Chapter 1 – Electric Circuit Variables

Exercises

Exercise 1.2-1 Find the charge that has entered an element by time t when $i = 8t^2 - 4t$ A, $t \geq 0$. Assume $q(t) = 0$ for $t < 0$.

Answer: $q(t) = \frac{8}{3}t^3 - 2t^2 \quad \text{C}$

Solution:

$$i(t) = 8t^2 - 4t \text{ A}$$

$$q(t) = \int_0^t i\,d\tau + q(0) = \int_0^t (8\tau^2 - 4\tau)\,d\tau + 0 = \frac{8}{3}\tau^3 - 2\tau^2\Big|_0^t = \underline{\frac{8}{3}t^3 - 2t^2 \quad \text{C}}$$

Exercise 1.2-2 The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

Answer: $i(t) = \frac{d}{dt} 4\sin 3t = 12\cos 3t \quad \text{A}$

Solution: $$i(t) = \frac{dq}{dt} = \frac{d}{dt} 4\sin 3t = 12\cos 3t \quad \text{A}$$

Exercise 1.3-1 Which of the three currents, $i_1 = 45\ \mu\text{A}$, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

Answer: i_3 is largest.

Solution:

$$i_1 = 45\ \mu\text{A} = 45 \times 10^{-6} \text{ A} < i_2 = 0.03 \text{ mA} = .03 \times 10^{-3} \text{ A} = 3 \times 10^{-5} \text{ A} < i_3 = 25 \times 10^{-4} \text{ A}$$

Exercise 1.5-1 Figure E 1.5-1 shows four circuit elements identified by the letters A, B, C, and D.

(a) Which of the devices supply 12 W?
(b) Which of the devices absorb 12 W?
(c) What is the value of the power received by device B?
(d) What is the value of the power delivered by device B?
(e) What is the value of the power delivered by device D?

Answers: (a) B and C, (b) A and D, (c)–12 W, (c) 12 W, (e)–12 W

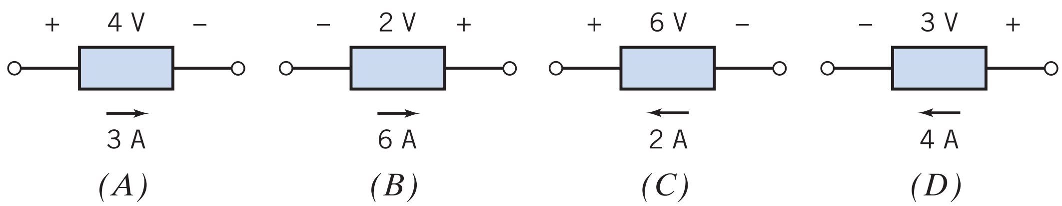


Figure E 1.5-1

Solution:

(a) B and C. The element voltage and current do not adhere to the passive convention in B and C so the product of the element voltage and current is the power supplied by these elements.

(b) A and D. The element voltage and current adhere to the passive convention in A and D so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) −12 W. The element voltage and current do not adhere to the passive convention in B, so the product of the element voltage and current is the power received by this element: (2 V)(6 A) = −12 W. The power supplied by the element is the negative of the power delivered to the element, 12 W.

(d) 12 W

(e) –12 W. The element voltage and current adhere to the passive convention in D, so the product of the element voltage and current is the power received by this element: (3 V)(4 A) = 12 W. The power supplied by the element is the negative of the power received to the element, −12 W.

Problems

Section 1-2 Electric Circuits and Current Flow

P1.2.1 The total charge that has entered a circuit element is $q(t) = 1.25(1–e^{-5t})$ when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t \geq 0$.

Answer: $i(t) = 6.25e^{-5t}$ A

Solution:

$$i(t) = \frac{d}{dt}1.25\left(1-e^{-5t}\right) = \underline{6.25\,e^{-5t}}\ \textbf{A}$$

P 1.2-2 The current in a circuit element is $i(t) = 4(1–e^{-5t})$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^{t} i(\tau) = \int_{-\infty}^{0} 0 d\tau = 0$

Answer: $q(t) = 4t + 0.8e^{-5t} – 0.8$ C for $t \geq 0$

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\left(1-e^{-5\tau}\right)d\tau + 0 = \int_0^t 4\,d\tau - \int_0^t 4e^{-5\tau}\,d\tau = \underline{4t + \frac{4}{5}e^{-5t} - \frac{4}{5}}\ \text{C}$$

P 1.2-3 The current in a circuit element is $i(t) = 4 \sin 3t$ A when $t \geq 0$ and $i(t) = 0$ when $t < 0$. Determine the total charge that has entered a circuit element for $t \geq 0$.

