assets were negatively impacted.

7. Apparently not! In hindsight, the firm may have underestimated costs and also underestimated the extra demand from the lower price.
8. Financing possibly could have been arranged if the company had taken quick enough action. Sometimes it becomes apparent that help is needed only when it is too late, again emphasizing the need for planning.
9. All three were important, but the lack of cash or, more generally, financial resources ultimately spelled doom. An inadequate cash resource is usually cited as the most common cause of small business failure.
10. Demanding cash up front, increasing prices, subcontracting production, and improving financial resources via new owners or new sources of credit are some of the options. When orders exceed capacity, price increases may be especially beneficial.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. It is important to remember that equity will not increase by the same percentage as the other assets. If every other item on the income statement and balance sheet increases by 15 percent, the pro forma income statement and balance sheet will look like this:

<u>Pro forma income statement</u>		<u>Pro forma balance sheet</u>			
Sales	\$ 45,310	Assets	\$33,580	Debt	\$ 10,810
Costs	<u>39,905</u>			Equity	<u>22,770</u>
Net income	<u>\$ 5,405</u>	Total	<u>\$33,580</u>	Total	<u>\$ 33,580</u>

In order for the balance sheet to balance, equity must be:

$$\text{Equity} = \text{Total liabilities and equity} - \text{Debt}$$

$$\text{Equity} = \$33,580 - 10,810$$

$$\text{Equity} = \$22,770$$

Equity increased by:

$$\text{Equity increase} = \$22,770 - 19,800$$

$$\text{Equity increase} = \$2,970$$

Net income is \$5,405 but equity only increased by \$2,970; therefore, a dividend of:

$$\text{Dividend} = \$5,405 - 2,970$$

$$\text{Dividend} = \$2,435$$

must have been paid. Dividends paid is the plug variable.

2. Here we are given the dividend amount, so dividends paid is not a plug variable. If the company pays out one-half of its net income as dividends, the pro forma income statement and balance sheet will look like this:

<u>Pro forma income statement</u>		<u>Pro forma balance sheet</u>			
Sales	\$45,310	Assets	\$33,580	Debt	\$ 9,400
Costs	<u>39,905</u>			Equity	<u>22,503</u>
Net income	\$ 5,405	Total	<u>\$33,580</u>	Total	<u>\$31,903</u>
Dividends	\$2,703				
Add. to RE	\$2,703				

Note that the balance sheet does not balance. This is due to EFN. The EFN for this company is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$33,580 - 31,903$$

$$\text{EFN} = \$1,678$$

3. An increase of sales to \$9,164 is an increase of:

$$\text{Sales increase} = (\$9,164 - 7,900)/\$7,900$$

$$\text{Sales increase} = .16, \text{ or } 16\%$$

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

<u>Pro forma income statement</u>		<u>Pro forma balance sheet</u>			
Sales	\$ 9,164.00	Assets	\$26,564.00	Debt	\$ 9,100.00
Costs	<u>6,217.60</u>			Equity	<u>16,746.40</u>
Net income	<u>\$ 2,946.40</u>	Total	<u>\$26,564.00</u>	Total	<u>\$25,846.40</u>

If no dividends are paid, the equity account will increase by the net income, so:

$$\text{Equity} = \$13,800 + 2,946.40$$

$$\text{Equity} = \$16,746.40$$

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$26,564 - 25,846.40$$

$$\text{EFN} = \$717.60$$

4. An increase of sales to \$23,345 is an increase of:

$$\text{Sales increase} = (\$23,345 - 20,300)/\$20,300$$

$$\text{Sales increase} = .15, \text{ or } 15\%$$

Assuming costs and assets increase proportionally, the pro forma financial statements will look like this:

<u>Pro forma income statement</u>		<u>Pro forma balance sheet</u>			
Sales	\$23,345.00	Assets	\$ 54,050	Debt	\$ 26,900.00
Costs	<u>19,665.00</u>			Equity	<u>21,397.20</u>
EBIT	3,680.00	Total	<u>\$ 54,050</u>	Total	<u>\$48,297.20</u>
Taxes (21%)	<u>772.80</u>				
Net income	<u>\$ 2,907.20</u>				

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$1,400/\$2,528)(\$2,907.20)$$

$$\text{Dividends} = \$1,610$$

The addition to retained earnings is:

$$\text{Addition to retained earnings} = \$2,907.20 - 1,610$$

$$\text{Addition to retained earnings} = \$1,297.20$$

And the new equity balance is:

$$\text{Equity} = \$20,100 + 1,297.20$$

$$\text{Equity} = \$21,397.20$$

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$54,050 - 48,297.20$$

$$\text{EFN} = \$5,752.80$$

5. Assuming costs, current liabilities, and assets increase proportionally, the pro forma financial statements will look like this:

<u>Pro forma income statement</u>		<u>Pro forma balance sheet</u>			
Sales	\$10,005.00	CA	\$4,830.00	CL	\$2,185.00
Costs	<u>6,440.00</u>	FA	11,960.00	LTD	3,800.00
Taxable income	\$3,565.00			Equity	<u>10,504.25</u>
Taxes (25%)	<u>891.25</u>	TA	<u>\$16,790.00</u>	Total D&E	<u>\$16,489.25</u>
Net income	<u>\$2,673.75</u>				

The payout ratio is 40 percent, so dividends will be:

$$\begin{aligned}\text{Dividends} &= .40(\$2,673.75) \\ \text{Dividends} &= \$1,069.50\end{aligned}$$

The addition to retained earnings is:

$$\begin{aligned}\text{Addition to retained earnings} &= \$2,673.75 - 1,069.50 \\ \text{Addition to retained earnings} &= \$1,604.25\end{aligned}$$

