Section 1.1.1 Solutions

Reserve Problem 1.1.1R

"Drilling mud" is used in hydrocarbon drilling operations to carry debris to the surface and to both cool and lubricate the drill bit. It is frequently modeled as a Bingham plastic. Explain why a substance that behaves according to this model does not satisfy the definition of a fluid.

SOLUTION:

The relationship between the applied shear stress and the strain rate of a Bingham plastic can be obtained from many sources, including Section 1.6 of this text. Although it behaves like a Newtonian fluid for an applied shear stress larger than its yield stress, it exhibits a strain rate of zero (like a solid) if the applied shear stress is lower than the yield stress.

From Section 1.1 of this text: "A fluid is defined as a substance that deforms continuously when acted on by a **shearing stress of any magnitude**."

Because a Bingham plastic has a finite yield stress, its behavior does not satisfy the definition of a fluid. Q.E.D.

NOTE: Although the question did not require you to think about the implications of a finite yield stress, they can be significant. Consider the start-up of a drill surrounded by such a fluid. In general terms, what might the flow look like? Would it require more torque to start the drill than it would for a Newtonian fluid?

Reserve Problem 1.1.2R

Many lava flows are well modeled as the flow of a Bingham plastic. Explain why lava flowing in this manner does not satisfy the definition of a fluid. Describe the influence of cooling as the lava flows away from the eruption site.

SOLUTION:

The relationship between the applied shear stress and the strain rate of a Bingham plastic can be obtained from many sources, including Section 1.6 of this text. Although it behaves like a Newtonian fluid for an applied shear stress larger than its yield stress, it exhibits a strain rate of zero (like a solid) if the applied shear stress is lower than the yield stress.

From Section 1.1 of this text: "A fluid is defined as a substance that deforms continuously when acted on by a **shearing stress of any magnitude**."

Because a Bingham plastic has a finite yield stress, its behavior does not satisfy the definition of a fluid. Q.E.D

As the lava cools, its viscosity will increase
$\rightarrow$ reduced speed $\rightarrow$ reduced rate of shearing $\rightarrow$ may transition to a solid
$\rightarrow$ reduced speed $\rightarrow$ pileup of fluid $\rightarrow$ decreased slope $\rightarrow$ further reduced speed

As the lava cools, the temperature at the surface of the flow may fall to the solidification temperature resulting in the formation of a solid crust. This crust will slow the rate of cooling and present a second no-slip boundary, resulting in significant changes in the fluid mechanics and the heat transfer processes occurring in the flow.

NOTE: Once on the surface of the earth, lava flows are gravity driven. If the slope on which the lava sits is sufficiently steep, the shear stress due to gravity causes the lava to flow downhill. If the slope is not sufficiently steep, the lava builds into a mound until the surface of the mound is sufficiently steep to cause the lava flow outward until the surface is once again not sufficiently steep to induce flow.

Section 1.2 Solutions

Problem 1.2.1

The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

SOLUTION:

$$F = C_D \rho V^2 \frac{A}{2}$$

or

$$C_D = \frac{2F}{\rho V^2 A}, \text{ where}$$

$$F \doteq MLT^{-2}, \ \rho \doteq ML^{-3}, V \doteq LT^{-1}, A \doteq L^2$$

Thus,

$$C_D \doteq \frac{\left(MLT^{-2}\right)}{\left[\left(ML^{-3}\right)\left(LT^{-1}\right)^2\left(L^2\right)\right]} = M^0 L^0 T^0$$

Hence, $\boxed{C_D \text{ is dimensionless}}$.

Problem 1.2.2

The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\mathrm{Ma} = \frac{V}{\sqrt{kRT}}$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

SOLUTION:

We denote the dimension of temperature by θ and use Newton's second law to get $F = \frac{ML}{T^2}$. Then

$$[\mathrm{Ma}] = \frac{\left(\frac{L}{T}\right)}{\sqrt{(1)\left(\frac{FL}{M\theta}\right)\theta\left(\frac{ML}{T^2F}\right)}} = \frac{\left(\frac{L}{T}\right)}{\sqrt{\frac{L^2}{T^2}}}$$

or

$$\boxed{[\mathrm{Ma}] = [1]}$$

Problem 1.2.3

Verify the dimensions, in both the FLT and the MLT systems, of the given quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

SOLUTION:

(a) Volume $\doteq \boxed{L^3}$

(b) Acceleration = time rate of change of velocity $\doteq \dfrac{LT^{-1}}{T} \doteq \boxed{LT^{-2}}$

(c) Mass $\doteq \boxed{M}$

or with $F \doteq MLT^{-2}$

Mass $\doteq \boxed{FL^{-1}T^2}$

(d) Moment of inertia (area) = second moment of area $\doteq \left(L^2\right)\left(L^2\right) \doteq \boxed{L^4}$

(e) Work = force $\times$ distance $\doteq \boxed{FL}$

or with $F \doteq MLT^{-2}$

Work $\doteq \boxed{ML^2T^{-2}}$

Problem 1.2.4

Verify the dimensions, in both the FLT and the MLT systems, of the given quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

SOLUTION:

(a) $\text{Angular velocity} = \dfrac{\text{angular displacement}}{\text{time}} \doteq \boxed{T^{-1}}$

(b) Energy ~ capacity of body to do work

$\text{Single work} = \text{force} \times \text{distance} \rightarrow \text{energy} \doteq \boxed{FL}$

or with $F \doteq MLT^{-2} \rightarrow \text{energy} \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

(c) $\text{Moment of inertia } (\text{area}) = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \boxed{L^4}$

(d) $\text{Power} = \text{rate of doing work} \doteq \dfrac{FL}{T} \doteq FLT^{-1} \doteq \left(MLT^{-2}\right)(L)\left(T^{-1}\right) \doteq \boxed{ML^2T^{-3}}$

(e) $\text{Pressure} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq FL^{-2} \doteq \left(MLT^{-2}\right)\left(L^{-2}\right) \doteq \boxed{ML^{-1}T^{-2}}$

Problem 1.2.5

Verify the dimensions, in both the FLT system and the MLT system, of the given quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

SOLUTION:

(a) $\text{Frequency} = \dfrac{\text{cycles}}{\text{time}} \doteq \boxed{T^{-1}}$

(b) $\text{Stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \boxed{FL^{-2}}$

Since $F \doteq MLT^{-2}$

$\text{Stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \boxed{ML^{-1}T^{-2}}$

(c) $\text{Strain} = \dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \boxed{L^0 \ \ (\text{dimensionless})}$

(d) $\text{Torque} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

(e) $\text{Work} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

Problem 1.2.6

If u is velocity, x is length, and t is time, what are the dimensions (in the MLT system) of **(a)** $\partial u/\partial t$, **(b)** $\partial^2 u/\partial x \partial t$, and **(c)** $\int (\partial u/\partial t)dx$?

SOLUTION:

(a) $$\frac{\partial u}{\partial t} \doteq \frac{LT^{-1}}{T} \doteq \boxed{LT^{-2}}$$

(b) $$\frac{\partial^2 u}{\partial x \partial t} \doteq \frac{LT^{-1}}{(L)(T)} \doteq \boxed{T^{-2}}$$

(c) $$\int \frac{\partial u}{\partial t} \partial x \doteq \frac{\left(LT^{-1}\right)}{T}(L) \doteq \boxed{L^2T^{-2}}$$

Problem 1.2.7

Verify the dimensions, in both the FLT system and the MLT system, of the given quantities, which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

SOLUTION:

(a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \boxed{LT^{-2}}$

(b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \boxed{FL^{-2}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \boxed{ML^{-1}T^{-2}}$

(c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)L \doteq \boxed{ML^2T^{-2}}$

(d) $\text{volume} = (\text{length})^3 \doteq \boxed{L^3}$

(e) $\text{work} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)L \doteq \boxed{ML^2T^{-2}}$

Problem 1.2.8

If p is pressure, V is velocity, and ρ is fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ, **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

SOLUTION:

(a) $$\frac{p}{\rho} \doteq \frac{FL^{-2}}{ML^{-3}} = \frac{MLT^{-2}L^{-2}}{ML^{-3}} = \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \boxed{L^2T^{-2}}$$

(b) $$pV\rho \doteq \left(ML^{-1}T^{-2}\right)\left(LT^{-1}\right)\left(ML^{-3}\right) \doteq \boxed{M^2L^{-3}T^{-3}}$$

(c) $$\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{\left(ML^{-3}\right)\left(LT^{-1}\right)^2} \doteq \boxed{M^0L^0T^0\,(\text{dimensionless})}$$

Problem 1.2.9

If P is force and x is length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

SOLUTION:

(a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \boxed{FL^{-1}}$

(b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \boxed{FL^{-3}}$

(c) $\int P dx \doteq \boxed{FL}$.

