

Chapter 3

1. THINK In this problem we're given the magnitude and direction of a vector in two dimensions and asked to calculate its x and y components.

EXPRESS The x and the y components of a vector $\vec{a}$ lying in the xy plane are given by

$$a_x = a \cos \theta, \quad a_y = a \sin \theta,$$

where $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ is the magnitude and the angle $\theta = \tan^{-1}(a_y/a_x)$ is between $\vec{a}$ and the $+x$ direction. Given that $\theta = 250^\circ$, we see that the vector is in the third quadrant, and we expect both the x and the y components of $\vec{a}$ to be negative.

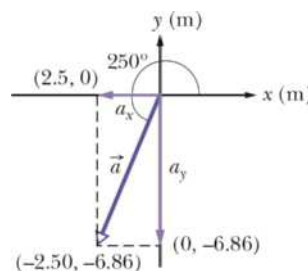
ANALYZE (a) The x component of $\vec{a}$ is

$$a_x = a \cos \theta = (7.3 \text{ m}) \cos 250^\circ = -2.5 \text{ m}.$$

(b) The y component is

$$a_y = a \sin \theta = (7.3 \text{ m}) \sin 250^\circ = -6.86 \text{ m} \approx -6.9 \text{ m}.$$

The results are depicted in the figure below:



LEARN In considering the variety of ways to compute these, we note that the vector is 70° below the $-x$ direction, so the components could also have been found from

$$a_x = -(7.3 \text{ m}) \cos 70^\circ = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m}) \sin 70^\circ = -6.86 \text{ m}.$$

Similarly, we note that the vector is 20° to the left from the $-y$ direction, so one could also achieve the same results by using

$$a_x = -(7.3 \text{ m}) \sin 20^\circ = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m}) \cos 20^\circ = -6.86 \text{ m}.$$

As a consistency check, we note that

$$\sqrt{a_x^2 + a_y^2} = \sqrt{(-2.50 \text{ m})^2 + (-6.86 \text{ m})^2} = 7.3 \text{ m}$$

and

$$\tan^{-1}(a_y/a_x) = \tan^{-1}[(-6.86 \text{ m})/(-2.50 \text{ m})] = 250^\circ,$$

which are indeed the values given in the problem statement.

3. THINK In this problem we're given the x and y components of a vector $\vec{A}$ in two dimensions and asked to calculate its magnitude and direction.

EXPRESS Vector $\vec{A}$ can be represented in *magnitude-angle* notation (A, θ), where

$$A = \sqrt{A_x^2 + A_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

is the angle $\vec{A}$ makes with the positive direction of the x axis. Given that $A_x = -25.0 \text{ m}$ and $A_y = 40.0 \text{ m}$, the above formulas can be used to calculate A and θ .

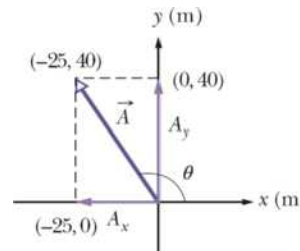
ANALYZE (a) The magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0 \text{ m})^2 + (40.0 \text{ m})^2} = 47.2 \text{ m}.$$

(b) Because $\tan \theta = \tan(\theta + 180^\circ)$,

$$\tan^{-1}[(40.0 \text{ m})/(-25.0 \text{ m})] = -58^\circ \text{ or } 122^\circ.$$

Noting that the vector is in the second quadrant (by the signs of its x and y components), we see that 122° is the correct answer. The results are depicted in the figure here



LEARN We can check our answers by noting that the x and the y components of $\vec{A}$ can be written as

$$A_x = A \cos \theta, \quad A_y = A \sin \theta.$$

Substituting the results calculated above, we obtain

$$A_x = (47.2 \text{ m}) \cos 122^\circ = -25.0 \text{ m}, \quad A_y = (47.2 \text{ m}) \sin 122^\circ = +40.0 \text{ m},$$

which indeed are the values given in the problem statement.

11. THINK This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$. We want to find the magnitude and direction of the resulting vector.

EXPRESS In two dimensions, vector $\vec{a}$ can be written in unit-vector notation as

$$\vec{a} = a_x \hat{i} + a_y \hat{j}.$$

Similarly, vector $\vec{b}$ can be expressed as $\vec{b} = b_x \hat{i} + b_y \hat{j}$. Adding the two vectors gives

$$\vec{r} = \vec{a} + \vec{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} = r_x \hat{i} + r_y \hat{j}.$$

ANALYZE (a) Given that

$$\vec{a} = (4.0 \text{ m}) \hat{i} + (3.0 \text{ m}) \hat{j}$$

and

$$\vec{b} = (-13.0 \text{ m}) \hat{i} + (7.0 \text{ m}) \hat{j},$$

we find the x and the y components of $\vec{r}$ to be

$$\begin{aligned} r_x &= a_x + b_x = (4.0 \text{ m}) + (-13 \text{ m}) = -9.0 \text{ m} \\ r_y &= a_y + b_y = (3.0 \text{ m}) + (7.0 \text{ m}) = 10.0 \text{ m}. \end{aligned}$$

Thus, $\vec{r} = (-9.0 \text{ m}) \hat{i} + (10 \text{ m}) \hat{j}$.

(b) The magnitude of $\vec{r}$ is

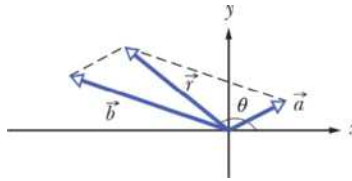
$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0 \text{ m})^2 + (10 \text{ m})^2} = 13 \text{ m}.$$

(c) The angle between the resultant and the $+x$ direction is

$$\theta = \tan^{-1} \left(\frac{r_y}{r_x} \right) = \tan^{-1} \left(\frac{10.0 \text{ m}}{-9.0 \text{ m}} \right) = -48^\circ \text{ or } 132^\circ.$$

Since the x component of the resultant is negative and the y component is positive, making the vector point into the second quadrant, we find the angle is 132° (measured counterclockwise from the $+x$ direction).

LEARN The addition of the two vectors is depicted in the figure below (not to scale). Indeed, since $r_x < 0$ and $r_y > 0$, we expect $\vec{r}$ to point into the second quadrant.



15. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$ in two dimensions. We're asked to find the components, magnitude, and direction of the resulting vector.

EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit-vector notation,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = (a \cos \alpha) \hat{i} + (a \sin \alpha) \hat{j}.$$

Similarly, a second vector $\vec{b}$ can be expressed as

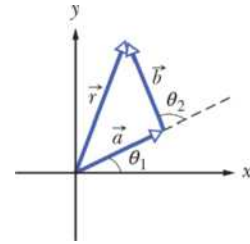
$$\vec{b} = b_x \hat{i} + b_y \hat{j} = (b \cos \beta) \hat{i} + (b \sin \beta) \hat{j}.$$

From the figure, we have $\alpha = \theta_1$ and $\beta = \theta_1 + \theta_2$ (since the angles are measured from the $+x$ direction) and the resulting vector is

$$\vec{r} = \vec{a} + \vec{b} = [a \cos \theta_1 + b \cos(\theta_1 + \theta_2)] \hat{i} + [a \sin \theta_1 + b \sin(\theta_1 + \theta_2)] \hat{j} = r_x \hat{i} + r_y \hat{j}.$$

ANALYZE (a) Given that $a = b = 10 \text{ m}$, $\theta_1 = 30^\circ$, and $\theta_2 = 105^\circ$, the x component of $\vec{r}$ is

$$r_x = a \cos \theta_1 + b \cos(\theta_1 + \theta_2) = (10 \text{ m}) \cos 30^\circ + (10 \text{ m}) \cos(30^\circ + 105^\circ) = 1.59 \text{ m}.$$



(b) Similarly, the y component of $\vec{r}$ is

$$r_y = a \sin \theta_1 + b \sin(\theta_1 + \theta_2) = (10 \text{ m}) \sin 30^\circ + (10 \text{ m}) \sin(30^\circ + 105^\circ) = 12.1 \text{ m}.$$

(c) The magnitude of $\vec{r}$ is

$$r = |\vec{r}| = \sqrt{(1.59 \text{ m})^2 + (12.1 \text{ m})^2} = 12.2 \text{ m}.$$

(d) The angle between $\vec{r}$ and the $+x$ direction is

$$\theta = \tan^{-1} \left(\frac{r_y}{r_x} \right) = \tan^{-1} \left(\frac{12.1 \text{ m}}{1.59 \text{ m}} \right) = 82.5^\circ.$$

LEARN As depicted in the figure, the resultant $\vec{r}$ lies in the first quadrant. This is what we expect. Note that the magnitude of $\vec{r}$ can also be calculated by using the law of cosines ($\vec{a}$, $\vec{b}$, and $\vec{r}$ form an isosceles triangle):

$$\begin{aligned} r &= \sqrt{a^2 + b^2 - 2ab \cos(180 - \theta_2)} = \sqrt{(10 \text{ m})^2 + (10 \text{ m})^2 - 2(10 \text{ m})(10 \text{ m}) \cos 75^\circ} \\ &= 12.2 \text{ m}. \end{aligned}$$

41. **THINK** The angle between two vectors can be calculated using the definition of a scalar product.

EXPRESS Since the scalar product of two vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \cdot \vec{b} = ab \cos \phi = a_x b_x + a_y b_y + a_z b_z,$$

we can find the angle between them:

$$\cos \phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab} \Rightarrow \phi = \cos^{-1} \left(\frac{a_x b_x + a_y b_y + a_z b_z}{ab} \right).$$

Once the magnitudes and components of the vectors are known, angle ϕ can be calculated.

ANALYZE Given that

$$\vec{a} = (3.0)\hat{i} + (3.0)\hat{j} + (3.0)\hat{k}$$

and

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$$\vec{b} = (2.0)\hat{i} + (1.0)\hat{j} + (3.0)\hat{k},$$

the magnitudes of the vectors are

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.20$$

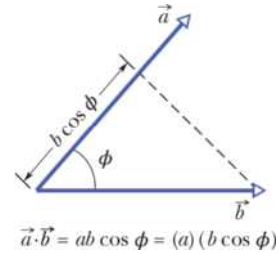
$$b = |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.74.$$

The angle between them is found from

$$\cos \phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.20)(3.74)} = 0.926$$

to be $\phi = 22^\circ$.

LEARN As the name indicates, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated next (see also Fig. 3.3.1 in the textbook):



43. **THINK** In this problem we are given three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ on the xy plane and asked to calculate their components.

EXPRESS From the figure, we note that $\vec{c} \perp \vec{b}$, which indicates that the angle between $\vec{c}$ and the $+x$ direction is $\theta + 90^\circ$. In unit-vector notation, the three vectors can be written as

$$\vec{a} = a_x \hat{i}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} = (b \cos \theta) \hat{i} + (b \sin \theta) \hat{j}$$

$$\vec{c} = c_x \hat{i} + c_y \hat{j} = [c \cos(\theta + 90^\circ)] \hat{i} + [c \sin(\theta + 90^\circ)] \hat{j}.$$

The above expressions allow us to evaluate the components of the vectors.

ANALYZE (a) The x component of $\vec{a}$ is $a_x = a \cos 0^\circ = a = 3.00$ m.

(b) Similarly, the y component of $\vec{a}$ is $a_y = a \sin 0^\circ = 0$.

(c) The x component of $\vec{b}$ is $b_x = b \cos 30^\circ = (4.00 \text{ m}) \cos 30^\circ = 3.46 \text{ m}$.

(d) The y component is $b_y = b \sin 30^\circ = (4.00 \text{ m}) \sin 30^\circ = 2.00 \text{ m}$.

(e) The x component of $\vec{c}$ is $c_x = c \cos 120^\circ = (10.0 \text{ m}) \cos 120^\circ = -5.00 \text{ m}$.

(f) The y component is $c_y = c \sin 120^\circ = (10.0 \text{ m}) \sin 120^\circ = 8.66 \text{ m}$.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ indicates

$$\vec{c} = c_x \hat{i} + c_y \hat{j} = p(a_x \hat{i}) + q(b_x \hat{i} + b_y \hat{j}) = (pa_x + qb_x) \hat{i} + qb_y \hat{j}$$

or

$$c_x = pa_x + qb_x, \quad c_y = qb_y.$$

Substituting the values found above, we have

$$\begin{aligned} -5.00 \text{ m} &= p(3.00 \text{ m}) + q(3.46 \text{ m}) \\ 8.66 \text{ m} &= q(2.00 \text{ m}). \end{aligned}$$

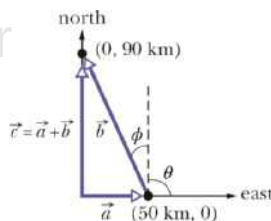
Solving these equations, we find $p = -6.67$.

(h) Similarly, $q = 4.33$ (note that it's easier to solve for q first). The numbers p and q have no units.

LEARN This exercise shows that given two nonparallel vectors in two dimensions, the third vector can always be written as a linear combination of the first two.

49. **THINK** This problem deals with the displacement of a sailboat. We want to find the displacement vector between two locations.

EXPRESS The situation is depicted in this next figure. Let $\vec{a}$ represent the first part of his actual voyage (50.0 km east) and $\vec{c}$ represent the intended voyage (90.0 km north). We look for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.



ANALYZE (a) Using the Pythagorean theorem, we find that the distance traveled by the sailboat is

$$b = \sqrt{(50.0 \text{ km})^2 + (90.0 \text{ km})^2} = 103 \text{ km}.$$

(b) The direction is

$$\phi = \tan^{-1} \left(\frac{50.0 \text{ km}}{90.0 \text{ km}} \right) = 29.1^\circ$$

west of due north (which is equivalent to 60.9° north of due west).