So the EFN is:

$$\begin{aligned}\text{EFN} &= \text{Total assets} - \text{Total liabilities and equity} \\ \text{EFN} &= \$16,790 - 16,489.25 \\ \text{EFN} &= \$300.75\end{aligned}$$

6. To calculate the internal growth rate, we first need to calculate the ROA, which is:

$$\begin{aligned}\text{ROA} &= \text{NI/TA} \\ \text{ROA} &= \$4,108/\$35,500 \\ \text{ROA} &= .1157, \text{ or } 11.57\%\end{aligned}$$

The plowback ratio, b , is one minus the payout ratio, so:

$$\begin{aligned}b &= 1 - .30 \\ b &= .70\end{aligned}$$

Now we can use the internal growth rate equation to get:

$$\begin{aligned}\text{Internal growth rate} &= (\text{ROA} \times b)/[1 - (\text{ROA} \times b)] \\ \text{Internal growth rate} &= [.1157(.70)]/[1 - .1157(.70)] \\ \text{Internal growth rate} &= .0881, \text{ or } 8.81\%\end{aligned}$$

7. To calculate the sustainable growth rate, we first need to calculate the ROE, which is:

$$\begin{aligned}\text{ROE} &= \text{NI/TE} \\ \text{ROE} &= \$4,108/\$21,900 \\ \text{ROE} &= .1876, \text{ or } 18.76\%\end{aligned}$$

The plowback ratio, b , is one minus the payout ratio, so:

$$\begin{aligned}b &= 1 - .30 \\ b &= .70\end{aligned}$$

Now we can use the sustainable growth rate equation to get:

$$\begin{aligned}\text{Sustainable growth rate} &= (\text{ROE} \times b)/[1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.1876(.70)]/[1 - .1876(.70)] \\ \text{Sustainable growth rate} &= .1512, \text{ or } 15.12\%\end{aligned}$$

8. The maximum percentage sales increase is the sustainable growth rate. To calculate the sustainable growth rate, we first need to calculate the ROE, which is:

$$\text{ROE} = \text{NI}/\text{TE}$$

$$\text{ROE} = \$4,774/\$51,700$$

$$\text{ROE} = .0923, \text{ or } 9.23\%$$

The plowback ratio, b , is one minus the payout ratio, so:

$$b = 1 - .40$$

$$b = .60$$

Now we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.0923(.60)]/[1 - .0923(.60)]$$

$$\text{Sustainable growth rate} = .0587, \text{ or } 5.87\%$$

So, the maximum dollar increase in sales is:

$$\text{Maximum increase in sales} = \$38,600(.0587)$$

$$\text{Maximum increase in sales} = \$2,264.04$$

9. Assuming costs vary with sales and a 20 percent increase in sales, the pro forma income statement will look like this:

HEIR JORDAN CORPORATION
Pro Forma Income Statement

Sales	\$63,600
Costs	49,080
Taxable income	<u>\$14,520</u>
Taxes (22%)	<u>3,194</u>
Net income	<u><u>\$11,326</u></u>

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$3,500/\$9,438)(\$11,326)$$

$$\text{Dividends} = \$4,200$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$11,326 - 4,200$$

$$\text{Addition to retained earnings} = \$7,126$$

10. Below is the balance sheet with the percentage of sales for each account on the balance sheet. Notes payable, total current liabilities, long-term debt, and all equity accounts do not vary directly with sales.

HEIR JORDAN CORPORATION				
Balance Sheet				
	(\$)	(%)	(\$)	(%)
Assets			Liabilities and Owners' Equity	
Current assets			Current liabilities	
Cash	\$ 3,140	5.92	Accounts payable	\$ 2,600 4.91
Accounts receivable	4,200	7.92	Notes payable	<u>5,700</u> n/a
Inventory	<u>6,500</u>	<u>12.26</u>	Total	\$ 8,300 n/a
Total	<u>\$13,840</u>	<u>26.11</u>	Long-term debt	\$28,000 n/a
Fixed assets			Owners' equity	
Net plant and equipment	<u>\$43,200</u>	<u>81.51</u>	Common stock and paid-in surplus	\$ 5,000 n/a
			Retained earnings	<u>15,740</u> n/a
			Total	<u>\$20,740</u> n/a
Total assets	<u>\$57,040</u>	<u>107.62</u>	Total liabilities and owners' equity	<u>\$57,040</u> n/a

11. Assuming costs vary with sales and a 15 percent increase in sales, the pro forma income statement will look like this:

HEIR JORDAN CORPORATION	
Pro Forma Income Statement	
Sales	\$60,950.00
Costs	<u>47,035.00</u>
Taxable income	\$13,915.00
Taxes (22%)	<u>3,061.30</u>
Net income	<u>\$10,853.70</u>

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$3,500/\$9,438)(\$10,853.70)$$

$$\text{Dividends} = \$4,025$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$10,853.70 - 4,025$$

$$\text{Addition to retained earnings} = \$6,828.70$$

The new accumulated retained earnings on the pro forma balance sheet will be:

$$\text{New accumulated retained earnings} = \$15,740 + 6,828.70$$

$$\text{New accumulated retained earnings} = \$22,568.70$$

The pro forma balance sheet will look like this:

HEIR JORDAN CORPORATION
Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 3,611.00	Accounts payable	\$ 2,990.00
Accounts receivable	4,830.00	Notes payable	<u>5,700.00</u>
Inventory	<u>7,475.00</u>	Total	\$ 8,690.00
Total	\$ 15,916.00	Long-term debt	\$ 28,000.00
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 49,680.00</u>	Common stock and paid-in surplus	\$ 5,000.00
		Retained earnings	<u>22,568.70</u>
		Total	<u>\$ 27,568.70</u>
Total assets	<u>\$ 65,596.00</u>	Total liabilities and owners' equity	<u>\$ 64,258.70</u>