Problem 1.2.10

If V is velocity, ℓ is length, and ν is a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the given combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, and **(d)** $V/\ell\nu$?

SOLUTION:

(a) $V\ell\nu \doteq \left(LT^{-1}\right)(L)\left(L^2T^{-1}\right) \doteq \boxed{L^4T^{-2}\,(\text{not dimensionless})}$

(b) $\dfrac{V\ell}{\nu} \doteq \dfrac{\left(LT^{-1}\right)(L)}{\left(L^2T^{-1}\right)} \doteq \boxed{L^0T^0\,(\text{dimensionless})}$

(c) $V^2\nu \doteq \left(LT^{-1}\right)^2\left(L^2T^{-1}\right) \doteq \boxed{L^4T^{-3}\,(\text{not dimensionless})}$

(d) $\dfrac{V}{\ell\nu} \doteq \dfrac{\left(LT^{-1}\right)}{(L)\left(L^2T^{-1}\right)} \doteq \boxed{L^{-2}\,(\text{not dimensionless})}$

Problem 1.2.11

The momentum flux is given by the product $\dot{m}V$, where $\dot{m}$ is mass flowrate and V is velocity. If mass flowrate is given in units of mass per unit time, show that the momentum flux can be expressed in units of force.

SOLUTION:

$$[\dot{m}V] \doteq \left(\frac{M}{T}\right)\left(\frac{L}{T}\right) \doteq M\frac{L}{T^2}\left[\frac{F\,T^2}{M\,L}\right] \doteq \boxed{F}$$

where $\dfrac{1}{g_c} \doteq \left[\dfrac{F\,T^2}{M\,L}\right]$ comes from Newton's Second Law.

Problem 1.2.12

An equation for the frictional pressure loss $\Delta p\ (\text{in. H}_2\text{O})$ in a circular duct of inside diameter $d(\text{in.})$ and length $L(\text{ft})$ for air flowing with velocity $V(\text{ft/min})$ is

$$\Delta p = 0.027\left(\frac{L}{d^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82}$$

where V_0 is a reference velocity equal to 1000 ft/min. Find the units of the "constant" 0.027.

SOLUTION:

Solving for the constant gives

$$0.027 = \frac{\Delta p_L}{\left(\frac{L}{D^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82}}$$

The units give

$$[0.027] \doteq \frac{(\text{in. H}_2\text{O})}{\left(\frac{\text{ft}}{\text{in.}^{1.22}}\right)\left(\frac{\frac{\text{ft}}{\text{min}}}{\frac{\text{ft}}{\text{min}}}\right)^{1.82}}$$

$$[0.027] \doteq \boxed{\frac{\text{in. H}_2\text{O}\cdot\text{in.}^{1.22}}{\text{ft}}}$$

Problem 1.2.13

The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ is a fluid property called viscosity $\left(FL^{-2}T\right)$, and ℓ is the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

SOLUTION:

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right] \frac{\left[L^4\right]\left[FL^{-2}\right]}{\left[FL^{-2}T\right][L]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

$$\boxed{\text{The constant } \frac{\pi}{8} \text{ is dimensionless.}}$$

Yes. This is a general homogeneous equation because it is valid in any consistent units system.

Problem 1.2.14

Show that each term in the given equation has units of lb/ft^3. Consider u as velocity, y as length, x as length, p as pressure, and μ as absolute viscosity.

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

SOLUTION:

$$\left[\frac{\partial p}{\partial x}\right] = \frac{\left[\dfrac{\text{lb}}{\text{ft}^2}\right]}{[\text{ft}]} \quad \text{or} \quad \boxed{\left[\frac{\partial p}{\partial x}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]}$$

and

$$\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right] \frac{\left[\dfrac{\text{ft}}{\text{s}}\right]}{\left[\text{ft}^2\right]} \quad \text{or} \quad \boxed{\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]}.$$

Problem 1.2.15

The pressure difference, Δp, across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ is the blood viscosity $(FL^{-2}T)$, ρ is the blood density $\left(ML^{-3}\right)$, D is the artery diameter, A_0 is the area of the unobstructed artery, and A_1 is the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

SOLUTION:

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1 \right]^2 \rho V^2$$

$$FL^{-2} \doteq [K_v] \frac{FT}{L^2} \frac{L}{T} \frac{1}{L} + [K_u] \left(\frac{L^2}{L^2} - 1 \right)^2 \left(\frac{FT^2}{L} \frac{1}{L^3} \right) \left(\frac{L}{T} \right)^2$$

$$FL^{-2} \doteq [K_v]\left(FL^{-2}\right) + [K_u]\left(FL^{-2}\right)$$

$\boxed{K_v \text{ and } K_u \text{ are dimensionless}}$ because each term in the equation must have the same dimensions. $\boxed{\text{Yes}}$, the equation would be valid in any consistent system of units.

Problem 1.2.16

Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a (\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See the equation $c = \sqrt{\frac{E_v}{\rho}}$.)

SOLUTION:

Substituting $[c] \doteq LT^{-1}$, $[E_v] \doteq FL^{-2}$, $[\rho] \doteq FL^{-4}T^2$ into the equation provided yields:

$$\left[LT^{-1}\right] = \left[\left(FL^{-2}\right)^a\right]\left[\left(FL^{-4}T^2\right)^b\right] = F^{a+b}L^{-2a-4b}T^{2b}$$

Dimensional homogeneity requires that the exponent of each dimension on both sides of the equal sign be the same.

F: 0 = a + b
L: 1 = −2a − 4b
T: −1 = 2b

Therefore,

T: −1 = 2b → b = −1/2
F: a = −b → a = 1/2
L: 1 = −2a−4b = −2(1/2) −4(−1/2) = 1✓

$$\boxed{a = \frac{1}{2}; \quad b = -\frac{1}{2}}$$

Yes, this is consistent with the standard formula for the speed of sound.

Problem 1.2.17

A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09\ BH^{\frac{3}{2}}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

SOLUTION:

$$Q = 3.09\ BH^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L][L]^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L]^{\frac{5}{2}}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{1/2}\ T^{-1}$ and is, therefore, not dimensionless. No.
Since the constant has dimensions its value will change with a change in units.
No.

Problem 1.2.18

A commercial advertisement shows a pearl falling in a bottle of shampoo. If the diameter D of the pearl is quite small and the shampoo is sufficiently viscous, the drag $\mathcal{D}$ on the pearl is given by Stokes's law,

$$\mathcal{D} = 3\pi\mu VD$$

where V is the speed of the pearl and μ is the fluid viscosity. Show that the term on the right side of Stokes's law has units of force.

SOLUTION:

$$[\mathcal{D}] = [3\pi\mu VD] \doteq \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)L = M\frac{L}{T^2} = M\underbrace{\left[\frac{FT^2}{ML}\right]}_{\frac{1}{g_c}}\frac{L}{T^2} = \boxed{F}$$

where $\dfrac{1}{g_c} = \left[\dfrac{FT^2}{ML}\right]$ comes from Newton's Second Law.