Hint: $q(0) = \int_{-\infty}^{t} i(\tau)d\tau = \int_{-\infty}^{0} 0\,d\tau = 0$

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 4\sin 5\tau\,d\tau + 0 = -\frac{4}{5}\cos 3\tau\Big|_0^t = \underline{-\frac{4}{5}\cos 3t + \frac{4}{5}}\ \text{C}$$

P 1.2-4 The current in a circuit element is $i(t)=\begin{cases}0 & t<2\\ 2 & 2<t<4\\ -1 & 4<t<8\\ 0 & 8<t\end{cases}$ where the units of current are A and the units of time are s. Determine the total charge that has entered a circuit element for $t \ge 0$.

Answer:

$$q(t)=\begin{cases}0 & t<2\\ 2t-4 & 2<t<4\\ 8-t & 4<t<8\\ 0 & 8<t\end{cases}\quad \text{where the units of charge are C.}$$

Solution:

$q(t)=\int_{-\infty}^{t} i(\tau)\,d\tau=\int_{-\infty}^{t} 0\,d\tau=$ $\underline{0\text{ C for } t\le 2}$ so $q(2) = 0$.

$q(t)=\int_{2}^{t} i(\tau)\,d\tau+q(2)=\int_{2}^{t} 2\,d\tau=2\tau\big|_2^t=$ $\underline{2t-4 \text{ C for } 2\le t\le 4}$. In particular, $q(4) = 4$ C.

$q(t)=\int_{4}^{t} i(\tau)\,d\tau+q(4)=\int_{4}^{t} -1\,d\tau+4=-\tau\big|_4^t+4=$ $\underline{8-t \text{ C for } 4\le t\le 8}$. In particular, $q(8) = 0$ C.

$q(t)=\int_{8}^{t} i(\tau)\,d\tau+q(8)=\int_{8}^{t} 0\,d\tau+0=$ $\underline{0\text{ C for } 8\le t}$.

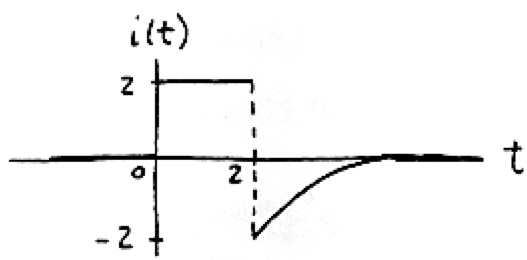
P 1.2-5 The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t)=\begin{cases}0 & t<0\\ 2t & 0\le t\le 2\\ 3+e^{-2t(t-2)} & t>2\end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \ge 0$.

Solution:

$$i(t)=\frac{dq(t)}{dt}\qquad i(t)=\begin{cases}0 & t<0\\ 2 & 0<t<2\\ \underline{-2e^{-2(t-2)}} & t>2\end{cases}$$



P 1.2-6 An electroplating bath, as shown in Figure P 1.2-6, is used to plate silver uniformly onto objects such as kitchen ware and plates. A current of 600 A flows for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams?

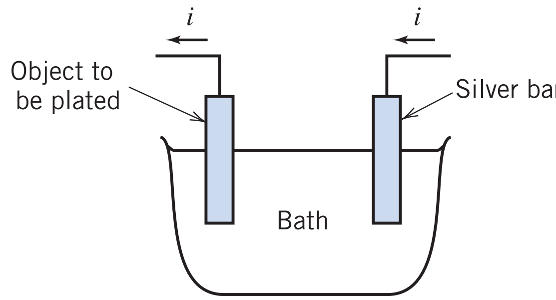


Figure P 1.2-6

Solution:

$$i = 450 \text{ A} = 450 \ \frac{\text{C}}{\text{s}}$$

$$\text{Silver deposited} = 450\frac{\text{C}}{\text{s}}\times 20\,\text{min}\times 60\ \frac{\text{s}}{\text{min}}\times 1.118\ \frac{\text{mg}}{\text{C}} = 6.0372\times 10^{5}\ \text{mg} = \underline{603.72\ \text{g}}$$

P1.2-7 Find the charge $q(t)$ and sketch its waveform when the current entering a terminal of ab element is as shown in Figure P1.2-7. Assume that $q(t) = 0$ for $t < 0$.