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$65,596 - 64,258.70$$

$$\text{EFN} = \$1,337.30$$

12. We need to calculate the retention ratio to calculate the internal growth rate. The retention ratio is:

$$b = 1 - .35$$

$$b = .65$$

Now we can use the internal growth rate equation to get:

$$\text{Internal growth rate} = (\text{ROA} \times b) / [1 - (\text{ROA} \times b)]$$

$$\text{Internal growth rate} = [.083(.65)] / [1 - .083(.65)]$$

$$\text{Internal growth rate} = .0570, \text{ or } 5.70\%$$

13. We need to calculate the retention ratio to calculate the sustainable growth rate. The retention ratio is:

$$b = 1 - .25$$

$$b = .75$$

Now we can use the sustainable growth rate equation to get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.141(.75)] / [1 - .141(.75)]$$

$$\text{Sustainable growth rate} = .1183, \text{ or } 11.83\%$$

14. We first must calculate the ROE to calculate the sustainable growth rate. To do this we must realize two other relationships. The total asset turnover is the inverse of the capital intensity ratio, and the equity multiplier is $1 + D/E$. Using these relationships, we get:

$$\begin{aligned} \text{ROE} &= (\text{PM})(\text{TAT})(\text{EM}) \\ \text{ROE} &= (.073)(1/.95)(1 + 1.05) \\ \text{ROE} &= .1575, \text{ or } 15.75\% \end{aligned}$$

The plowback ratio is one minus the dividend payout ratio, so:

$$\begin{aligned} b &= 1 - (\$24,000/\$84,000) \\ b &= .7143 \end{aligned}$$

Now we can use the sustainable growth rate equation to get:

$$\begin{aligned} \text{Sustainable growth rate} &= (\text{ROE} \times b)/[1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.1575(.7143)]/[1 - .1575(.7143)] \\ \text{Sustainable growth rate} &= .1268, \text{ or } 12.68\% \end{aligned}$$

15. We must first calculate the ROE using the DuPont ratio to calculate the sustainable growth rate. The ROE is:

$$\begin{aligned} \text{ROE} &= (\text{PM})(\text{TAT})(\text{EM}) \\ \text{ROE} &= (.052)(3.2)(.95) \\ \text{ROE} &= .1581, \text{ or } 15.81\% \end{aligned}$$

The plowback ratio is one minus the dividend payout ratio, so:

$$\begin{aligned} b &= 1 - .35 \\ b &= .65 \end{aligned}$$

Now we can use the sustainable growth rate equation to get:

$$\begin{aligned} \text{Sustainable growth rate} &= (\text{ROE} \times b)/[1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.1581(.65)]/[1 - .1581(.65)] \\ \text{Sustainable growth rate} &= .1145, \text{ or } 11.45\% \end{aligned}$$

Intermediate

16. To determine full capacity sales, we divide the current sales by the capacity the company is currently using, so:

$$\begin{aligned} \text{Full capacity sales} &= \$830,000/.94 \\ \text{Full capacity sales} &= \$882,979 \end{aligned}$$

The maximum sales growth is the full capacity sales divided by the current sales, so:

$$\begin{aligned} \text{Maximum sales growth} &= (\$882,979/\$830,000) - 1 \\ \text{Maximum sales growth} &= .0638, \text{ or } 6.38\% \end{aligned}$$

17. To find the new level of fixed assets, we need to find the current percentage of fixed assets to full capacity sales. Doing so, we find:

$$\text{Fixed assets/Full capacity sales} = \$590,000/\$882,979$$

$$\text{Fixed assets/Full capacity sales} = .6682$$

Next, we calculate the total dollar amount of fixed assets needed at the new sales figure.

$$\text{Total fixed assets} = .6682(\$910,000)$$

$$\text{Total fixed assets} = \$608,055.42$$

The new fixed assets necessary is the total fixed assets at the new sales figure minus the current level of fixed assets.

$$\text{New fixed assets} = \$608,055.42 - 590,000$$

$$\text{New fixed assets} = \$18,055.42$$

18. We have all the variables to calculate ROE using the DuPont identity except the profit margin. If we find ROE, we can solve the DuPont identity for profit margin. We can calculate ROE from the sustainable growth rate equation. For this equation we need the retention ratio, so:

$$b = 1 - .25$$

$$b = .75$$

Using the sustainable growth rate equation and solving for ROE, we get:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$.11 = [\text{ROE}(.75)]/[1 - \text{ROE}(.75)]$$

$$\text{ROE} = .1321, \text{ or } 13.21\%$$

Now we can use the DuPont identity to find the profit margin as:

$$\text{ROE} = \text{PM}(\text{TAT})(\text{EM})$$

$$.1321 = \text{PM}(1/.65)(1 + .75)$$

$$\text{PM} = (.1321)/[(1/.65)(1.75)]$$

$$\text{PM} = .0491, \text{ or } 4.91\%$$

19. We are given the profit margin. Remember that:

$$\text{ROA} = \text{PM}(\text{TAT})$$

We can calculate the ROA from the internal growth rate formula, and then use the ROA in this equation to find the total asset turnover. The retention ratio is:

$$b = 1 - .25$$

$$b = .75$$

Using the internal growth rate equation to find the ROA, we get:

$$\begin{aligned}\text{Internal growth rate} &= (\text{ROA} \times b) / [1 - (\text{ROA} \times b)] \\ .064 &= [\text{ROA}(.75)] / [1 - \text{ROA}(.75)] \\ \text{ROA} &= .0802, \text{ or } 8.02\%\end{aligned}$$