Problem 1.2.20

Express the given quantities in SI units: **(a)** 10.2 $\text{in.}/\text{min}$, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s^2, and **(e)** $0.0234\,\text{lb}\cdot\text{s}/\text{ft}^2$.

SOLUTION:

(a) $$10.2\,\frac{\text{in.}}{\text{min}}=\left(10.2\,\frac{\text{in.}}{\text{min}}\right)\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)=4.32\times10^{-3}\,\frac{\text{m}}{\text{s}}=\boxed{4.32\,\frac{\text{mm}}{\text{s}}}$$

(b) $$4.81\,\text{slugs}=(4.81\,\text{slugs})\left(1.459\times10\,\frac{\text{kg}}{\text{slug}}\right)=\boxed{70.2\,\text{kg}}$$

(c) $$3.02\,\text{lb}=(3.02\,\text{lb})\left(4.448\,\frac{\text{N}}{\text{lb}}\right)=\boxed{13.4\,\text{N}}$$

(d) $$73.1\,\frac{\text{ft}}{\text{s}^2}=\left(73.1\,\frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times10^{-1}\,\frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right)=\boxed{22.3\,\frac{\text{m}}{\text{s}^2}}$$

(e) $$0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}=\left(0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times10\,\frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right)=\boxed{1.12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

Problem 1.2.21

Express the given quantities in BG units: **(a)** 14.2 km , **(b)** $8.14\,\mathrm{N/m^3}$, **(c)** $1.61\,\mathrm{kg/m^3}$, **(d)** $0.0320\,\mathrm{N\cdot m/s}$, and **(e)** $5.67\,\mathrm{mm/hr}$.

SOLUTION:

(a) $14.2\,\mathrm{km} = \left(14.2\times10^{3}\ \mathrm{m}\right)\left(3.281\,\dfrac{\mathrm{ft}}{\mathrm{m}}\right) = \boxed{4.66\times10^{4}\ \mathrm{ft}}$

(b) $8.14\,\dfrac{\mathrm{N}}{\mathrm{m^3}} = \left(8.14\,\dfrac{\mathrm{N}}{\mathrm{m^3}}\right)\left(6.366\times10^{-3}\,\dfrac{\dfrac{\mathrm{lb}}{\mathrm{ft^3}}}{\dfrac{\mathrm{N}}{\mathrm{m^3}}}\right) = \boxed{5.18\times10^{-2}\,\dfrac{\mathrm{lb}}{\mathrm{ft^3}}}$

(c) $1.61\,\dfrac{\mathrm{kg}}{\mathrm{m^3}} = \left(1.61\,\dfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(1.940\times10^{-3}\,\dfrac{\dfrac{\mathrm{slugs}}{\mathrm{ft^3}}}{\dfrac{\mathrm{kg}}{\mathrm{m^3}}}\right) = \boxed{3.12\times10^{-3}\,\dfrac{\mathrm{slugs}}{\mathrm{ft^3}}}$

(d) $0.0320\,\dfrac{\mathrm{N\cdot m}}{\mathrm{s}} = \left(0.0320\,\dfrac{\mathrm{N\cdot m}}{\mathrm{s}}\right)\left(7.376\times10^{-1}\,\dfrac{\dfrac{\mathrm{ft\cdot lb}}{\mathrm{s}}}{\dfrac{\mathrm{N\cdot m}}{\mathrm{s}}}\right) = \boxed{2.36\times10^{-2}\,\dfrac{\mathrm{ft\cdot lb}}{\mathrm{s}}}$

(e) $5.67\,\dfrac{\mathrm{mm}}{\mathrm{hr}} = \left(5.67\times10^{-3}\,\dfrac{\mathrm{m}}{\mathrm{hr}}\right)\left(3.281\,\dfrac{\mathrm{ft}}{\mathrm{m}}\right)\left(\dfrac{1\,\mathrm{hr}}{3600\,\mathrm{s}}\right) = \boxed{5.17\times10^{-6}\,\dfrac{\mathrm{ft}}{\mathrm{s}}}$

Problem 1.2.22

Express the given quantities in SI units: **(a)** 160 acres , **(b)** 15 gallons (US), **(c)** 240 miles, **(d)** 79.1 hp , and **(e)** 60.3 °F.

SOLUTION:

(a) $160\text{ acres} = (160\text{ acres})\left(4.356\times10^{4}\ \frac{\text{ft}^2}{\text{acre}}\right)\left(9.290\times10^{-2}\ \frac{\text{m}^2}{\text{ft}^2}\right) = \boxed{6.47\times10^{5}\ \text{m}^2}$

(b) $15\text{ gallons} = (15\text{ gal})\left(3.785\ \frac{\text{liters}}{\text{gal}}\right)\left(10^{-3}\ \frac{\text{m}^3}{\text{liter}}\right) = \boxed{56.8\times10^{-2}\ \text{m}^3}$

(c) $240\text{ mi} = (240\text{ mi})\left(5280\ \frac{\text{ft}}{\text{mi}}\right)\left(3.048\times10^{-1}\ \frac{\text{m}}{\text{ft}}\right) = \boxed{3.86\times10^{5}\ \text{m}}$

(d) $79.1\text{ hp} = (79.1\text{ hp})\left(550\ \frac{\frac{\text{ft}\cdot\text{lb}}{\text{s}}}{\text{hp}}\right)\left(1.356\ \frac{\text{J}}{\text{ft}\cdot\text{lb}}\right) = 5.90\times10^{4}\ \frac{\text{J}}{\text{s}}\times\frac{1\text{ W}}{1\ \frac{\text{J}}{\text{s}}} = \boxed{5.90\times10^{4}\ \text{W}}$

(e) Relationship between units of temperature:

$$\text{K} = {}^{\circ}\text{C} + 273 = \frac{5}{9}\left({}^{\circ}\text{F} - 32\right) + 273$$

$$\frac{5}{9}\left(60.3\ {}^{\circ}\text{F} - 32\right) + 273 = \boxed{289\text{ K}}$$

Problem 1.2.23

Water flows from a large drainage pipe at a rate of 1200 gal/min . What is the volume rate of flow in **(a)** m^3/s, **(b)** liters/min , and **(c)** ft^3/s?

SOLUTION:

(a) $$\text{Flowrate} = \left(1200\,\frac{\text{gal}}{\text{min}}\right)\left(6.309\times10^{-5}\,\frac{\frac{\text{m}^3}{\text{s}}}{\frac{\text{gal}}{\text{min}}}\right) = \boxed{7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}}$$

(b) Since $1\,\text{liter} = 10^{-3}\,\text{m}^3$,

$$\text{Flowrate} = \left(7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}\right)\left(\frac{10^3\,\text{liters}}{\text{m}^3}\right)\left(\frac{60\,\text{s}}{\text{min}}\right) = \boxed{4540\,\frac{\text{liters}}{\text{min}}}$$

(c) $$\text{Flowrate} = \left(7.57\times10^{-2}\,\frac{\text{m}^3}{\text{s}}\right)\left(3.531\times10\,\frac{\frac{\text{ft}^3}{\text{s}}}{\frac{\text{m}^3}{\text{s}}}\right) = \boxed{2.67\,\frac{\text{ft}^3}{\text{s}}}$$

Problem 1.2.24

The universal gas constant R_0 is equal to $49{,}700\ \text{ft}^2/\left(\text{s}^2\cdot°\text{R}\right)$, or $8310\ \text{m}^2/\left(\text{s}^2\cdot\text{K}\right)$. Show that these two magnitudes are equal.