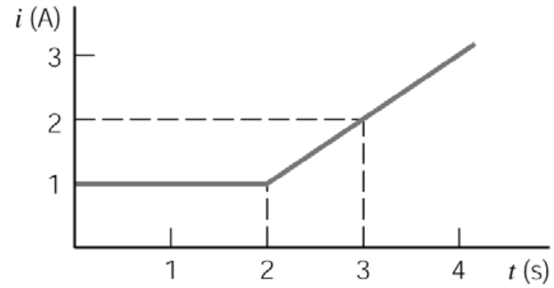


Figure P1.2-7

Solution:

$$i(t) = \begin{cases} 1 & \text{when} \quad 0 < t \le 2 \\ t-1 & \text{when} \quad 2 \le t \end{cases}$$

and
$$q(t) = \int_0^t i(\tau)d\tau + q(0) = \int_0^t i(\tau)d\tau$$

since $q(0) = 0$.

When $0 < t \le 2$, we have

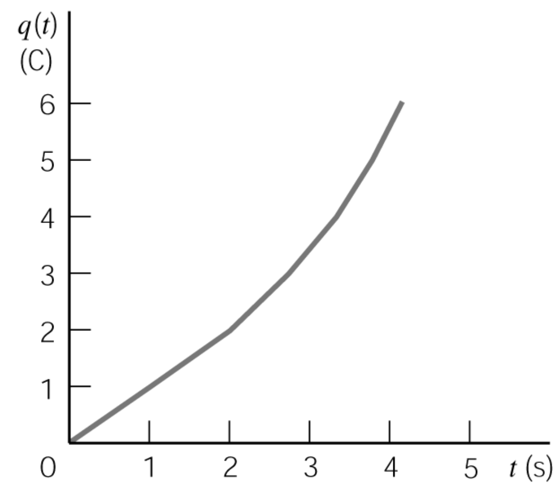
$$q = \int_0^t 1\, d\tau = t \quad \text{C}$$

When $t \ge 2$, we have

$$q = \int_0^t i(\tau)d\tau = \int_0^2 1\, d\tau + \int_2^t (\tau - 1)\, d\tau$$

$$= \tau\Big|_0^2 + \frac{\tau^2}{2}\Big|_2^t - \tau\Big|_2^t = \frac{t^2}{2} - t + 2 \quad \text{C}$$

The sketch of $q(t)$ is shown to the right:.



Section 1-3 Systems of Units

P 1.3-1 A constant current of 3.2 μA flows through an element. What is the charge that has passed through the element in the first millisecond?
Answer: 3.2 nC

Solution:

$$\Delta q = i\,\Delta t = \left(3.2\times10^{-6}\ \text{A}\right)\left(1\times10^{-3}\ \text{s}\right) = 3.2\times10^{-9}\ \text{As} = 3.2\times10^{-9}\ \text{C} = \underline{3.2\times10^{-9}\ \text{nC}}$$

P 1.3-2 A charge of 45 nC passes through a circuit element during a particular interval of time that is 5 ms in duration. Determine the average current in this circuit element during that interval of time.
Answer: $i = 9\ \mu\text{A}$

Solution:

$$i = \frac{\Delta q}{\Delta t} = \frac{45\times10^{-9}}{5\times10^{-3}} = 9\times10^{-6} = \underline{9\ \mu\text{A}}$$

P 1.3-3 Ten billion electrons per second pass through a particular circuit element. What is the average current in that circuit element?
Answer: $i = 1.602$ nA

Solution

$$i = \left[10\,\text{billion}\ \frac{\text{electron}}{\text{s}}\right]\left[1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right] = \left[10\times10^{9}\ \frac{\text{electron}}{\text{s}}\right]\left[1.602\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right]$$

$$= 10^{10}\times1.602\times10^{-19}\ \frac{\text{electron}}{\text{s}}\ \frac{\text{C}}{\text{electron}}$$

$$= 1.602\times10^{-9}\ \frac{\text{C}}{\text{s}} = \underline{1.602\ \ \text{nA}}$$

P1.3-4 The charge flowing in a wire is plotted in Figure P1.3-4. Sketch the corresponding current.