LEARN This problem could also be solved by first expressing the vectors in unit-vector notation: $\vec{a} = (50.0 \text{ km})\hat{i}$, $\vec{c} = (90.0 \text{ km})\hat{j}$. This gives

$$\vec{b} = \vec{c} - \vec{a} = -(50.0 \text{ km})\hat{i} + (90.0 \text{ km})\hat{j}.$$

The angle between $\vec{b}$ and the $+x$ direction is

$$\theta = \tan^{-1} \left(\frac{90.0 \text{ km}}{-50.0 \text{ km}} \right) = 119.1^\circ.$$

The angle θ is related to ϕ by $\theta = 90^\circ + \phi$.

53. **THINK** This problem involves finding scalar and vector products between two vectors $\vec{a}$ and $\vec{b}$.

EXPRESS We apply Eqs. 3.3.1 and 3.3.8 to calculate the scalar product and the magnitude of the vector product of two vectors:

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

$$|\vec{a} \times \vec{b}| = ab \sin \phi.$$

ANALYZE (a) Given that $a = |\vec{a}| = 10$, $b = |\vec{b}| = 6.0$, and $\phi = 60^\circ$, the scalar (dot) product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \cdot \vec{b} = ab \cos \phi = (10)(6.0) \cos 60^\circ = 30.$$

(b) Similarly, the magnitude of the vector (cross) product of the two vectors is

$$|\vec{a} \times \vec{b}| = ab \sin \phi = (10)(6.0) \sin 60^\circ = 52.$$

LEARN When two vectors $\vec{a}$ and $\vec{b}$ are parallel ($\phi = 0$), their scalar and vector products are $\vec{a} \cdot \vec{b} = ab \cos \phi = ab$ and $|\vec{a} \times \vec{b}| = ab \sin \phi = 0$, respectively. However, when they are perpendicular ($\phi = 90^\circ$), we have $\vec{a} \cdot \vec{b} = ab \cos \phi = 0$ and $|\vec{a} \times \vec{b}| = ab \sin \phi = ab$.

54. From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $\vec{a} \cdot \vec{b} = 0$ since the angle between them is 90°

(b) $\vec{a} \cdot \vec{c} = \vec{a} \cdot (-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$

(c) Similarly, $\vec{b} \cdot \vec{c} = -9.0$.

57. **THINK** This problem deals with the addition and subtraction of two vectors.

EXPRESS From the problem statement, we have

$$\vec{A} + \vec{B} = (6.0)\hat{i} + (1.0)\hat{j}$$

$$\vec{A} - \vec{B} = -(4.0)\hat{i} + (7.0)\hat{j}.$$

Solving the simultaneous equations gives $\vec{A}$ and $\vec{B}$.

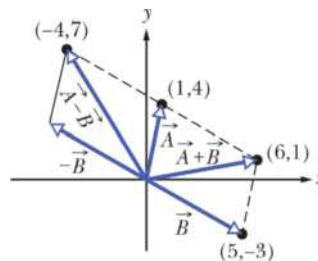
ANALYZE Adding the above equations and dividing by 2 leads to $\vec{A} = (1.0)\hat{i} + (4.0)\hat{j}$. The magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.0)^2 + (4.0)^2} = 4.1.$$

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LEARN The vector $\vec{B}$ is $\vec{B} = (5.0)\hat{i} + (-3.0)\hat{j}$, and its magnitude is

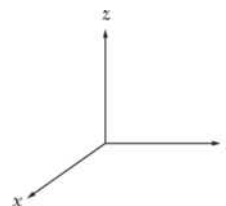
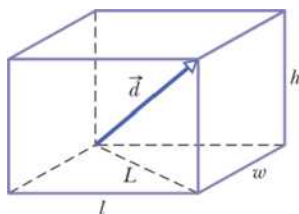
$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.0)^2 + (-3.0)^2} = 5.8.$$



The results are summarized in the figure.

64. **THINK** This problem deals with the displacement and distance traveled by a fly from one corner of a room to the diagonally opposite corner. The displacement vector is three-dimensional.

EXPRESS The displacement $\vec{d}$ of the fly is illustrated in the figure on the left.



The coordinate system shown on the right allows us to express the displacement as a three-dimensional vector.

ANALYZE (a) The magnitude of the displacement from one corner to the diagonally opposite corner is

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2}.$$

Substituting the values given, we obtain

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2} = \sqrt{(3.70 \text{ m})^2 + (4.30 \text{ m})^2 + (3.00 \text{ m})^2} = 6.42 \text{ m}.$$

(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than d , the magnitude of the displacement.

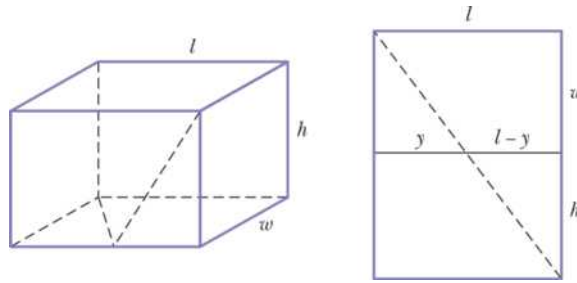
(c) The length of the path of the fly can be greater than d , however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0 \text{ m}$.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward (as shown in the figure above). Then the x component of the displacement is $w = 3.70$ m, the y component of the displacement is 4.30 m, and the z component is 3.00 m. Thus, the displacement vector can be written as

$$\vec{d} = (3.70 \text{ m})\hat{i} + (4.30 \text{ m})\hat{j} + (3.00 \text{ m})\hat{k}.$$

(f) Suppose the path of the fly is as shown by the dashed lines on the following diagram on the left. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the right.



The shortest walking distance between the lower left back corner of the room and the upper right front corner is the dashed straight line shown on the diagram. Its length is

$$s_{\min} = \sqrt{(w+h)^2 + l^2} = \sqrt{(3.70 \text{ m} + 3.00 \text{ m})^2 + (4.30 \text{ m})^2} = 7.96 \text{ m}.$$

LEARN To show that the shortest path is indeed given by $s_{\min}$, we again use the figure on the right and write the length of the path as

$$s = \sqrt{y^2 + w^2} + \sqrt{(l-y)^2 + h^2}.$$

The condition for minimum is given by

$$\frac{ds}{dy} = \frac{y}{\sqrt{y^2 + w^2}} - \frac{l-y}{\sqrt{(l-y)^2 + h^2}} = 0.$$

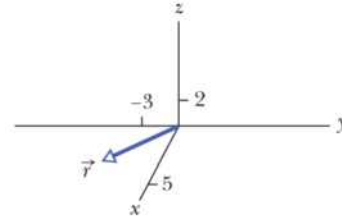
A little algebra shows that the condition is satisfied when $y = lw / (w + h)$, which gives

$$s_{\min} = \sqrt{w^2 \left(1 + \frac{l^2}{(w+h)^2} \right)} + \sqrt{h^2 \left(1 + \frac{l^2}{(w+h)^2} \right)} = \sqrt{(w+h)^2 + l^2}.$$

Any other path would be longer than 7.96 m.

Chapter 4

5. **THINK** This problem deals with the motion of a train in two dimensions. The entire trip consists of three parts, and we're interested in the overall average velocity.