Plugging ROA and PM into the equation we began with and solving for TAT, we get:

$$\begin{aligned}\text{ROA} &= (\text{PM})(\text{TAT}) \\ .0802 &= .057(\text{TAT}) \\ \text{TAT} &= .0802 / .057 \\ \text{TAT} &= 1.41 \text{ times}\end{aligned}$$

- 20.** We should begin by calculating the D/E ratio. We calculate the D/E ratio as follows:

$$\text{Total debt ratio} = .45 = \text{TD}/\text{TA}$$

Inverting both sides we get:

$$1/.45 = \text{TA}/\text{TD}$$

Next, we need to recognize that:

$$\text{TA}/\text{TD} = 1 + \text{TE}/\text{TD}$$

Substituting this into the previous equation, we get:

$$1/.45 = 1 + \text{TE}/\text{TD}$$

Subtracting 1 (one) from both sides and inverting again, we get:

$$\begin{aligned}\text{D}/\text{E} &= 1 / [(1/.45) - 1] \\ \text{D}/\text{E} &= .82\end{aligned}$$

With the D/E ratio, we can calculate the EM and solve for ROE using the DuPont identity:

$$\begin{aligned}\text{ROE} &= (\text{PM})(\text{TAT})(\text{EM}) \\ \text{ROE} &= (.059)(1.15)(1 + .82) \\ \text{ROE} &= .1234, \text{ or } 12.34\%\end{aligned}$$

Now we can calculate the retention ratio as:

$$\begin{aligned}b &= 1 - .40 \\ b &= .60\end{aligned}$$

Finally, putting all the numbers we have calculated into the sustainable growth rate equation, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= (\text{ROE} \times b) / [1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.1234(.60)] / [1 - .1234(.60)] \\ \text{Sustainable growth rate} &= .0799, \text{ or } 7.99\%\end{aligned}$$

21. To calculate the sustainable growth rate, we first must calculate the retention ratio and ROE. The retention ratio is:

$$b = 1 - \$7,500/\$19,200$$

$$b = .6094$$

And the ROE is:

$$\text{ROE} = \$19,200/\$77,000$$

$$\text{ROE} = .2494, \text{ or } 24.94\%$$

So, the sustainable growth rate is:

$$\text{Sustainable growth rate} = (\text{ROE} \times b)/[1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = [.2494(.6094)]/[1 - .2494(.6094)]$$

$$\text{Sustainable growth rate} = .1792, \text{ or } 17.92\%$$

If the company grows at the sustainable growth rate, the new level of total assets is:

$$\text{New TA} = 1.1792(\$67,000 + 77,000) = \$169,800.92$$

To find the new level of debt in the company's balance sheet, we take the percentage of debt in the capital structure times the new level of total assets. The additional borrowing will be the new level of debt minus the current level of debt. So:

$$\text{New TD} = [D/(D + E)](\text{TA})$$

$$\text{New TD} = [\$67,000/(\$67,000 + 77,000)](\$169,800.92)$$

$$\text{New TD} = \$79,004.59$$

And the additional borrowing will be:

$$\text{Additional borrowing} = \$79,004.59 - 67,000$$

$$\text{Additional borrowing} = \$12,004.59$$

The growth rate that can be supported with no outside financing is the internal growth rate. To calculate the internal growth rate, we first need the ROA, which is:

$$\text{ROA} = \$19,200/(\$67,000 + 77,000)$$

$$\text{ROA} = .1333, \text{ or } 13.33\%$$

This means the internal growth rate is:

$$\text{Internal growth rate} = (\text{ROA} \times b)/[1 - (\text{ROA} \times b)]$$

$$\text{Internal growth rate} = [.1333(.6094)]/[1 - .1333(.6094)]$$

$$\text{Internal growth rate} = .0884, \text{ or } 8.84\%$$

22. Since the company issued no new equity, shareholders' equity increased by retained earnings. Retained earnings for the year were:

$$\begin{aligned}\text{Retained earnings} &= \text{NI} - \text{Dividends} \\ \text{Retained earnings} &= \$29,000 - 6,400 \\ \text{Retained earnings} &= \$22,600\end{aligned}$$

So, the equity at the end of the year was:

$$\begin{aligned}\text{Ending equity} &= \$153,000 + 22,600 \\ \text{Ending equity} &= \$175,600\end{aligned}$$

The ROE based on the end of period equity is:

$$\begin{aligned}\text{ROE} &= \$29,000/\$175,600 \\ \text{ROE} &= .1651, \text{ or } 16.51\%\end{aligned}$$

The plowback ratio is:

$$\begin{aligned}\text{Plowback ratio} &= \text{Addition to retained earnings}/\text{NI} \\ \text{Plowback ratio} &= \$22,600/\$29,000 \\ \text{Plowback ratio} &= .7793, \text{ or } 77.93\%\end{aligned}$$

Using the equation presented in the text for the sustainable growth rate, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= (\text{ROE} \times b)/[1 - (\text{ROE} \times b)] \\ \text{Sustainable growth rate} &= [.1651(.7793)]/[1 - .1651(.7793)] \\ \text{Sustainable growth rate} &= .1477, \text{ or } 14.77\%\end{aligned}$$

The ROE based on the beginning of period equity is

$$\begin{aligned}\text{ROE} &= \$29,000/\$153,000 \\ \text{ROE} &= .1895, \text{ or } 18.95\%\end{aligned}$$

Using the shortened equation for the sustainable growth rate and the beginning of period ROE, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= \text{ROE} \times b \\ \text{Sustainable growth rate} &= .1895 \times .7793 \\ \text{Sustainable growth rate} &= .1477, \text{ or } 14.77\%\end{aligned}$$

Using the shortened equation for the sustainable growth rate and the end of period ROE, we get:

$$\begin{aligned}\text{Sustainable growth rate} &= \text{ROE} \times b \\ \text{Sustainable growth rate} &= .1651 \times .7793 \\ \text{Sustainable growth rate} &= .1287, \text{ or } 12.87\%\end{aligned}$$

Using the end of period ROE in the shortened sustainable growth rate equation results in a growth rate that is too low. This will always occur whenever the equity increases. If equity increases, the ROE based on end of period equity is lower than the ROE based on the beginning of period equity. The ROE (and sustainable growth rate) in the abbreviated equation is based on equity that did not exist when the net income was earned.