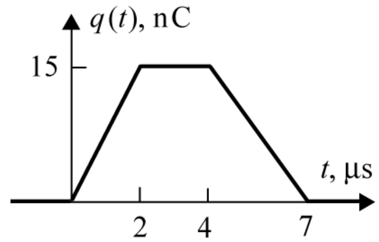
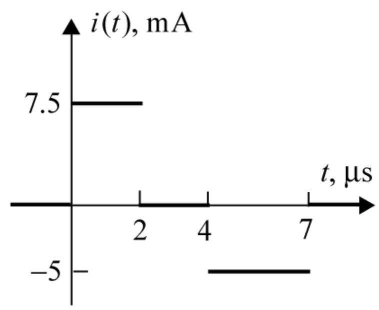


Figure P1.3-4

P1.3-4

$$i(t)=\frac{d}{dt}q(t)=\text{the slope of the } q \text{ versus } t \text{ plot}=\begin{cases}\dfrac{15\times10^{-9}}{2\times10^{-6}}=7.5\times10^{-3}=7.5\text{ mA when } 0<t<2\mu s\\ -\dfrac{15\times10^{-9}}{3\times10^{-6}}=-5\times10^{-3}=-5\text{ mA when } 4\mu s<t<7\mu s\\ 0\quad\text{otherwise}\end{cases}$$



P1.3-5 The current in a circuit element is plotted in Figure P1.3-5. Sketch the corresponding charge flowing through the element for $t > 0$.

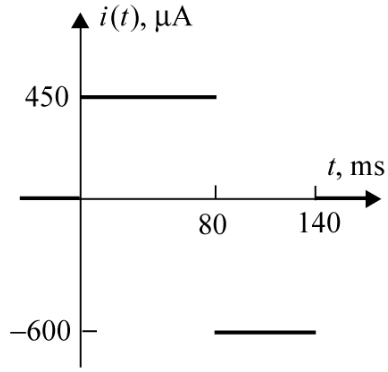


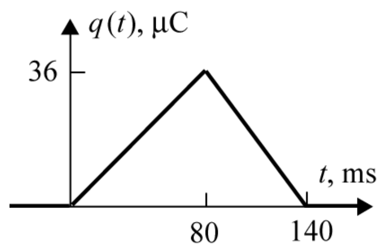
Figure P1.3-5

Solution:

$$q(t)=\int_0^t i(\tau)\,d\tau=\begin{cases}\int_0^t 450\mu\text{A}\,d\tau \quad \text{when } 0<t<80 \text{ ms}\\ \left(450\times10^{-6}\right)\left(80\times10^{-3}\right)+\int_{80\text{ ms}}^{t}\left(-600\mu\text{A}\right)d\tau \quad \text{when } 80\text{ ms}<t<140\text{ ms}\\ \left(450\times10^{-6}\right)\left(80\times10^{-3}\right)+\left(-600\times10^{-6}\right)\left(60\times10^{-3}\right)+\int_{140\text{ ms}}^{t}0\,d\tau\end{cases}$$

$$=\begin{cases}\left(450\times10^{-6}\right)t \quad \text{when } 0<t<80\text{ ms}\\ \left(36\times10^{-6}\right)+\left(-600\times10^{-6}\right)t \quad \text{when } 80\text{ ms}<t<140\text{ ms}\\ 0\text{ C} \quad \text{when } 140\text{ ms}<t\end{cases}$$

While $0 < t < 80$ ms $q(t)$ increases linearly from 0 to 36 μC and while $80 < t < 140$ ms $q(t)$ decreases linearly from 36 to 0 μC. Here's the sketch:



P1.3-6 The current in a circuit element is plotted in Figure P1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 μs.