EXPRESS The average velocity of the entire trip is given by Eq. 4.2.1, $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$, where the total displacement $\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$ is the sum of three displacements (each a result of a constant velocity during a given time), and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. We use a coordinate system with $+x$ for east and $+y$ for north.

ANALYZE (a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0 \frac{\text{km}}{\text{h}}\right) \left(\frac{40.0 \text{ min}}{60 \text{ min/h}}\right) \hat{i} = (40.0 \text{ km})\hat{i}.$$

The second displacement has a magnitude of $(60.0 \frac{\text{km}}{\text{h}}) (\frac{20.0 \text{ min}}{60 \text{ min/h}}) = 20.0 \text{ km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0 \text{ km}) \cos(40.0^\circ) \hat{i} + (20.0 \text{ km}) \sin(40.0^\circ) \hat{j} = (15.3 \text{ km})\hat{i} + (12.9 \text{ km})\hat{j}.$$

Similarly, the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0 \frac{\text{km}}{\text{h}}\right) \left(\frac{50.0 \text{ min}}{60 \text{ min/h}}\right) \hat{i} = (-50.0 \text{ km})\hat{i}.$$

Thus, the total displacement is

$$\begin{aligned} \Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0 \text{ km})\hat{i} + (15.3 \text{ km})\hat{i} + (12.9 \text{ km})\hat{j} - (50.0 \text{ km})\hat{i} \\ &= (5.30 \text{ km})\hat{i} + (12.9 \text{ km})\hat{j}. \end{aligned}$$

The time for the trip is

$$\Delta t = (40.0 + 20.0 + 50.0) \text{ min} = 110 \text{ min},$$

which is equivalent to 1.83 h. Equation 4.2.1 then yields

$$\vec{v}_{\text{avg}} = \frac{(5.30 \text{ km})\hat{i} + (12.9 \text{ km})\hat{j}}{1.83 \text{ h}} = (2.90 \text{ km/h})\hat{i} + (7.01 \text{ km/h})\hat{j}.$$

The magnitude of $\vec{v}_{\text{avg}}$ is

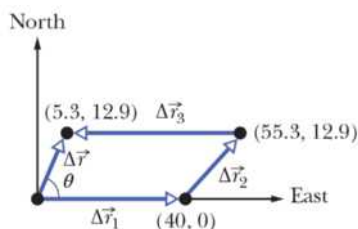
$$|\vec{v}_{\text{avg}}| = \sqrt{(2.90 \text{ km/h})^2 + (7.01 \text{ km/h})^2} = 7.59 \text{ km/h}.$$

(b) The angle is

$$\theta = \tan^{-1} \left(\frac{v_{\text{avg},y}}{v_{\text{avg},x}} \right) = \tan^{-1} \left(\frac{7.01 \text{ km/h}}{2.90 \text{ km/h}} \right) = 67.5^\circ \text{ (north of east),}$$

or 22.5° east of due north.

LEARN The displacement of the train is depicted in the figure below:



Note that the net displacement $\Delta \vec{r}$ is found by vectorially adding $\Delta \vec{r}_1$, $\Delta \vec{r}_2$, and $\Delta \vec{r}_3$.

13. THINK Knowing the position of a particle as a function of time allows us to calculate the particle's velocity and acceleration by taking time derivatives.

EXPRESS From the position vector $\vec{r}(t)$, the velocity and acceleration of the particle can be found by differentiating $\vec{r}(t)$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}.$$

ANALYZE (a) Taking the derivative of the position vector $\vec{r}(t) = \hat{i} + 4t^2\hat{j} + t\hat{k}$ with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{i} + 4t^2\hat{j} + t\hat{k}) = 8t\hat{j} + \hat{k}.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s²),

$$\vec{a} = \frac{d}{dt}(8t\hat{j} + \hat{k}) = 8\hat{j}.$$

LEARN The particle undergoes constant acceleration in the $+y$ direction. This can be seen by noting that the y component of $\vec{r}(t)$ is $4t^2$, which is quadratic in t .

15. **THINK** Given the initial velocity and acceleration of a particle, we're interested in finding its velocity and position at a later time.

EXPRESS Since the acceleration,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = (-1.0 \text{ m/s}^2) \hat{i} + (-0.50 \text{ m/s}^2) \hat{j},$$

is constant along both x and y axes, we may use Table 2.4.1 for the motion along each axis. This can be handled individually (for x and y) or together with unit-vector notation (for $\Delta \vec{r}$).

Since the particle started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$. The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a} t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. Along the x axis, we have

$$x(t) = v_{0x} t + \frac{1}{2} a_x t^2, \quad v_x(t) = v_{0x} + a_x t.$$

Similarly, along the y axis, we get

$$y(t) = v_{0y} t + \frac{1}{2} a_y t^2, \quad v_y(t) = v_{0y} + a_y t.$$

Known: $v_{0x} = 3.0 \text{ m/s}$, $v_{0y} = 0$, $a_x = -1.0 \text{ m/s}^2$, $a_y = -0.5 \text{ m/s}^2$

ANALYZE (a) Substituting these values, we find that the components of the velocity are

$$\begin{aligned} v_x(t) &= v_{0x} + a_x t = (3.0 \text{ m/s}) - (1.0 \text{ m/s}^2) t \\ v_y(t) &= v_{0y} + a_y t = -(0.50 \text{ m/s}^2) t. \end{aligned}$$

When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.0 - 1.0 t_m = 0$ or $t_m = 3.0 \text{ s}$. The y component of the velocity at this time is

$$v_y = -(0.50 \text{ m/s}^2)(3.0) = -1.5 \text{ m/s}.$$

Thus, $\vec{v}_m = (-1.5 \text{ m/s}) \hat{j}$.

(b) At $t = 3.0$ s, the components of the position are

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (3.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-1.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 4.50 \text{ m}$$

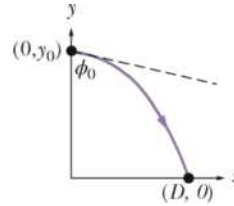
$$y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5 \text{ m/s}^2)(3.0 \text{ s})^2 = -2.25 \text{ m}.$$

Using unit-vector notation, we can write the results as $\vec{r}_m = (4.50 \text{ m})\hat{i} - (2.25 \text{ m})\hat{j}$.

LEARN The motion of the particle in this problem is two-dimensional, and the kinematics in the x and y directions can be analyzed separately.

33. **THINK** This problem deals with projectile motion. We're interested in the horizontal displacement and velocity of the projectile before it strikes the ground.

EXPRESS We adopt the positive direction choices used in the textbook, so equations such as Eq. 4.4.4 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0^\circ$ for the angle measured from the $+x$ direction, since the angle $\phi_0 = 53.0^\circ$ given in the problem is measured from the $-y$ direction. The initial setup of the problem is shown in the figure (not to scale).



ANALYZE (a) The initial speed of the projectile is the plane's speed at the moment of release. Given that $y_0 = 730$ m and $y = 0$ at $t = 5.00$ s, we use Eq. 4.4.4 to find v_0 :

$$y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2$$

$$0 - 730 \text{ m} = v_0 \sin(-37.0^\circ)(5.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(5.00 \text{ s})^2,$$

which yields $v_0 = 202$ m/s.