23. The ROA using end of period assets is:

$$\text{ROA} = \$29,000/\$215,000$$

$$\text{ROA} = .1349, \text{ or } 13.49\%$$

The beginning of period assets had to have been the ending assets minus the addition to retained earnings, so:

$$\text{Beginning assets} = \text{Ending assets} - \text{Addition to retained earnings}$$

$$\text{Beginning assets} = \$215,000 - 22,600$$

$$\text{Beginning assets} = \$192,400$$

And the ROA using beginning of period assets is:

$$\text{ROA} = \$29,000/\$192,400$$

$$\text{ROA} = .1507, \text{ or } 15.07\%$$

Using the internal growth rate equation presented in the text, we get:

$$\text{Internal growth rate} = (\text{ROA} \times b)/[1 - (\text{ROA} \times b)]$$

$$\text{Internal growth rate} = [.1349(.7793)]/[1 - .1349(.7793)]$$

$$\text{Internal growth rate} = .1175, \text{ or } 11.75\%$$

Using the formula $\text{ROA} \times b$, and beginning of period assets:

$$\text{Internal growth rate} = .1507 \times .7793$$

$$\text{Internal growth rate} = .1175, \text{ or } 11.75\%$$

Using the formula $\text{ROA} \times b$, and end of period assets:

$$\text{Internal growth rate} = .1349 \times .7793$$

$$\text{Internal growth rate} = .1051, \text{ or } 10.51\%$$

Using the end of period ROA in the shortened internal growth rate equation results in a growth rate that is too low. This will always occur whenever the assets increase. If assets increase, the ROA based on end of period assets is lower than the ROA based on the beginning of period assets. The ROA (and internal growth rate) in the abbreviated equation is based on assets that did not exist when the net income was earned.

24. Assuming costs vary with sales and a 20 percent increase in sales, the pro forma income statement will look like this:

Pro Forma Income Statement	
Sales	\$ 823,836
Costs	665,304
Other expenses	<u>16,824</u>
EBIT	\$ 141,708
Interest	<u>12,090</u>
Taxable income	\$ 129,618
Taxes (21%)	<u>27,220</u>
Net income	<u><u>\$ 102,398</u></u>

The payout ratio is constant, so the dividends paid this year is the payout ratio from last year times net income, or:

$$\text{Dividends} = (\$27,475/\$83,740)(\$102,398)$$

$$\text{Dividends} = \$33,597$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$102,398 - 33,597$$

$$\text{Addition to retained earnings} = \$68,801$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$166,705 + 68,801$$

$$\text{New retained earnings} = \$235,506$$

The pro forma balance sheet will look like this:

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 25,128	Accounts payable	\$ 64,548
Accounts receivable	38,364	Notes payable	<u>13,215</u>
Inventory	<u>85,584</u>	Total	\$ 77,763
Total	\$ 149,076	Long-term debt	\$ 127,500
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 410,376</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>235,506</u>
		Total	<u>\$ 340,506</u>
Total assets	<u><u>\$ 559,452</u></u>	Total liabilities and owners' equity	<u><u>\$ 545,769</u></u>

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$559,452 - 545,769$$

$$\text{EFN} = \$13,683$$

25. First, we need to calculate full capacity sales, which is:

$$\text{Full capacity sales} = \$686,530 / .80$$

$$\text{Full capacity sales} = \$858,163$$

The full capacity ratio at full capacity sales is:

$$\text{Full capacity ratio} = \text{Fixed assets} / \text{Full capacity sales}$$

$$\text{Full capacity ratio} = \$341,980 / \$858,163$$

$$\text{Full capacity ratio} = .39850$$

The fixed assets required at the projected sales figure is the full capacity ratio times the projected sales level:

$$\text{Total fixed assets} = .39850(\$823,836) = \$328,301$$

So, EFN is:

$$\text{EFN} = (\$149,076 + 328,301) - \$545,769 = -\$68,393$$

Note that this solution assumes that fixed assets are decreased (sold) so the company has a 100 percent fixed asset utilization. If we assume fixed assets are not sold, the answer becomes:

$$\text{EFN} = (\$149,076 + 341,980) - \$545,769 = -\$54,713$$

26. The D/E ratio of the company is:

$$\text{D/E} = (\$67,005 + 127,500) / \$271,705$$

$$\text{D/E} = .7159$$

So the new total debt amount will be:

$$\text{New total debt} = .7159(\$340,506)$$

$$\text{New total debt} = \$243,758$$

This is the new total debt for the company. Given that our calculation for EFN is the amount that must be raised externally and does not increase spontaneously with sales, we need to subtract the spontaneous increase in accounts payable. The new level of accounts payable, which is the current accounts payable times the sales growth, will be:

$$\text{Spontaneous increase in accounts payable} = \$53,790(.20)$$

$$\text{Spontaneous increase in accounts payable} = \$10,758$$

This means that \$10,758 of the new total debt is not raised externally. So, the debt raised externally, which will be the EFN, is:

$$\text{EFN} = \text{New total debt} - (\text{Beginning LTD} + \text{Beginning CL} + \text{Spontaneous increase in AP})$$

$$\text{EFN} = \$243,758 - (\$127,500 + 67,005 + 10,758) = \$38,495$$

The pro forma balance sheet with the new long-term debt will be:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 25,128	Accounts payable	\$ 64,548
Accounts receivable	38,364	Notes payable	<u>13,215</u>
Inventory	<u>85,584</u>	Total	\$ 77,763
Total	\$ 149,076	Long-term debt	\$ 165,995
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 410,376</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>235,506</u>
		Total	<u>\$ 340,506</u>
Total assets	<u>\$ 559,452</u>	Total liabilities and owners' equity	<u>\$ 584,264</u>