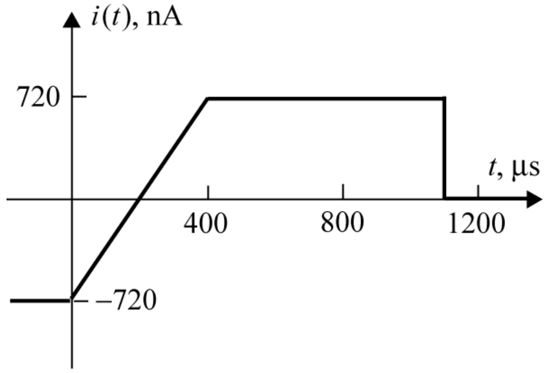
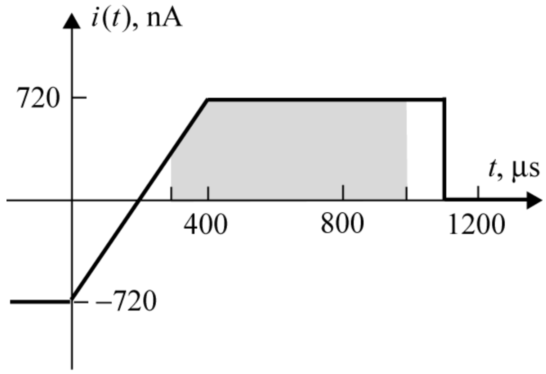


Figure P1.3-6

Solution:

$$q(t)=\int_{300\ \mu s}^{1000\ \mu s} i(\tau)\,d\tau = \text{"area under the curve between 300 } \mu\text{s and 1000 } \mu\text{s"}$$



$$q(t)=\left(\frac{360+720}{2}\times10^{-9}\right)\left(100\times10^{-6}\right)+\left(720\times10^{-9}\right)\left(600\times10^{-6}\right)=\left(54+432\right)\times10^{-12}=486\text{ pC}$$

Section 1-5 Power and Energy

P1.5-1 Figure P1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.
(*a*) Which of the devices supply 30 mW?
(*b*) Which of the devices absorb 0.03 W?
(*c*) What is the value of the power received by device *B*?
(*d*) What is the value of the power delivered by device *B*?
(*e*) What is the value of the power delivered by device *C*?

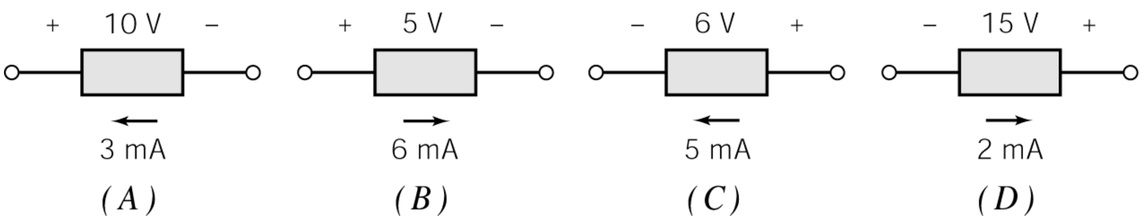


Figure P1.5-1

Solution:

(a) *A* and *D.* The element voltage and current do not adhere to the passive convention in Figures P1.5- *A* and *D* so the product of the element voltage and current is the power supplied by these elements.

(b) *B* and *C*. The element voltage and current adhere to the passive convention in Figures P1.5-1 *B* and *C* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) 30 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*B*, so the product of the element voltage and current is the power received by this element: (5 V)(6 mA) = 30 mW. The power supplied by the element is the negative of the power received to the element, −30 W.

(d) −30 mW

(e) –30 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*C*, so the product of the element voltage and current is the power received by this element: (5 V)(6 mA) = 30 mW. The power supplied by the element is the negative of the power received to the element, −30 W.

P 1.5-2 An electric range has a constant current of 10 A entering the positive voltage terminal with a voltage of 110 V. The range is operated for two hours. (a) Find the charge in coulombs that passes through the range. (b) Find the power absorbed by the range. (c) If electric energy costs 12 cents per kilowatt-hour, determine the cost of operating the range for two hours.