(b) The horizontal distance traveled is

$$D = v_x t = (v_0 \cos \theta_0)t = [(202 \text{ m/s})\cos(-37.0^\circ)](5.00 \text{ s}) = 806 \text{ m}.$$

(c) The x component of the velocity (just before impact) is

$$v_x = v_0 \cos \theta_0 = (202 \text{ m/s})\cos(-37.0^\circ) = 161 \text{ m/s}.$$

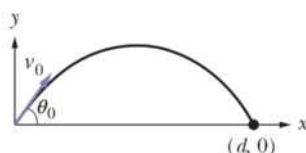
(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin \theta_0 - gt = (202 \text{ m/s}) \sin(-37.0^\circ) - (9.80 \text{ m/s}^2)(5.00 \text{ s}) = -171 \text{ m/s}.$$

LEARN In this projectile problem, we analyzed the kinematics in the vertical and horizontal directions separately since they do not affect each other. The x component of the velocity, $v_x = v_0 \cos \theta_0$, remains unchanged throughout since there's no horizontal acceleration during the flight.

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook, so equations such as Eq. 4.4.4 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of Module 4.5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$.



The projectile-motion equations lead to

$$d = (v_0 \cos \theta_0)t, \quad 0 = v_0 t \sin \theta_0 - \frac{1}{2}gt^2,$$

where θ_0 is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

ANALYZE The time at which the bullet strikes the target is given by $t = d/(v_0 \cos \theta_0)$. Eliminating t leads to $2v_0^2 \sin \theta_0 \cos \theta_0 - gd = 0$. Using $\sin \theta_0 \cos \theta_0 = \frac{1}{2} \sin(2\theta_0)$, we obtain

$$v_0^2 \sin(2\theta_0) = gd$$

$$\sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80 \text{ m/s}^2)(45.7 \text{ m})}{(460 \text{ m/s})^2},$$

which yields $\sin(2\theta_0) = 2.11 \times 10^{-3}$, or $\theta_0 = 0.0606^\circ$. If the gun is aimed at a point a distance ℓ above the target, then $\tan \theta_0 = \ell/d$ and

$$\ell = d \tan \theta_0 = (45.7 \text{ m}) \tan(0.0606^\circ) = 0.0484 \text{ m} = 4.84 \text{ cm}.$$

LEARN Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

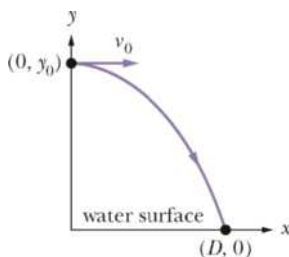
37. **THINK** The trajectory of the diver is a projectile motion. We are interested in the displacement of the diver at a later time.

EXPRESS The initial velocity has no vertical component ($\theta_0 = 0$), but only an x component. Equations 4.4.3 and 4.4.4 can be simplified to

$$x - x_0 = v_{0x}t$$

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2,$$

where $x_0 = 0$, $v_{0x} = v_0 = +2.0$ m/s, and $y_0 = +10.0$ m (taking the water surface to be at $y = 0$). The setup of the problem is shown in the figure.



ANALYZE (a) At $t = 0.80$ s, the horizontal distance of the diver from the edge is

$$x = x_0 + v_{0x}t = 0 + (2.0 \text{ m/s})(0.80 \text{ s}) = 1.60 \text{ m}.$$

(b) Similarly, using the second equation for the vertical motion, we obtain

$$y = y_0 - \frac{1}{2}gt^2 = 10.0 \text{ m} - \frac{1}{2}(9.80 \text{ m/s}^2)(0.80 \text{ s})^2 = 6.86 \text{ m}.$$

(c) At the instant the diver strikes the water surface, $y = 0$. Solving for t with the equation $y = y_0 - \frac{1}{2}gt^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0 \text{ m})}{9.80 \text{ m/s}^2}} = 1.43 \text{ s}.$$

During this time, the x displacement of the diver is $D = x = (2.00 \text{ m/s})(1.43 \text{ s}) = 2.86 \text{ m}$.

LEARN Using Eq. 4.4.7 with $\theta_0 = 0$, we see that the trajectory of the diver can also be written as

$$y = y_0 - \frac{gx^2}{2v_0^2}.$$

Part (c) can also be solved by setting $y = 0$:

$$y = y_0 - \frac{gx^2}{2v_0^2} = 0$$

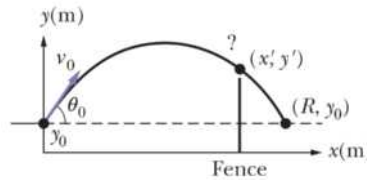
$$x = D = \sqrt{\frac{2v_0^2 y_0}{g}} = \sqrt{\frac{2(2.0 \text{ m/s})^2 (10.0 \text{ m})}{9.8 \text{ m/s}^2}} = 2.86 \text{ m}.$$

47. THINK The baseball undergoes projectile motion after being hit by the batter. We want to know if the ball clears a high fence at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook, so equations such as Eq. 4.4.4 are directly applicable. The coordinate origin is at ground level directly below the impact point between bat and ball. In the absence of a fence, with $\theta_0 = 45^\circ$, the range is $R = 107 \text{ m}$ (the ball returns to its launch height). We want to know how high the ball is when it is at $x' = 97.5 \text{ m}$, which requires knowing the initial velocity. The trajectory of the baseball can be described by Eq. 4.4.7:

$$y - y_0 = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}.$$

The setup of the problem is shown in the figure below (not to scale).



ANALYZE (a) We first solve for the initial speed v_0 . Using the range information ($y = y_0$ when $x = R$) and $\theta_0 = 45^\circ$, we find that Eq. 4.4.7 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(107 \text{ m})}{\sin(90^\circ)}} = 32.4 \text{ m/s}.$$

Thus, to find the time when the ball flies over the fence we write:

$$x' = (v_0 \cos \theta_0) t'$$

$$t' = \frac{x'}{v_0 \cos \theta_0} = \frac{97.5 \text{ m}}{(32.4 \text{ m/s}) \cos 45^\circ} = 4.26 \text{ s}.$$

At this moment, the ball is at a height of

$$y' = y_0 + (v_0 \sin \theta_0) t' - \frac{1}{2} g t'^2$$

$$= 1.22 \text{ m} + [(32.4 \text{ m/s}) \sin 45^\circ](4.26 \text{ s}) - \frac{1}{2} (9.8 \text{ m/s}^2)(4.26 \text{ s})^2$$

$$= 9.88 \text{ m},$$

which indicates that it does clear the 7.32 m high fence.

(b) At $t' = 4.26 \text{ s}$, the center of the ball is $9.88 \text{ m} - 7.32 \text{ m} = 2.56 \text{ m}$ above the fence.

LEARN Using the trajectory equation, we can show that the minimum initial velocity required to clear the fence is about 31.9 m/s.