SOLUTION:

$$R_0=\left(\frac{8310\,\text{m}^2}{\text{s}^2\cdot\text{K}}\right)\left(\frac{3.281\,\text{ft}}{1\,\text{m}}\right)^2\left(\frac{(5/9)\,\text{K}}{1°\text{R}}\right)=\boxed{49{,}700\ \frac{\text{ft}^2}{\left(\text{s}^2\cdot°\text{R}\right)}}$$

Problem 1.2.25

Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table B.1 Physical Properties of Water (BG/EE Units).

SOLUTION:

Some possible examples:

$$\frac{\text{acceleration}\times\text{time}}{\text{velocity}} \doteq \frac{\left(LT^{-2}\right)(T)}{\left(LT^{-1}\right)} \doteq \boxed{L^0T^0}$$

$$\text{frequency}\times\text{time} \doteq \left(T^{-1}\right)(T) \doteq \boxed{T^0}$$

$$\frac{(\text{velocity})^2}{\text{length}\times\text{acceleration}} \doteq \frac{\left(LT^{-1}\right)^2}{(L)\left(LT^{-2}\right)} \doteq \boxed{L^0T^0}$$

$$\frac{\text{force}\times\text{time}}{\text{momentum}} \doteq \frac{(F)(T)}{\left(MLT^{-1}\right)} \doteq \frac{(F)(T)}{\left(FT^2L^{-1}\right)\left(LT^{-1}\right)} \doteq \boxed{F^0L^0T^0}$$

$$\frac{\text{density}\times\text{velocity}\times\text{length}}{\text{dynamic viscosity}} \doteq \frac{\left(ML^{-3}\right)\left(LT^{-1}\right)(L)}{ML^{-1}T^{-1}} \doteq \boxed{M^0L^0T^0}$$

Problem 1.2.26

An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is velocity, g is the acceleration of gravity, and ℓ is length. Determine the value of the Froude number for $V = 10$ ft/s, $g = 32.2$ ft/s^2, and $\ell = 2$ ft. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

SOLUTION:

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10 \ \frac{\text{ft}}{\text{s}}}{\sqrt{\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)(2 \text{ ft})}} = \boxed{1.25}$$

In SI units:

$$V = \left(10 \frac{\text{ft}}{\text{s}}\right)\left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 3.05 \frac{\text{m}}{\text{s}}$$

$$g = 9.81 \ \frac{\text{m}}{\text{s}^2}$$

$$\ell = (2 \text{ ft})\left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 0.610 \text{ m}$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05 \ \frac{\text{m}}{\text{s}}}{\sqrt{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.610 \text{ m})}} = \boxed{1.25}$$

The value of a dimensionless parameter is independent of the unit system.

Reserve Problem 1.2.1R

With the exception of the 410 bore, the gauge of a shotgun barrel indicates the number of round lead balls, each having the diameter of the barrel, that in aggregate weigh 1 lb. For example, a 1/10th-lb lead ball will completely fill the barrel of a 10-gauge shotgun. Using a density of 0.411 $lb_m/in.^3$ for lead, determine the diameter of a 10-gauge shotgun in inches and millimeters.

SOLUTION:

$$W = \gamma \cancel{V} = \rho g \cancel{V} = \rho g \left(\frac{4}{3} \pi R^3 \right)$$

$$R = \sqrt[3]{\left(\frac{3W}{4\pi\rho g} \right)} = \left[\frac{3\left(\frac{1}{10}\,\text{lb}\right)}{4\pi \left(0.411 \frac{\text{lb}_\text{m}}{\text{in.}^3} \right) \left(32.2 \frac{\text{ft}}{\text{s}^2} \right)} \right]^{1/3} = \left[\frac{0.3}{4\pi(0.411)(32.2)} \left(\frac{\text{lb} \cdot \text{in.}^3 \cdot \text{s}^2}{\text{lb}_\text{m} \cdot \text{ft}} \right) \times \frac{32.2\ \text{lb}_\text{m} \cdot \text{ft}}{1\ \text{lb} \cdot \text{s}^2} \right]^{1/3}$$

$$= \left[\frac{0.33}{4\pi(0.411)} \text{in.}^3 \right]^{1/3} = 0.0387 \text{ in.}$$

$$\boxed{D = 0.775 \text{ in.}}$$

$$D = 0.775 \text{ in.} \times \frac{25.4 \text{ mm}}{\text{in.}}$$

$$\boxed{D = 19.7 \text{ mm}}$$

Reserve Problem 1.2.2R

The given equation is an expression of Newton's Second Law in which the force due to a pressure gradient is required to be equal to, and in the opposite direction to, the viscous force on an infinitesimal particle of fluid resulting in the particle moving at a constant velocity. Show that this equation is dimensionally homogeneous. $0 = -\dfrac{\partial p}{\partial x} + \mu \dfrac{\partial^2 u}{\partial y^2}$

SOLUTION:

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\left.\begin{aligned} p &= \text{pressure}\left(\text{e.g.}, \frac{\text{lb}}{\text{ft}^2}\right) \quad \rightarrow \quad p \doteq \frac{F}{L^2} = FL^{-2} \\ x &= \text{position}(\text{e.g., ft}) \quad \rightarrow \quad x \doteq L \end{aligned}\right\} \frac{\partial p}{\partial x} \doteq FL^{-3}$$

$$\left.\begin{aligned} \mu &= \text{viscosity} \quad \left(\text{e.g.}, \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right) \rightarrow \quad \mu \doteq \frac{FT}{L^2} = FTL^{-2} \\ u &= \text{velocity} \quad \left(\text{e.g.}, \frac{\text{ft}}{\text{s}}\right) \quad \rightarrow \quad u \doteq \frac{L}{T} = LT^{-1} \\ y &= \text{position} \quad (\text{e.g., ft}) \quad \rightarrow \quad y \doteq L \end{aligned}\right\} \mu \frac{\partial^2 u}{\partial y^2} \doteq \left(FTL^{-2}\right)\frac{\left(LT^{-1}\right)}{L^2} = FL^{-2+1-2}T^{1-1} = FL^{-3}$$

The two RHS-terms have the same dimensions.
The number zero can have those dimensions.
Therefore, the equation is dimensionally homogeneous. **Q.E.D.**

Reserve Problem 1.2.3R

The capillary number, $Ca = \mu \mathrm{V}/\sigma$, is a very useful dimensionless parameter in the study of flows with a gas/liquid interface or an interface between two immiscible liquids. **(a)** Use the basic dimensions of *FLT* to show that *Ca* is a dimensionless parameter. **(b)** Use the basic dimensions of *MLT* to show that this is a dimensionless parameter.

SOLUTION:

(a) For *FLT*:

$$Ca = \frac{\mu \mathrm{V}}{\sigma}$$

$$Ca \doteq \frac{\left(\cancel{F} L^{-2} \cancel{T}\right)\left(L \cancel{T^{-1}}\right)}{\cancel{F} L^{-1}} \doteq \frac{L^{-1}}{L^{-1}} \quad \rightarrow \quad \boxed{Ca \doteq 1 \text{ therefore, dimensionless}}$$

(b) For *MLT*:

$$Ca \doteq \frac{\left(\cancel{M} \cancel{L^{-1}} T^{-1}\right)\left(\cancel{L} T^{-1}\right)}{\cancel{M} T^{-2}} \doteq \frac{T^{-2}}{T^{-2}} \quad \rightarrow \quad \boxed{Ca \doteq 1 \text{ therefore, dimensionless}}$$

Reserve Problem 1.2.4R

Neglecting aerodynamic drag, friction, and other losses, how much power (in units of horsepower), must an engine provide for a 3500 lb_m automobile to maintain a speed of 60 mph on a 4% grade (4 ft of rise for 100 ft of run)?