The funds raised by the debt issue can be put into an excess cash account to make the balance sheet balance. The excess debt will be:

$$\text{Excess debt} = \$584,264 - 559,452 = \$24,812$$

To make the balance sheet balance, the company will have to increase its assets. We will put this amount in an account called excess cash, which will give us the following balance sheet:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 25,128	Accounts payable	\$ 64,548
Excess cash	24,812	Notes payable	<u>13,215</u>
Accounts receivable	38,364	Total	\$ 77,763
Inventory	<u>85,584</u>	Long-term debt	\$ 165,995
Total	\$ 173,888		
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 410,376</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>235,506</u>
		Total	<u>\$ 340,506</u>
Total assets	<u>\$ 584,264</u>	Total liabilities and owners' equity	<u>\$ 584,264</u>

The excess cash has an opportunity cost that we discussed earlier. Increasing fixed assets would also not be a good idea since the company already has enough fixed assets. A likely scenario would be the repurchase of debt and equity in its current capital structure weights. The company's debt-assets and equity-assets are:

$$\text{Debt-assets} = .7159 / (1 + .7159) = .42$$

$$\text{Equity-assets} = 1 / (1 + .7159) = .58$$

So, the amount of debt and equity needed will be:

$$\text{Total debt needed} = .42(\$559,452) = \$233,406$$

$$\text{Equity needed} = .58(\$559,452) = \$326,046$$

So, the repurchases of debt and equity will be:

$$\text{Debt repurchase} = (\$77,763 + 165,995) - 233,406 = \$10,352$$

$$\text{Equity repurchase} = \$340,506 - 326,046 = \$14,460$$

Assuming all of the debt repurchase is from long-term debt, and the equity repurchase is entirely from the retained earnings, the final pro forma balance sheet will be:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 25,128	Accounts payable	\$ 64,548
Accounts receivable	38,364	Notes payable	<u>13,215</u>
Inventory	<u>85,584</u>	Total	\$ 77,763
Total	\$ 149,076	Long-term debt	\$ 155,643
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 410,376</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>221,046</u>
		Total	<u>\$ 326,046</u>
Total assets	<u>\$ 559,452</u>	Total liabilities and owners' equity	<u>\$ 559,452</u>

Challenge

27. The pro forma income statements for all three growth rates will be:

	Pro Forma Income Statement		
	15 % Sales Growth	20% Sales Growth	25% Sales Growth
Sales	\$789,510	\$823,836	\$858,163
Costs	637,583	665,304	693,025
Other expenses	16,123	16,824	17,525
EBIT	<u>\$135,804</u>	<u>\$141,708</u>	<u>\$147,613</u>
Interest	12,090	12,090	12,090
Taxable income	<u>\$123,714</u>	<u>\$129,618</u>	<u>\$135,523</u>
Taxes (21%)	25,980	27,220	28,460
Net income	<u>\$97,734</u>	<u>\$102,398</u>	<u>\$107,063</u>
Dividends	\$32,066	\$33,597	\$35,127
Add to RE	65,667	68,801	71,936

We will calculate the EFN for the 15 percent growth rate first. Assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$27,475/\$83,740)(\$97,734)$$

$$\text{Dividends} = \$32,066$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$97,734 - 32,066$$

$$\text{Addition to retained earnings} = \$65,667$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$166,705 + 65,667$$

$$\text{New retained earnings} = \$232,372$$

The pro forma balance sheet will look like this:

15% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 24,081	Accounts payable	\$ 61,859
Accounts receivable	36,766	Notes payable	<u>13,215</u>
Inventory	<u>82,018</u>	Total	\$ 75,074
Total	\$ 142,865	Long-term debt	\$ 127,500
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 393,277</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>232,372</u>
		Total	<u>\$ 337,372</u>
Total assets	<u>\$ 536,142</u>	Total liabilities and owners' equity	<u>\$ 539,946</u>

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$536,142 - 539,946$$

$$\text{EFN} = -\$3,804$$

At a 20 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$27,475/\$83,740)(\$102,398)$$

$$\text{Dividends} = \$33,597$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$102,398 - 33,597$$

$$\text{Addition to retained earnings} = \$68,801$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$166,705 + 68,801$$

$$\text{New retained earnings} = \$235,506$$

The pro forma balance sheet will look like this:

20% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 25,128	Accounts payable	\$ 64,548
Accounts receivable	38,364	Notes payable	<u>13,215</u>
Inventory	<u>85,584</u>	Total	\$ 77,763
Total	\$ 149,076	Long-term debt	\$ 127,500
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 410,376</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>235,506</u>
		Total	<u>\$ 340,506</u>
Total assets	<u>\$ 559,452</u>	Total liabilities and owners' equity	<u>\$ 545,769</u>

So the EFN is:

$$\text{EFN} = \text{Total assets} - \text{Total liabilities and equity}$$

$$\text{EFN} = \$559,452 - 545,769$$

$$\text{EFN} = \$13,683$$

At a 25 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$27,475 / \$83,740)(\$107,063)$$

$$\text{Dividends} = \$35,127$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$107,063 - 35,127$$

$$\text{Addition to retained earnings} = \$71,936$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$166,705 + 71,936$$

$$\text{New retained earnings} = \$238,641$$

The pro forma balance sheet will look like this:

25% Sales Growth:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 26,175	Accounts payable	\$ 67,238
Accounts receivable	39,963	Notes payable	<u>13,215</u>
Inventory	<u>89,150</u>	Total	\$ 80,453
Total	\$ 155,288	Long-term debt	\$ 127,500
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 427,475</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>238,641</u>
		Total	<u>\$ 343,641</u>
Total assets	<u>\$ 582,763</u>	Total liabilities and owners' equity	<u>\$ 551,593</u>

So the EFN is:

EFN = Total assets – Total liabilities and equity

EFN = \$582,763 – 551,593

EFN = \$31,169

28. The pro forma income statements for all three growth rates will be:

	Pro Forma Income Statement		
	20% Sales Growth	30% Sales Growth	35% Sales Growth
Sales	\$823,836	\$892,489	\$926,816
Costs	665,304	720,746	748,467
Other expenses	16,824	18,226	18,927
EBIT	<u>\$141,708</u>	<u>\$153,517</u>	<u>\$159,422</u>
Interest	12,090	12,090	12,090
Taxable income	<u>\$129,618</u>	<u>\$141,427</u>	<u>\$147,332</u>
Taxes (21%)	27,220	29,700	30,940
Net income	<u>\$102,398</u>	<u>\$111,727</u>	<u>\$116,392</u>
Dividends	\$33,597	\$36,658	\$38,188
Add to RE	68,801	75,070	78,204

At a 30 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = (\$27,475/\$83,740)(\$111,727)

Dividends = \$36,658

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$111,727 - 36,658$$

$$\text{Addition to retained earnings} = \$75,070$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New addition to retained earnings} = \$166,705 + 75,070$$

$$\text{New addition to retained earnings} = \$241,775$$

The new total debt will be:

$$\text{New total debt} = .7159(\$346,775)$$

$$\text{New total debt} = \$248,245$$

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

$$\text{New long-term debt} = \$248,245 - 83,142$$

$$\text{New long-term debt} = \$165,103$$

The pro forma balance sheet will look like this:

$$\text{Sales growth rate} = 30\% \text{ and debt/equity ratio} = .7159$$

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 27,222	Accounts payable	\$ 69,927
Accounts receivable	41,561	Notes payable	<u>13,215</u>
Inventory	<u>92,716</u>	Total	\$ 83,142
Total	\$ 161,499	Long-term debt	\$ 165,103
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 444,574</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>241,775</u>
		Total	<u>\$ 346,775</u>
Total assets	<u>\$ 606,073</u>	Total liabilities and owners' equity	<u>\$ 595,020</u>

So the excess debt raised is:

$$\text{Excess debt} = \$595,020 - 606,073$$

$$\text{Excess debt} = -\$11,053$$

And the EFN is:

$$\text{EFN} = \$37,603 + 11,053$$

$$\text{EFN} = \$48,656$$

At a 35 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

$$\text{Dividends} = (\$27,475/\$83,740)(\$116,392)$$

$$\text{Dividends} = \$38,188$$

And the addition to retained earnings will be:

$$\text{Addition to retained earnings} = \$116,392 - 38,188$$

$$\text{Addition to retained earnings} = \$78,204$$

The new retained earnings on the pro forma balance sheet will be:

$$\text{New retained earnings} = \$166,705 + 78,204$$

$$\text{New retained earnings} = \$244,909$$

The new total debt will be:

$$\text{New total debt} = .7159(\$349,909)$$

$$\text{New total debt} = \$250,489$$

So, the new long-term debt will be the new total debt minus the new short-term debt, or:

$$\text{New long-term debt} = \$250,489 - 85,832$$

$$\text{New long-term debt} = \$164,657$$

Sales growth rate = 35% and debt/equity ratio = .7159:

Pro Forma Balance Sheet

Assets		Liabilities and Owners' Equity	
Current assets		Current liabilities	
Cash	\$ 28,269	Accounts payable	\$ 72,617
Accounts receivable	43,160	Notes payable	<u>13,215</u>
Inventory	<u>96,282</u>	Total	\$ 85,832
Total	\$ 167,711	Long-term debt	\$ 164,657
Fixed assets		Owners' equity	
Net plant and equipment	<u>\$ 461,673</u>	Common stock and paid-in surplus	\$ 105,000
		Retained earnings	<u>244,909</u>
		Total	<u>\$ 349,909</u>
Total assets	<u>\$ 629,384</u>	Total liabilities and owners' equity	<u>\$ 600,397</u>

So the excess debt raised is:

$$\text{Excess debt} = \$600,397 - 629,384$$

$$\text{Excess debt} = -\$28,986$$

At a 35 percent growth rate, the firm will need funds in the amount of \$28,986 in addition to the external debt already raised. So, the EFN will be:

$$\text{EFN} = \$37,157 + 28,986$$

$$\text{EFN} = \$66,143$$

29. We need the ROE to calculate the sustainable growth rate. The ROE is:

$$\text{ROE} = (\text{PM})(\text{TAT})(\text{EM})$$

$$\text{ROE} = (.062)(1/1.05)(1 + .55)$$

$$\text{ROE} = .0915, \text{ or } 9.15\%$$

Now we can use the sustainable growth rate equation to find the retention ratio as:

$$\text{Sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Sustainable growth rate} = .12 = [.0915(b)] / [1 - .0915(b)]$$

$$b = 1.17$$

This implies the payout ratio is:

$$\text{Payout ratio} = 1 - b$$

$$\text{Payout ratio} = 1 - 1.17$$

$$\text{Payout ratio} = -.17$$

This answer indicates a dividend payout ratio of negative 17 percent, which is impossible. So, the growth rate is not consistent with the other constraints. The lowest possible payout rate is 0 (without

issuing stock), which corresponds to a retention ratio of 1, or total earnings retention. This problem illustrates a key point we made in the chapter: Sustainable growth analysis forces the user to make internally consistent assumptions.