49. **THINK** In this problem a football is given an initial speed and undergoes projectile motion. We want to know the smallest and greatest angles at which a field goal can be scored.

EXPRESS We adopt the positive direction choices used in the textbook, so equations such as Eq. 4.4.4 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of the ball at the goalpost and try to find the kicking angles θ_0 such that $y = 3.44 \text{ m}$ when $x = 50 \text{ m}$. Writing the kinematics equations for projectile motion,

$$x = v_0 \cos \theta_0 t, \quad y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2,$$

we see the first equation gives $t = x/(v_0 \cos \theta_0)$, and when this is substituted into the second equation, the result is

$$y = x \tan \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta_0}.$$

ANALYZE We might solve this equation by trial and error, systematically trying values of θ_0 until we find the two angles that satisfy the equation. A little manipulation, however, will give an algebraic solution. Using the trigonometric identity

$$1/\cos^2 \theta_0 = 1 + \tan^2 \theta_0,$$

we obtain

$$\frac{1}{2} \frac{gx^2}{v_0^2} \tan^2 \theta_0 - x \tan \theta_0 + y + \frac{1}{2} \frac{gx^2}{v_0^2} = 0,$$

which is a quadratic equation for the variable $\tan \theta_0$. To simplify writing the solution, we denote

$$c = \frac{1}{2} gx^2/v_0^2 = \frac{1}{2} (9.80 \text{ m/s}^2)(50 \text{ m})^2/(25 \text{ m/s})^2 = 19.6 \text{ m}.$$

Then the quadratic equation becomes

$$c \tan^2 \theta_0 - x \tan \theta_0 + y + c = 0.$$

Using the general quadratic solution, we obtain

$$\tan \theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50 \text{ m} \pm \sqrt{(50 \text{ m})^2 - 4(3.44 \text{ m} + 19.6 \text{ m})(19.6 \text{ m})}}{2(19.6 \text{ m})}.$$

The solutions are $\tan \theta_0 = 1.95$ and $\tan \theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63^\circ$ and $\theta_0 = 31^\circ$. Thus,

(a) the smallest elevation angle is $\theta_0 = 31^\circ$, and

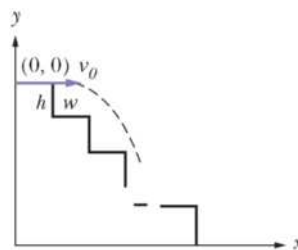
(b) the greatest elevation angle is $\theta_0 = 63^\circ$.

LEARN If kicked at any angle between 31° and 63° , the ball will travel above the crossbar on the goalposts.

55. THINK In this problem a ball rolls off the top of a stairway with an initial speed, and we want to know on which step it lands first.

EXPRESS We denote h as the height of a step and w as the width. To hit step n , the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw . We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction, as shown in the figure.

The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$).



ANALYZE We equate y to $-nh$ and solve for the time to reach the level of step n :

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

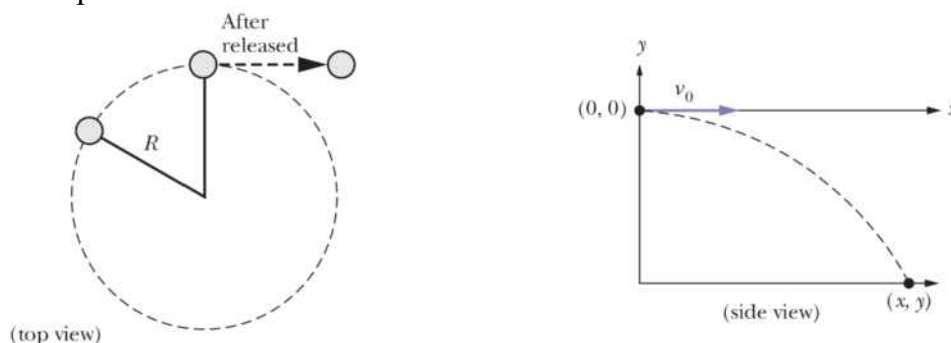
$$x = v_{0x} \sqrt{\frac{2nh}{g}} = (1.52 \text{ m/s}) \sqrt{\frac{2n(0.203 \text{ m})}{9.8 \text{ m/s}^2}} = (0.309 \text{ m}) \sqrt{n}.$$

Let's try values of n until we find one for which x/w is less than n but greater than $n - 1$. For $n = 1$, $x = 0.309 \text{ m}$, and $x/w = 1.52$, which is greater than n . For $n = 2$, $x = 0.437 \text{ m}$, and $x/w = 2.15$, which is also greater than n . For $n = 3$, $x = 0.535 \text{ m}$, and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

LEARN To check the consistency of our calculation, we can substitute $n = 3$ into our equations. The results are $t = 0.353 \text{ s}$, $y = 0.609 \text{ m}$, and $x = 0.535 \text{ m}$. This indeed corresponds to the third step.

67. **THINK** In this problem we have a stone whirled in a horizontal circle. After the string breaks, the stone undergoes projectile motion.

EXPRESS The stone moves in a circular path (overhead view shown in the figure) initially but undergoes projectile motion after the string breaks (side view in the figure). Since $a = v^2/R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematics equations of projectile motion (discussed in Module 4.6) to find that speed.



Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, we find that the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2} g t^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10 \text{ m}$ and $y = -2.0 \text{ m}$.

ANALYZE Solving the equation involving y for the stone's flight time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x \sqrt{-\frac{g}{2y}} = (10 \text{ m}) \sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

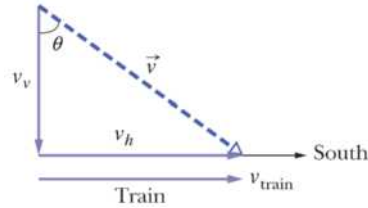
Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$

LEARN The equations can be combined to give $a = \frac{gx^2}{-2yR}$, which indicates that the greater the centripetal acceleration, the greater the initial speed of the projectile and the greater the distance traveled by the stone. This is precisely what we expect.

75. **THINK** This problem deals with relative motion in two dimensions. Raindrops appear to fall vertically by an observer on a moving train.

EXPRESS Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop, $v_h = 30 \text{ m/s}$, must be the same as the speed of the train, that is $v_h = v_{\text{train}}$ (see the figure).



On the other hand, if v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then $\tan \theta = v_h/v_v$. Knowing v_v and v_h allows us to determine the speed of the raindrops.

ANALYZE With $\theta = 70^\circ$, we find the vertical component of the velocity to be

$$v_v = v_h/\tan \theta = (30 \text{ m/s})/\tan 70^\circ = 10.9 \text{ m/s}.$$

Therefore, the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

LEARN As long as the horizontal component of the velocity of the raindrops matches the speed of the train, the passenger on board will see the rain falling perfectly vertically.

77. THINK This problem deals with relative motion in two dimensions. Snowflakes falling vertically downward are seen to fall at an angle by a moving observer.