SOLUTION:

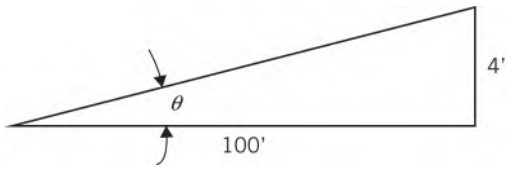
The work required to lift a mass in a gravitational field is: $\text{Work} = mg\Delta h$

The power required to lift a mass in a gravitational field at a constant rate is

$$P = \frac{d}{dt}(\text{Work}) = \frac{\Delta \text{Work}}{\Delta t} = mg\frac{\Delta h}{\Delta t}$$

For a 4% grade:

$\tan(\theta) = 0.04 \quad \rightarrow \quad \theta = 2.39°$



$$\frac{\Delta h}{\Delta t} = V_y = \left|\vec{\text{V}}\right|\sin(\theta) = (60 \text{ mph})\sin(2.39°) = 2.40 \text{ mph}$$

The power required for the stated conditions is

$$P = mg\frac{\Delta h}{\Delta t} = (3500 \text{ lb}_\text{m})\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(2.40\frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \times \frac{1\ \cancel{\text{hr}}}{3600 \text{ s}} \times \frac{5280 \text{ ft}}{1\ \cancel{\text{mi}}}\right)$$

$$= (3500)\left(\cancel{32.2}\right)(3.52)\frac{\text{lb}_\text{m} - \text{ft}^2}{\text{s}^3} \times \frac{1}{\cancel{32.2}\ \frac{\text{ft} - \text{lb}_\text{m}}{\text{lb} - \text{s}^2}}$$

$$= 12{,}320\frac{\text{ft} - \text{lb}}{\text{s}} \times \frac{1 \text{ hp}}{550\frac{\text{ft} - \text{lb}}{\text{s}}} = 22.40 \text{ hp} \qquad \rightarrow \qquad \boxed{P = 22.4 \text{ hp}}$$

Reserve Problem 1.2.5R

Bernoulli's Equation for the steady incompressible flow of an inviscid fluid will be developed in Chapter 3. It is very useful for modeling many flows of practical interest. One form of the equation is $\frac{p}{\rho}+\frac{1}{2}V^2+gz=\Pi,$ where V is the speed of the fluid, the z-coordinate is pointed upward, and Π is a constant. **(a)** Show that this equation is dimensionally homogeneous and determine the dimension of Π using **(a)** the FLT system of dimensions, and **(b)** the MLT system.

SOLUTION:

(a)

$$FLT:\quad \left.\begin{aligned}\frac{p}{\rho}&\doteq\frac{FL^{-2}}{FL^{-4}T^2}=\frac{L^2}{T^2}\\ \frac{1}{2}V^2&\doteq\frac{L^2}{T^2}\\ gz&\doteq\frac{L^2}{T^2}\end{aligned}\right\}\rightarrow$$

a) for FLT, equation is dimensionally homogenous with $\Pi\doteq L^2/T^2$

(b)

$$MLT:\quad \left.\begin{aligned}\frac{p}{\rho}&\doteq\frac{ML^{-1}T^{-2}}{ML^{-3}}=\frac{L^2}{T^2}\\ \frac{1}{2}V^2&\doteq\frac{L^2}{T^2}\\ gz&\doteq\frac{L^2}{T^2}\end{aligned}\right\}\rightarrow$$

b) for MLT, equation is dimensionally homogenous with $\Pi\doteq L^2/T^2$

Section 1.4 Solutions

Problem 1.4.2

A tank contains 500 kg of a liquid whose specific gravity is 2. Determine the volume of the liquid in the tank.

SOLUTION:

$m = \rho V = SG\rho_{\mathrm{H_2O}} V$

Thus,

$$V = \frac{m}{\left(SG\rho_{\mathrm{H_2O}}\right)} = \frac{500\,\mathrm{kg}}{\left((2)\left(999\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\right)} = \boxed{0.250\,\mathrm{m}^3}$$

Problem 1.4.3

A stick of butter at $35\ °\text{F}$ measures $1.25\ \text{in.} \times 1.25\ \text{in.} \times 4.65\ \text{in.}$ and weighs $4\ \text{oz}$. Find its specific weight.

SOLUTION:

$$\gamma = \frac{W}{V} = \frac{(4\ \text{oz})\left(\frac{1\ \text{lb}}{16\ \text{oz}}\right)}{(1.25\ \text{in.})^2 (4.65\ \text{in.})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3} = \boxed{59.5 \frac{\text{lb}}{\text{ft}^3}}$$

Problem 1.4.4

Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter $\left(\text{g/m}^3\right)$. Assume that a cumulus cloud occupies a volume of 1 cubic kilometer, and its liquid water content is 0.2 g/m^3. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

SOLUTION:

(a) $$\text{Volume} = (1\ \text{km})^3\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)^3 = 10^9\ \text{m}^3\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\times\frac{1\ \text{m}}{5280\ \text{ft}}\right)^3 = \boxed{0.240\ \text{mi}^3}$$

(b) $W = \gamma \times \text{Volume}$

$$\gamma = \rho g = \left(0.2\ \frac{\text{g}}{\text{m}^3}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}}\right) = 1.962\times10^{-3}\ \frac{\text{N}}{\text{m}^3}$$

$$W = \left(1.962\times10^{-3}\ \frac{\text{N}}{\text{m}^2}\right)\left(10^9\ \text{m}^3\right)\left(2.248\times10^{-1}\ \frac{\text{lb}}{\text{N}}\right) = \boxed{4.4\times10^5\ \text{lb}}$$

Problem 1.4.5

A tank of oil has a mass of 25 slugs. **(a)** Determine its weight in pounds and in Newtons at the Earth's surface. **(b)** What would be its mass (in slugs) and its weight (in pounds) if located on the moon's surface, where the gravitational attraction is approximately one-sixth of that at the Earth's surface?

SOLUTION:

(a)

$$\text{Weight} = \text{mass} \times g$$

$$= (25 \text{ slugs})\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1 \text{ lb} \cdot \text{s}^2}{1 \text{ slug} \cdot \text{ft}}\right) = \boxed{805 \text{ lb}}$$

$$= (25 \text{ slugs})\left(\frac{14.59 \text{ kg}}{1 \text{ slug}}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1 \text{ N} \cdot \text{s}^2}{1 \text{ kg} \cdot \text{m}}\right) = \boxed{3580 \text{ N}}$$

(b)

$$\text{Mass} = \boxed{25 \text{ slugs} \left(\text{mass does not depend on gravitational attraction}\right)}$$

$$\text{Weight} = \frac{805 \text{ lb}}{6} = \boxed{134 \text{ lb}}$$

Problem 1.4.6

A certain object weighs $300\ \text{N}$ at the Earth's surface. Determine the mass of the object (in kilograms) and its weight (in Newtons) when located on a planet with an acceleration of gravity equal to $4.0\ \text{ft/s}^2$.

SOLUTION:

$$\text{Mass} = \frac{\text{weight}}{g} = \frac{300\ \text{N}}{9.81 \dfrac{\text{m}}{\text{s}^2}} = \boxed{30.6\ \text{kg}}$$

$$\text{For } g = 4.0 \frac{\text{ft}}{\text{s}^2}$$

$$\text{Weight} = (30.6\ \text{kg})\left(4.0 \frac{\text{ft}}{\text{s}^2}\right)\left(0.3048 \frac{\text{m}}{\text{ft}}\right)\left(\frac{1\ \text{N} \cdot \text{s}^2}{1\ \text{kg} \cdot \text{m}}\right) = \boxed{37.3\ \text{N}}$$

Problem 1.4.7

The density of a certain type of jet fuel is $775\ \text{kg/m}^3$. Determine its specific gravity and specific weight.

SOLUTION:

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}}} = \frac{775\,\frac{\text{kg}}{\text{m}^3}}{1000\,\frac{\text{kg}}{\text{m}^3}} = \boxed{0.775}$$

$$\gamma = \rho g = \left(775\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^3}\right) = \boxed{7.60\,\frac{\text{kN}}{\text{m}^3}}$$

Problem 1.4.8

At $4\,°\text{C}$ a mixture of automobile antifreeze (50% water and 50% ethylene glycol by volume) has a density of $1064\,\text{kg/m}^3$. If the water density is $1000\,\text{kg/m}^3$, find the density of the ethylene glycol.