Solution:

a.) $q = \int i\,dt = i\Delta t = (10\text{ A})(2\text{ hrs})(3600\text{ s/hr}) = \underline{7.2\times10^4\text{ C}}$

b.) $P = v\,i = (110\text{ V})(10\text{ A}) = \underline{1100\text{ W}}$

c.) $\text{Cost} = \dfrac{0.12\,\$}{\text{kWhr}}\times 1.1\,\text{kW}\times 2\,\text{hrs} = \underline{0.264\,\$}$

P 1.5-3 A walker's cassette tape player uses four AA batteries in series to provide 6 V to the player circuit. The four alkaline battery cells store a total of 200 watt-seconds of energy. If the cassette player is drawing a constant 10 mA from the battery pack, how long will the cassette operate at normal power?

Solution:

$$P = (6\text{ V})(10\text{ mA}) = 0.06\text{ W}$$

$$\Delta t = \frac{\Delta w}{P} = \frac{200\text{ W}\cdot\text{s}}{0.06\text{ W}} = \underline{3.33\times10^3\text{ s}}$$

P 1.5-4 The current through and voltage across an element vary with time as shown in Figure P 1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

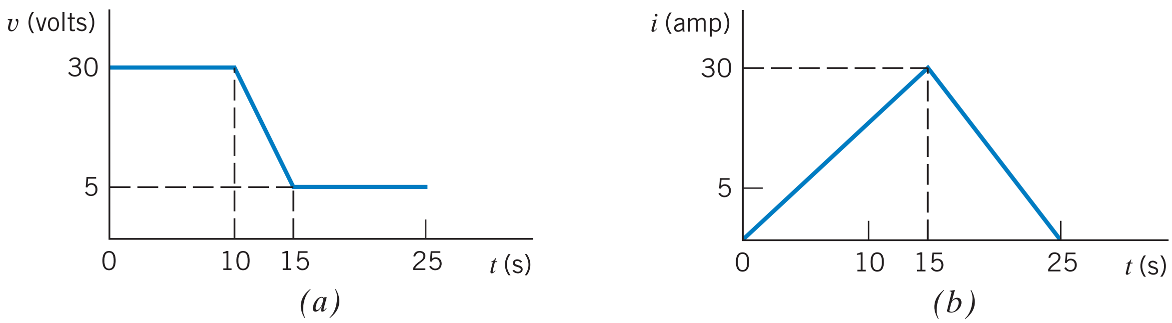


Figure P 1.5-4

Solution:

for $0 \le t \le 10$ s: $v = 30$ V and $i = \frac{30}{15}t = 2t$ A $\therefore$ $\underline{P = 30(2t) = 60t \text{ W}}$

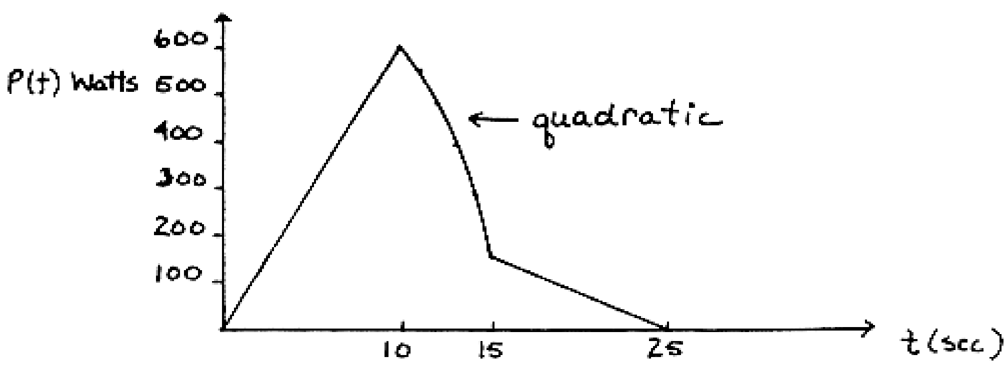
for $10 \le t \le 15$ s: $v(t) = -\frac{25}{5}t + b \Rightarrow v(10) = 30 \text{ V} \Rightarrow b = 80 \text{ V}$