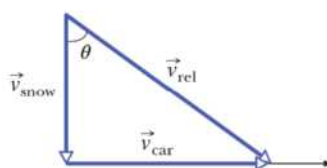
EXPRESS Relative to the car, the velocity of the snowflakes has a vertical component of $v_v = 8.0 \text{ m/s}$ and a horizontal component of $v_h = 50 \text{ km/h} = 13.9 \text{ m/s}$.

ANALYZE The angle θ from the vertical is found from

$$\tan \theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m/s}}{8.0 \text{ m/s}} = 1.74,$$

which yields $\theta = 60^\circ$.

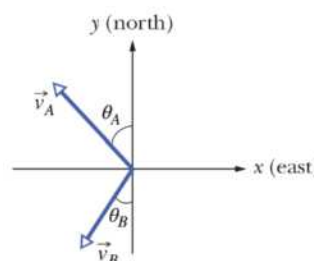
LEARN The problem can also be solved by expressing the velocity relation in vector notation: $\vec{v}_{\text{rel}} = \vec{v}_{\text{car}} + \vec{v}_{\text{snow}}$, as in the figure.



79. THINK This problem involves analyzing the relative motion of two ships sailing in different directions.

EXPRESS Given that $\theta_A = 45^\circ$ and $\theta_B = 40^\circ$, as defined in the figure, the velocity vectors (relative to the shore) for ships A and B are given by

$$\begin{aligned}\vec{v}_A &= -(v_A \cos 45^\circ)\hat{i} + (v_A \sin 45^\circ)\hat{j} \\ \vec{v}_B &= -(v_B \sin 40^\circ)\hat{i} - (v_B \cos 40^\circ)\hat{j},\end{aligned}$$



with $v_A = 24 \text{ knots}$ and $v_B = 28 \text{ knots}$. We take east as the $+\hat{i}$ direction and north as the $+\hat{j}$ direction.

The velocity of ship A relative to ship B is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

ANALYZE (a) The relative velocity is

$$\begin{aligned}\vec{v}_{AB} &= \vec{v}_A - \vec{v}_B = (v_B \sin 40^\circ - v_A \cos 45^\circ)\hat{i} + (v_B \cos 40^\circ + v_A \sin 45^\circ)\hat{j} \\ &= (1.03 \text{ knots})\hat{i} + (38.4 \text{ knots})\hat{j}.\end{aligned}$$

The magnitude is

$$|\vec{v}_{AB}| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38.4 \text{ knots}.$$

(b) The angle θ_{AB} that $\vec{v}_{AB}$ makes with north is given by

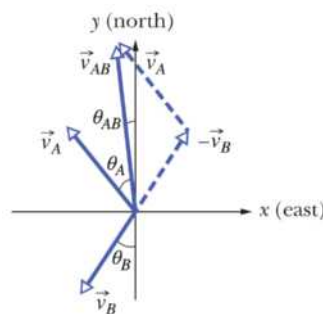
$$\theta_{AB} = \tan^{-1} \left(\frac{v_{AB,x}}{v_{AB,y}} \right) = \tan^{-1} \left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}} \right) = 1.5^\circ,$$

which is to say that $\vec{v}_{AB}$ points 1.5° east of north.

(c) Since the two ships started at the same time, their relative velocity is the rate that the distance between them increases. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160 \text{ nautical miles}}{38.4 \text{ knots}} = 4.2 \text{ h}.$$

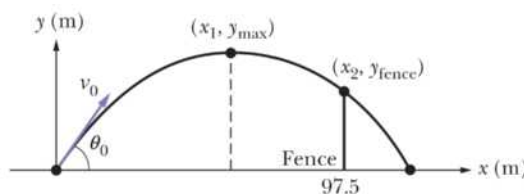
(d) The velocity $\vec{v}_{AB}$ does not change with time, and $\vec{r}_{AB}$ is in the same direction as $\vec{v}_{AB}$ since the ships started at the same time. Reversing the points of view, we have $\vec{v}_{AB} = -\vec{v}_{BA}$ and $\vec{r}_{AB} = -\vec{r}_{BA}$ (i.e., they are opposite to each other). Hence, we conclude that B stays at a bearing of 1.5° west of south relative to A during the journey (neglecting the curvature of Earth).



LEARN The relative velocity is depicted in the figure. When analyzing relative motion in two dimensions, such a vector diagram can be very helpful.

87. **THINK** This problem deals with the projectile motion of a baseball. Given the information on the position of the ball at two instants, we are asked to analyze its trajectory.

EXPRESS The trajectory of the baseball is shown in the figure. According to the problem statement, at $t_1 = 3.0 \text{ s}$, the ball reaches its maximum height $y_{\max}$, and at $t_2 = t_1 + 2.5 \text{ s} = 5.5 \text{ s}$, it barely clears a fence at $x_2 = 97.5 \text{ m}$.



Equation 2.4.5 can be applied to the vertical (y axis) motion related to reaching the maximum height (when $t_1 = 3.0 \text{ s}$ and $v_y = 0$):

$$y_{\max} - y_0 = v_y t - \frac{1}{2} g t^2.$$

ANALYZE (a) With ground level chosen so that $y_0 = 0$, this equation gives the result

$$y_{\max} = \frac{1}{2} g t_1^2 = \frac{1}{2} (9.8 \text{ m/s}^2) (3.0 \text{ s})^2 = 44.1 \text{ m}.$$

(b) After the moment it reached maximum height, it is falling; at $t_2 = t_1 + 2.5 \text{ s} = 5.5 \text{ s}$, it has fallen through a displacement given by Eq. 2.4.8:

$$y_{\text{fence}} - y_{\max} = 0 - \frac{1}{2} g (t_2 - t_1)^2.$$

Thus, the height of the fence is

$$y_{\text{fence}} = y_{\max} - \frac{1}{2} g (t_2 - t_1)^2 = 44.1 \text{ m} - \frac{1}{2} (9.8 \text{ m/s}^2) (2.5 \text{ s})^2 = 13.48 \text{ m}.$$

(c) Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find from $97.5 \text{ m} = v_{0x}(5.5 \text{ s})$ that $v_{0x} = 17.7 \text{ m/s}$. The total flight time of the ball is $T = 2t_1 = 2(3.0 \text{ s}) = 6.0 \text{ s}$. Thus, the range of the baseball is

$$R = v_{0x} T = (17.7 \text{ m/s})(6.0 \text{ s}) = 106.4 \text{ m},$$

which means that the ball travels an additional distance

$$\Delta x = R - x_2 = 106.4 \text{ m} - 97.5 \text{ m} = 8.86 \text{ m}$$

beyond the fence before striking the ground.

LEARN Part (c) can also be solved by noting that after passing the fence, the ball will strike the ground in 0.5 s (so the total “fall-time” equals the “rise-time”). With $v_{0x} = 17.7 \text{ m/s}$, we have $\Delta x = (17.7 \text{ m/s})(0.5 \text{ s}) = 8.86 \text{ m}$.

89. THINK We have a particle moving in a two-dimensional plane with a constant acceleration. Since the x and y components of the acceleration are constant, we can use Table 2.4.1 for the motion along both axes.