SOLUTION:

$$S = \frac{(\rho_{\text{mixture}})_{4°\text{C}}}{(\rho_{\text{water}})_{4°\text{C}}} = \frac{\left(\dfrac{m_{\text{eg}} + m_{\text{w}}}{\cancel{V}}\right)_{4°\text{C}}}{\left(\dfrac{m_{\text{w}}}{\cancel{V}}\right)_{4°\text{C}}} = \frac{m_{\text{eg}} + m_{\text{w}}}{m_{\text{w}}}$$

where m_{w} is the mass of the pure water in volume $\cancel{V}$ at $4\,°\text{C}$.
Then,

$$S = \frac{\rho_{\text{eg}}(0.5\cancel{V}) + \rho_{\text{w}}(0.5\cancel{V})}{\rho_{\text{w}}\cancel{V}} = 0.5\left(\frac{\rho_{\text{eg}} + \rho_{\text{w}}}{\rho_{\text{w}}}\right)$$

The problem statement gives

$$S = \frac{\rho_{\text{mixture}}}{\rho_{\text{w}}} = \frac{1064\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = 1.064 \;\rightarrow 1.064 = 0.5\left(\frac{\rho_{\text{eg}} + \rho_{\text{w}}}{\rho_{\text{w}}}\right) \rightarrow \rho_{\text{eg}} = \rho_{\text{w}}\left(\frac{1.064}{0.5} - 1\right)$$

$$= \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1.064}{0.5} - 1\right) \boxed{\rho_{\text{eg}} = 1130\,\frac{\text{kg}}{\text{m}^3}}$$

DISCUSSION: If the mixture were at some temperature T, then for equal volumes of mixture and $4\,°\text{C}$ water,

$$S = \frac{(m_{\text{eg}} + m_{\text{w}})_T}{(m_{\text{w}})_{4\,°\text{C}}} = \frac{0.5\,(\rho_{\text{eg}} + \rho_{\text{w}})_T}{(\rho_{\text{w}})_{4\,°\text{C}}}$$

Problem 1.4.9

A *hydrometer* is used to measure the specific gravity of liquids. For a certain liquid, a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

SOLUTION:

$$SG = \frac{\rho}{\rho_{H_2O\,@4\,^\circ C}}$$

$$1.15 = \frac{\rho}{1000\,\frac{\text{kg}}{\text{m}^3}}$$

$$\rho = (1.15)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) = \boxed{1150\,\frac{\text{kg}}{\text{m}^3}}$$

$$\gamma = \rho g = \left(1150\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}}\right) = \boxed{11.3\,\frac{\text{kN}}{\text{m}^3}}$$

Problem 1.4.10

An open, rigid-walled, cylindrical tank contains $4\ \text{ft}^3$ of water at $40\ °\text{F}$. Over a 24-hr period of time the water temperature varies from 40 to $90\ °\text{F}$. Make use of the data in Appendix B to determine how much the volume of water will change. For a tank diameter of 2 ft, would the corresponding change in water depth be very noticeable? Explain.

SOLUTION:

$$\text{mass of water} = \cancel{V} \times \rho$$

Amount of mass is not a function of temperature.

$$\cancel{V}_{40°} \times \rho_{40°} = m = \cancel{V}_{90°} \times \rho_{90°}$$

From Table B.1 Physical Properties of Water (BG/EE Units)

$$\rho_{\text{H}_2\text{O}@40\,°\text{F}} = 1.940 \frac{\text{slugs}}{\text{ft}^3}$$

$$\rho_{\text{H}_2\text{O}@90\,°\text{F}} = 1.931 \frac{\text{slugs}}{\text{ft}^3}$$

Therefore,

$$\cancel{V}_{90°} = \frac{\left(4\ \text{ft}^3\right)\left(1.940 \dfrac{\text{slugs}}{\text{ft}^3}\right)}{1.931 \dfrac{\text{slugs}}{\text{ft}^3}} = 4.0186\ \text{ft}^3$$

Thus, the increase in volume is: $\Delta\cancel{V} = 4.0186 - 4.000 = \boxed{0.0186\ \text{ft}^3}$

For a tank diameter of 2 ft: $\Delta h = \dfrac{\Delta\cancel{V}}{A} = \dfrac{0.0186\ \text{ft}^3}{\dfrac{\pi}{4}\left(2\ \text{ft}\right)^2} = 5.92 \times 10^{-3}\ \text{ft} = 0.0710\ \text{in.}$

This small change in depth would not be very noticeable. $\boxed{\text{No}}$.

Note: A slightly different value for Δh will be obtained if specific weight of water is used rather than its density. This is due to the fact that there is some uncertainty in the fourth significant figure of these two values, and the solution is sensitive to this uncertainty.

Problem 1.4.12

A mountain climber's oxygen tank contains 1 lb of oxygen when he begins his trip at sea level, where the acceleration of gravity is 32.174 ft/s^2. What is the weight of the oxygen in the tank when he reaches the top of Mt. Everest where the acceleration of gravity is 32.082 ft/s^2?? Assume that no oxygen has been removed from the tank; it will be used on the descent portion of the climb.

SOLUTION:

$W = mg$

Let $(\ )_{sl}$ denote sea level and $(\ )_{MtE}$ denote the top of Mt. Everest

Thus,

$W_{sl} = 1\,lb = m_{sl} g_{sl}$

and

$W_{MtE} = m_{MtE} g_{MtE}$

However, $m_{sl} = m_{MtE}$ so that since $m = \dfrac{W}{g}$

$$m_{sl} = \frac{W_{sl}}{g_{sl}} = m_{MtE} = \frac{W_{MtE}}{g_{MtE}}$$

or

$$W_{MtE} = W_{sl} \frac{g_{MtE}}{g_{sl}} = 1\,\text{lb} \frac{32.082 \dfrac{\text{ft}}{\text{s}^2}}{32.174 \dfrac{\text{ft}}{\text{s}^2}} = \boxed{0.9971\,\text{lb}}$$

Problem 1.4.13

The information on a can of pop indicates that the can contains 355 ml. The mass of a full can of pop is 0.369 kg, while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

SOLUTION:

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$\text{Total weight} = \text{mass} \times g = (0.369\,\text{kg})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}}\right) = 3.62\,\text{N}$$

$$\text{Weight of can} = 0.153\,\text{N}$$

$$\text{Volume of fluid} = \left(355\times10^{-3}\ \text{liter}\right)\left(\frac{1\,\text{m}^3}{1000\ \text{liter}}\right) = 355\times10^{-6}\ \text{m}^3$$

Therefore,

$$\gamma = \frac{3.62\,\text{N} - 0.153\,\text{N}}{355\times10^{-6}\ \text{m}^3} = \boxed{9770\,\frac{\text{N}}{\text{m}^3}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\,\dfrac{\text{N}}{\text{m}^3}}{9.81\,\dfrac{\text{m}}{\text{s}^2}} = 996\,\frac{\text{N}\cdot\text{s}^2}{\text{m}^4}\left(\frac{1\,\text{kg}\cdot\text{m}}{1\,\text{N}\cdot\text{s}^2}\right) = \boxed{996\,\frac{\text{kg}}{\text{m}^3}}$$

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}\,@4\,^\circ\text{C}}} = \frac{996\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = \boxed{0.996}$$

For water at 20 °C (see Table B.2 Physical Properties of Water [SI Units])

$$\gamma_{\text{H}_2\text{O}} = 9789\,\frac{\text{N}}{\text{m}^3};\quad \rho_{\text{H}_2\text{O}} = 998.2\,\frac{\text{kg}}{\text{m}^3};\quad SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

Problem 1.4.14

The variation in the density of water, ρ, with temperature, T, in the range $20\ °\text{C} \le T \le 50\ °\text{C}$, is given in the table here.