$v(t) = -5t + 80$ and $i(t) = 2t$ A $\Rightarrow$ $P = (2t)(-5t+80) = \underline{-10t^2+160t \text{ W}}$

for $15 \le t \le 25$ s: $v = 5$ V and $i(t) = -\frac{30}{10}t + b$ A

$i(25) = 0 \Rightarrow b = 75 \Rightarrow i(t) = -3t+75$ A

$\therefore \underline{P = (5)(-3t+75) = -15t+375 \text{ W}}$



$$\text{Energy} = \int P\,dt = \int_0^{10} 60t\,dt + \int_{10}^{15}\left(160t - 10t^2\right)dt + \int_{15}^{25}(375 - 15t)\,dt$$

$$= 30t^2\Big|_0^{10} + 80t^2 - \frac{10}{3}t^3\Big|_{10}^{15} + 375t - \frac{15}{2}t^2\Big|_{15}^{25} = \underline{5833.\bar{3} \text{ J}}$$

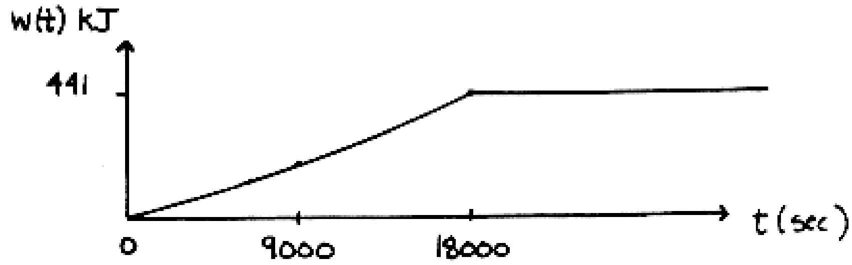
P 1.5-5 An automobile battery is charged with a constant current of 2 A for five hours. The terminal voltage of the battery is $v = 11 + 0.5t$ V for $t > 0$, where t is in hours. (a) Find the energy delivered to the battery during the five hours. (b) If electric energy costs 15 cents/kWh, find the cost of charging the battery for five hours.
Answer: (b) 1.84 cents

Solution:

a.) Assuming no more energy is delivered to the battery after 5 hours (battery is fully charged).

$$w = \int P dt = \int_0^t vi\, d\tau = \int_0^{5(3600)} 2\left(11 + \frac{0.5\,\tau}{3600}\right) d\tau = 22t + \frac{0.5}{3600}\tau^2 \Big|_0^{5(3600)}$$

$$= 441 \times 10^3 \text{ J} = \underline{441 \text{ kJ}}$$



b.) $\text{Cost} = 441\,\text{kJ} \times \dfrac{1 \text{ hr}}{3600\text{s}} \times \dfrac{15¢}{\text{kWhr}} = \underline{1.84¢}$

P 1.5-6 Find the power, $p(t)$, supplied by the element shown in Figure P 1.5-6 when $v(t) = 4 \sin 3t$ V and $i(t) = (1/12) \sin 3t$ A. Evaluate $p(t)$ at t=0.5 s and t = 1 s. Observe that the power supplied by this element has a positive value at some times and a negative value at other times.

Hint: $(\sin at)(\sin bt) = \frac{1}{2}(\cos(a-b)t - \cos(a+b)t)$

Answer: $p(t) = (1/6)\cos(6t)$ W, $p(0.5) = 0.0235$ W, $p(1) = -0.02466$ W

Solution:

$$p(t) = v(t)\,i(t) = (4\cos 3t)\left(\frac{1}{12}\sin 3t\right) = \frac{1}{6}(\sin 0 + \sin 6t) = \underline{\frac{1}{6}\sin 6t \quad \text{W}}$$

$$p(0.5) = \frac{1}{6}\sin 3 = \underline{0.0235 \quad \text{W}}$$

$$p(1) = \frac{1}{6}\sin 6 = \underline{-0.0466 \quad \text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear
t0=0;                   % initial time
tf=2;                   % final time
dt=0.02;                % time increment
t=t0:dt:tf;             % time

v=4*cos(3*t);           % device voltage
i=(1/12)*sin(3*t);      % device current

for k=1:length(t)
   p(k)=v(k)*i(k);      % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```