As an aside, we should note that it is possible to have a retention ratio greater than 1 if the company issues new stock. However, since the growth rate we are evaluating is perpetual, the company would have to issue stock every year, forever. But, doing so violates our underlying assumption that the sustainable growth rate requires no new equity.

The maximum sustainable growth rate for this company is:

$$\text{Maximum sustainable growth rate} = (\text{ROE} \times b) / [1 - (\text{ROE} \times b)]$$

$$\text{Maximum sustainable growth rate} = [.0915(1)] / [1 - .0915(1)]$$

$$\text{Maximum sustainable growth rate} = .1007, \text{ or } 10.07\%$$

30. We know that EFN is:

$$\text{EFN} = \text{Increase in assets} - \text{Addition to retained earnings}$$

The increase in assets is the beginning assets times the growth rate, so:

$$\text{Increase in assets} = A \times g$$

The addition to retained earnings next year is the current net income times the retention ratio, times one plus the growth rate, so:

$$\text{Addition to retained earnings} = (\text{NI} \times b)(1 + g)$$

And rearranging the profit margin to solve for net income, we get:

$$\text{NI} = \text{PM}(S)$$

Substituting the last three equations into the EFN equation we started with and rearranging, we get:

$$\text{EFN} = A(g) - \text{PM}(S)b(1 + g)$$

$$\text{EFN} = A(g) - \text{PM}(S)b - [\text{PM}(S)b]g$$

$$\text{EFN} = -\text{PM}(S)b + [A - \text{PM}(S)b]g$$

31. We start with the EFN equation we derived in Problem 30 and set it equal to zero:

$$\text{EFN} = 0 = -\text{PM}(S)b + [A - \text{PM}(S)b]g$$

Substituting the rearranged profit margin equation into the internal growth rate equation, we have:

$$\text{Internal growth rate} = [\text{PM}(S)b] / [A - \text{PM}(S)b]$$

Since:

$$ROA = NI/A$$

$$ROA = PM(S)/A$$

We can substitute this into the internal growth rate equation and divide both the numerator and denominator by A. This gives:

$$\text{Internal growth rate} = \{[PM(S)b]/A\} / \{[A - PM(S)b]/A\}$$

$$\text{Internal growth rate} = b(ROA)/[1 - b(ROA)]$$

To derive the sustainable growth rate, we must realize that to maintain a constant D/E ratio with no external equity financing, EFN must equal the addition to retained earnings times the D/E ratio:

$$EFN = (D/E)[PM(S)b(1 + g)]$$

$$EFN = A(g) - PM(S)b(1 + g)$$

Solving for g and then dividing the numerator and denominator by A:

$$\text{Sustainable growth rate} = PM(S)b(1 + D/E)/[A - PM(S)b(1 + D/E)]$$

$$\text{Sustainable growth rate} = [ROA(1 + D/E)b]/[1 - ROA(1 + D/E)b]$$

$$\text{Sustainable growth rate} = b(ROE)/[1 - b(ROE)]$$

32. In the following derivations, the subscript “E” refers to end of period numbers, and the subscript “B” refers to beginning of period numbers. TE is total equity and TA is total assets.

For the sustainable growth rate:

$$\text{Sustainable growth rate} = (ROE_E \times b)/(1 - ROE_E \times b)$$

$$\text{Sustainable growth rate} = (NI/TE_E \times b)/(1 - NI/TE_E \times b)$$

We multiply this equation by:

$$(TE_E/TE_E)$$

$$\text{Sustainable growth rate} = (NI/TE_E \times b)/(1 - NI/TE_E \times b) \times (TE_E/TE_E)$$

$$\text{Sustainable growth rate} = (NI \times b)/(TE_E - NI \times b)$$

Recognize that the numerator is equal to beginning of period equity, that is:

$$(TE_E - NI \times b) = TE_B$$

Substituting this into the previous equation, we get:

$$\text{Sustainable rate} = (NI \times b)/TE_B$$

Which is equivalent to:

$$\text{Sustainable rate} = (NI/TE_B) \times b$$

$$\text{Since } ROE_B = NI/TE_B$$

The sustainable growth rate equation is:

$$\text{Sustainable growth rate} = \text{ROE}_B \times b$$

For the internal growth rate:

$$\text{Internal growth rate} = (\text{ROA}_E \times b) / (1 - \text{ROA}_E \times b)$$

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_E \times b) / (1 - \text{NI}/\text{TA}_E \times b)$$

We multiply this equation by:

$$(\text{TA}_E/\text{TA}_E)$$

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_E \times b) / (1 - \text{NI}/\text{TA}_E \times b) \times (\text{TA}_E/\text{TA}_E)$$

$$\text{Internal growth rate} = (\text{NI} \times b) / (\text{TA}_E - \text{NI} \times b)$$

Recognize that the numerator is equal to beginning of period assets, that is:

$$(\text{TA}_E - \text{NI} \times b) = \text{TA}_B$$

Substituting this into the previous equation, we get:

$$\text{Internal growth rate} = (\text{NI} \times b) / \text{TA}_B$$

Which is equivalent to:

$$\text{Internal growth rate} = (\text{NI}/\text{TA}_B) \times b$$

$$\text{Since } \text{ROA}_B = \text{NI}/\text{TA}_B$$

The internal growth rate equation is:

$$\text{Internal growth rate} = \text{ROA}_B \times b$$