Density (kg/m^3)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2 T + c_3 T^2$, which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

SOLUTION:

Fit the data to a second order polynomial using a standard curve-fitting program such as found in Excel. Thus,

$$\rho = \boxed{1001 - 0.0533T - 0.0041T^2}$$

As shown in the table here, ρ (predicted) from Eq. 1 is in good agreement with ρ (given).

T (°C)	ρ-predict (kg/m^3)	ρ-data (kg/m^3)
20	998.3	998.2
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.3	992.2
45	990.3	990.2
50	988.1	988.1

At $T = 42.1\ °\text{C}$: $\rho = 1001 - 0.0533(42.1\ °\text{C}) - 0.0041(42.1\ °\text{C})^2 = \boxed{991.5\ \frac{\text{kg}}{\text{m}^3}}$

Problem 1.4.15

If one cup of cream having a density of $1005\ \text{kg/m}^3$ is turned into three cups of whipped cream, determine the specific gravity and specific weight of the whipped cream.

SOLUTION:

Mass of cream, $m = \left(1005\,\frac{\text{kg}}{\text{m}^3}\right) \times \left(\cancel{V}_{\text{cup}}\right)$, where $\cancel{V}$ = volume

Noting that the mass is the same in liquid and in "whipped" form,

$$\rho_{\substack{\text{whipped}\\\text{cream}}} = \frac{m_{\substack{\text{whipped}\\\text{cream}}}}{\cancel{V}_{3\text{ cups}}} = \frac{\left(1005\,\frac{\text{kg}}{\text{m}^3}\right)\cancel{V}_{\text{cup}}}{\cancel{V}_{3\text{ cups}}} = \frac{1005\,\frac{\text{kg}}{\text{m}^3}}{3} = 335\,\frac{\text{kg}}{\text{m}^3}$$

$$SG = \frac{\rho_{\substack{\text{whipped}\\\text{cream}}}}{\rho_{\text{H}_2\text{O}\,@\,4\,^\circ\text{C}}} = \frac{335\,\frac{\text{kg}}{\text{m}^3}}{1000\,\frac{\text{kg}}{\text{m}^3}} = \boxed{0.335}$$

$$\gamma_{\substack{\text{whipped}\\\text{cream}}} = \rho_{\substack{\text{whipped}\\\text{cream}}} \times g = \left(335\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}}\right) = \boxed{3290\,\frac{\text{N}}{\text{m}^3}}$$

Problem 1.4.16

With the exception of the 410 bore, the gauge of a shotgun barrel indicates the number of round lead balls, each having the bore diameter of the barrel, that together weigh 1 lb. For example, a shotgun is called a 12-gauge shotgun if a $\frac{1}{12}$-lb lead ball fits the bore of the barrel. Find the diameter of a 12-gauge shotgun in inches and millimeters. Lead has a specific weight of 0.411 lb/in.^{3}

SOLUTION:

$$\cancel{V}_{\text{ball}} = \frac{\text{weight}}{\gamma} = \frac{\frac{1}{12}\,\text{lb}}{\left(0.411\frac{\text{lb}}{\text{in.}^3}\right)} = 0.20276\ \text{in.}^3$$

For a sphere: $\cancel{V} = \dfrac{4\pi R^3}{3} \quad \rightarrow \quad R = \sqrt[3]{\dfrac{3\cancel{V}}{4\pi}}$

$$R = \sqrt[3]{\frac{3\left(0.20276\,\text{in.}^3\right)}{4\pi}} = 0.3644\,\text{in.}$$

$$\boxed{D = 0.729\ \text{in.}}$$

$$D = 0.729\,\text{in.}\left(\frac{25.4\,\text{mm}}{\text{in.}}\right)$$

$$\boxed{D = 18.5\,\text{mm}}$$

Reserve Problem 1.4.1R

The density of gasoline is typically between 0.71 and 0.77 kg/liter. The density increases with the volume fraction of aromatics. Using the marketable gasoline standard reference of 0.755 kg/liter:

(a) How much does 1 liter weigh?
(b) How much does one US gallon weight in units of lb?
(c) If a pool of gasoline is on fire, why is the fire not extinguished by pouring water on it?

SOLUTION:

(a)

$$W = \rho g \cancel{V} = \left(0.755\frac{\text{kg}}{\text{liter}}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(1\text{ liter}) = (0.755)(9.81)(1)\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\times\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot 1\text{ m}}$$

$$\boxed{W = 7.41\text{ N}}$$

(b)

$$W = \rho g \cancel{V}$$

$$= \left(0.755\frac{\cancel{\text{kg}}}{\cancel{\text{liter}}}\times\frac{2.2\text{ lb}_\text{m}}{1\ \cancel{\text{kg}}}\times\frac{1\ \cancel{\text{liter}}}{1000\ \cancel{\text{cm}^3}}\times\frac{1000\ \cancel{\text{cm}^3}}{0.001\ \cancel{\text{m}^3}}\times\frac{1\ \cancel{\text{m}^3}}{35.3\ \cancel{\text{ft}^3}}\times\frac{1\ \cancel{\text{ft}^3}}{7.48\ \cancel{\text{gal}}}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(1\ \cancel{\text{gal}}\right)$$

$$= \frac{(0.755)(2.2)}{(0.001)(35.3)(7.48)}(32.2)\frac{\text{lb}_\text{m}\cdot\text{ft}}{\text{s}^2}\times\frac{1\text{ lb}\cdot\text{s}^2}{32.2\text{ lb}_\text{m}\cdot\text{ft}} \quad\rightarrow\quad \boxed{W = 6.2\text{ lb}}$$

(c) Water extinguishes a fire by preventing the oxygen in air from combining with the fuel. Gasoline floats on water because it is less dense than water. Pouring water on a fire fueled by a less dense liquid such as gasoline does not create a barrier to the oxygen and therefore does not extinguish the fire.

Reserve Problem 1.4.2R

Although the density of milk depends on the relative proportions of its constituents, a value of ρ = 1.033 kg/liter is a reasonable estimate to use for modeling.

(a) What is weight of 1 l of milk, expressed in Newtons?

(b) What is the weight of 1 US gallon of milk, expressed in pounds?

SOLUTION:

(a)

$$W = \rho g \cancel{V} = \left(1.033 \frac{\text{kg}}{\text{liter}}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(1 \text{ liter}) = (1.033)(9.81)(1)\frac{\text{kg}\cdot\text{m}}{\text{s}^2} \times \frac{1 \text{ N}\cdot\text{s}^2}{1 \text{ kg}\cdot 1 \text{ m}}$$

$$\boxed{W = 10.1 \text{ N}}$$

(b)

$$W = \rho g \cancel{V}$$

$$= \left(1.033 \frac{\cancel{\text{kg}}}{\cancel{\text{liter}}} \times \frac{2.2 \text{ lb}_\text{m}}{1 \cancel{\text{kg}}} \times \frac{1 \cancel{\text{liter}}}{1000 \cancel{\text{cm}^3}} \times \frac{1000 \cancel{\text{cm}^3}}{0.001 \cancel{\text{m}^3}} \times \frac{1 \cancel{\text{m}^3}}{35.3 \cancel{\text{ft}^3}} \times \frac{1 \cancel{\text{ft}^3}}{7.48 \cancel{\text{gal}}}\right)\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)(1 \cancel{\text{gal}})$$

$$= \frac{(1.033)(2.2)}{(0.001)(35.3)(7.48)}(32.2)\frac{\text{lb}_\text{m}\cdot\text{ft}}{\text{s}^2} \times \frac{1 \text{ lb}\cdot\text{s}^2}{32.2 \text{ lb}_\text{m}\cdot\text{ft}} \quad \rightarrow \quad \boxed{W = 8.6 \text{lb}}$$

Reserve Problem 1.4.3R

Milk has a specific weight of approximately 64.4 lb/ft^3. **(a)** Express the density of milk in the base units of the English Engineering system. **(b)** Express the density of milk in the base units of the British Gravitational system. **(c)** Express the density of milk in the base units of the SI system. **(d)** Express the mass of a quart of milk in the base units of the English Engineering system.