EXPRESS Using vector notation with $\vec{r}_0 = 0$, the position and velocity of the particle as a function of time are given by $\vec{r}(t) = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$ and $\vec{v}(t) = \vec{v}_0 + \vec{a} t$. Where units are not shown, SI units are to be understood.

ANALYZE (a) Given the initial velocity $\vec{v}_0 = (8.0 \text{ m/s})\hat{j}$ and the acceleration $\vec{a} = (4.0 \text{ m/s}^2)\hat{i} + (2.0 \text{ m/s}^2)\hat{j}$, the position vector of the particle is

$$\vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 = (8.0\hat{j})t + \frac{1}{2}(4.0\hat{i} + 2.0\hat{j})t^2 = (2.0t^2)\hat{i} + (8.0t + 1.0t^2)\hat{j}.$$

Therefore, the time that corresponds to $x = 29 \text{ m}$ can be found by solving the equation $2.0t^2 = 29$, which leads to $t = 3.8 \text{ s}$. The y coordinate at that time is

$$y = (8.0 \text{ m/s})(3.8 \text{ s}) + (1.0 \text{ m/s}^2)(3.8 \text{ s})^2 = 45 \text{ m}.$$

(b) The velocity of the particle is given by $\vec{v} = \vec{v}_0 + \vec{a}t$. Thus, at $t = 3.8 \text{ s}$, the velocity is

$$\vec{v} = (8.0 \text{ m/s})\hat{j} + ((4.0 \text{ m/s}^2)\hat{i} + (2.0 \text{ m/s}^2)\hat{j})(3.8 \text{ s}) = (15.2 \text{ m/s})\hat{i} + (15.6 \text{ m/s})\hat{j},$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2 \text{ m/s})^2 + (15.6 \text{ m/s})^2} = 22 \text{ m/s}.$$

LEARN Instead of using vector notation, we can also deal with the x and the y components individually.

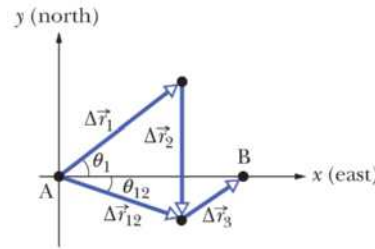
93. **THINK** This problem deals with the two-dimensional kinematics of a desert camel moving from oasis A to oasis B.

EXPRESS The journey of the camel is illustrated in the figure. We use the coordinate system shown there. Lengths are in kilometers and times are in hours. Using vector notation, we write the displacements for the first two segments of the trip as

$$\Delta\vec{r}_1 = (75 \text{ km})\cos(37^\circ)\hat{i} + (75 \text{ km})\sin(37^\circ)\hat{j}$$

$$\Delta\vec{r}_2 = (-65 \text{ km})\hat{j}.$$

The net displacement is $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2$. As can be seen from the figure, to reach oasis B requires an additional displacement $\Delta\vec{r}_3$.



ANALYZE (a) We add as *vectors* the individual displacements to find the net displacement of the camel: $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60 \text{ km})\hat{i} - (20 \text{ km})\hat{j}$. Its corresponding magnitude is

$$|\Delta\vec{r}_{12}| = \sqrt{(60 \text{ km})^2 + (-20 \text{ km})^2} = 63 \text{ km}.$$

(b) The direction of $\Delta\vec{r}_{12}$ is

$$\theta_{12} = \tan^{-1}[(-20 \text{ km})/(60 \text{ km})] = -18^\circ,$$

or 18° south of east.

(c) To calculate the average velocity for the first two segments of the journey (including the rest period), we use the result from part (a) in Eq. 4.2.1 along with the fact that

$$\Delta t_{12} = \Delta t_1 + \Delta t_2 + \Delta t_{\text{rest}} = 50 \text{ h} + 35 \text{ h} + 5.0 \text{ h} = 90 \text{ h}.$$

In unit-vector notation, we have

$$\vec{v}_{12, \text{avg}} = \frac{(60\hat{i} - 20\hat{j}) \text{ km}}{90 \text{ h}} = (0.67\hat{i} - 0.22\hat{j}) \text{ km/h}.$$

This leads to $|\vec{v}_{12, \text{avg}}| = 0.70 \text{ km/h}$.

(d) The direction of $\vec{v}_{12, \text{avg}}$ is

$$\theta_{12} = \tan^{-1}[(-0.22 \text{ km/h})/(0.67 \text{ km/h})] = -18^\circ,$$

or 18° south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain

$$(140 \text{ km})/(90 \text{ h}) = 1.56 \text{ km/h} \approx 1.6 \text{ km/h}.$$

(f) The required net displacement is the 90 km east from A to B . The displacement from the resting place to B is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90 \text{ km})\hat{i},$$

which produces $\Delta\vec{r}_3 = (30 \text{ km})\hat{i} + (20 \text{ km})\hat{j}$ in unit-vector notation, or $(36 \angle 33^\circ)$ in magnitude-angle notation. Therefore, using Eq. 4.2.1 we obtain

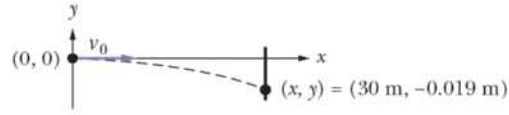
$$|\vec{v}_{3, \text{avg}}| = \frac{36 \text{ km}}{(120 - 90) \text{ h}} = 1.2 \text{ km/h}.$$

(g) The direction of $\vec{v}_{3,\text{avg}}$ is the same as $\Delta\vec{r}_3$ (i.e., 33° north of east).

LEARN With a vector-capable calculator in polar mode, we could perform the vector addition of the displacements as $(75 \angle 37^\circ) + (65 \angle -90^\circ) = (63 \angle -18^\circ)$. Note the distinction between average velocity and average speed.

97. **THINK** A bullet fired horizontally from a rifle strikes the target at some distance below its aiming point. We're asked to find its total flight time and speed.

EXPRESS The trajectory of the bullet is shown in the figure (not to scale). Note that the origin is chosen to be at the firing point. With this convention, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. Knowing the coordinates (x, y) at the target allows us to calculate the total flight time and speed of the bullet.



ANALYZE (a) If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-0.019 \text{ m})}{9.8 \text{ m/s}^2}} = 6.2 \times 10^{-2} \text{ s}.$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since $x = 30$ m is the horizontal position of the target, we have $x = v_0 t$. Thus,

$$v_0 = \frac{x}{t} = \frac{30 \text{ m}}{6.3 \times 10^{-2} \text{ s}} = 4.8 \times 10^2 \text{ m/s}.$$

LEARN Alternatively, we may use Eq. 4.4.7 to solve for the initial velocity. With $\theta_0 = 0$ and $y_0 = 0$, the equation simplifies to $y = -\frac{gx^2}{2v_0^2}$, from which we find

$$v_0 = \sqrt{-\frac{gx^2}{2y}} = \sqrt{-\frac{(9.8 \text{ m/s}^2)(30 \text{ m})^2}{2(-0.019 \text{ m})}} = 4.8 \times 10^2 \text{ m/s},$$

in agreement with what we calculated in part (b).