SOLUTION:

(a)

$$\rho_{\text{MILK}} = \frac{\gamma_{\text{MILK}}}{g} = \frac{64.4 \dfrac{\cancel{\text{lb}}}{\text{ft}^3}}{32.2 \dfrac{\cancel{\text{ft}}}{\cancel{s^2}}} \times 32.2 \frac{\text{lb}_\text{m} \cdot \cancel{\text{ft}}}{\cancel{\text{lb}} \cdot \cancel{s^2}} \quad \rightarrow \quad \boxed{\rho_{\text{MILK}} = 64.4 \frac{\text{lb}_\text{m}}{\text{ft}^3}}$$

NOTE: Although mass and weight are two distinctly different quantities, in the BG system, the numerical values for mass and weight are the same. The units are different!

(b)

$$\rho_{\text{MILK}} = \frac{\gamma_{\text{MILK}}}{g} = \frac{64.4 \dfrac{\cancel{\text{lb}}}{\text{ft}^3}}{32.2 \dfrac{\cancel{\text{ft}}}{\cancel{s^2}}} \times 1 \frac{\text{slug} \cdot \cancel{\text{ft}}}{\cancel{\text{lb}} \cdot \cancel{s^2}} \quad \rightarrow \quad \boxed{\rho_{\text{MILK}} = 2.0 \frac{\text{slug}}{\text{ft}^3}}$$

(c)

$$\rho_{\text{MILK}} = \frac{\gamma_{\text{MILK}}}{g} = \frac{64.4 \dfrac{\cancel{\text{lb}}}{\cancel{\text{ft}^3}} \times \dfrac{4.448 \ \cancel{\text{N}}}{1 \ \cancel{\text{lb}}} \times \left(\dfrac{3.281 \ \cancel{\text{ft}}}{1 \text{ m}} \right)^3}{9.81 \dfrac{\cancel{\text{m}}}{\cancel{s^2}}} \times 1 \frac{\text{kg} \cdot \cancel{\text{m}}}{\cancel{\text{N}} \cdot \cancel{s^2}} \quad \rightarrow \quad \boxed{\rho_{\text{MILK}} = 1031 \frac{\text{kg}}{\text{m}^3}}$$

(d)

$$m = \rho_{\text{MILK}} \cancel{V} = \frac{\gamma_{\text{MILK}}}{g} \forall = \frac{64.4 \dfrac{\cancel{\text{lb}}}{\cancel{\text{ft}^3}}}{32.2 \dfrac{\cancel{\text{ft}}}{\cancel{s^2}}} \left(32.2 \frac{\text{lb}_\text{m} \cdot \text{ft}}{\text{lb} \cdot \text{s}^2} \right) \left(1 \ \cancel{\text{qt}} \times \frac{1 \ \cancel{\text{ft}^3}}{29.92 \ \cancel{\text{qt}}} \right) \rightarrow \boxed{m_{\text{QUART MILK}} = 2.15 \text{ lb}}$$

Reserve Problem 1.4.4R

A container holds 0.15 m^3 of a liquid with a mass of 118 kg. **(a)** Determine the specific weight of the fluid. **(b)** Using the resources in this textbook, which fluid is most likely to be in the container?

SOLUTION:

(a) $\gamma = \rho g = \dfrac{m}{\cancel{V}} g = \dfrac{(118\ \text{kg})}{(0.15\ \text{m}^3)}\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)\times 1\dfrac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \quad \rightarrow \quad \boxed{\textbf{a)}\ \gamma = 7720\dfrac{\text{N}}{\text{m}^3}}$

(b) From text, specific weight of ethyl alcohol is 7740 N/m^3 $\rightarrow$ **b**)ethyl alcohol°

Reserve Problem 1.4.5R

Determine the weight of a 1 ft^3 volume of liquid that has a specific gravity of 1.2 in the base units of **(a)** the SI system, **(b)** the BG system, and **(c)** the EE system.

SOLUTION:

By definition: $\text{SG} = \dfrac{\rho}{\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}}}$

The density of water at its reference temperature, to three significant digits, is

$$\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} = 1000\,\frac{\text{kg}}{\text{m}^3} = 1.94\,\frac{\text{slugs}}{\text{ft}^3} = 62.4\,\frac{\text{lb}_\text{m}}{\text{ft}^3}$$

The weight in any units can be computed using

$$W = mg = \rho \cancel{V} g = \text{SG}\,\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} \cancel{V} g = 1.2\,\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} \cancel{V} g$$

(a) $W = 1.2\,\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} \cancel{V} g$

$$= 1.2\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(1\,\text{ft}^3 \times \frac{1\,\text{m}^3}{35.31\,\text{ft}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) \times 1\,\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \quad \rightarrow \quad \textbf{(a)}\ \boxed{W = 333\ \text{N}}$$

(b) $W = 1.2\,\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} \cancel{V} g$

$$= 1.2\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(1\,\text{ft}^3\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right) \times 1\,\frac{\text{lb}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \quad \rightarrow \quad \textbf{(b)}\ \boxed{W = 75.0\ \text{lb}}$$

(c) $W = 1.2\,\rho_{\text{H}_2\text{O}@4\,^\circ\text{C}} \cancel{V} g$

$$= 1.2\left(62.4\,\frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(1\,\text{ft}^3\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right) \times \frac{1}{32.2}\,\frac{\text{lb}\cdot\text{s}^2}{\text{lb}_\text{m}\cdot\text{ft}} \quad \rightarrow \quad \textbf{(c)}\ \boxed{W = 74.9\ \text{lb}}$$

NOTE: The difference between the answers for part “b” and part “c” is due to the finite number of digits use for the ρ, g, and g_c.

Section 1.5 Solutions

Problem 1.5.1

A regulation basketball is initially flat and is then inflated to a pressure of approximately 24 lb/in.^{2} absolute. Consider the air temperature to be constant at $70\ ^\circ\text{F}$. Find the mass of air required to inflate the basketball. The basketball's inside radius is 4.67 in.

SOLUTION:

$$m = \frac{p\cancel{V}}{RT} = \frac{p}{RT}\left(\frac{4}{3}\pi R^3\right) = \frac{\left(24\dfrac{\text{lb}}{\text{in.}^2}\right)\left(\dfrac{4\pi}{3}\right)(4.67\text{ in.})^3\left(\dfrac{1\text{ ft}}{12\text{ in.}}\right)}{\left(1716\dfrac{\text{ft}\cdot\text{lb}}{\text{slug}^\circ\text{R}}\right)(460+70)^\circ\text{R}}$$

$$\boxed{m = 0.000938\text{ slug}}$$

Problem 1.5.2

Nitrogen is compressed to a density of $4\ \text{kg/m}^3$ under an absolute pressure of $400\ \text{kPa}$. Determine the temperature in degrees Celsius.

SOLUTION:

$$T = \frac{p}{\rho R} = \frac{400\times10^3\ \frac{\text{N}}{\text{m}^2}}{\left(4\ \frac{\text{kg}}{\text{m}^3}\right)\left(296.8\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(1\ \frac{\frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)} = 337\ \text{K}$$

$$T_C = T_K - 273 = 337\ \text{K} - 273 = \boxed{64\ ^\circ\text{C}}$$

Problem 1.5.3

The temperature and pressure at the surface of Mars during a Martian spring day were determined to be $-50\ ^\circ\text{C}$ and $900\ \text{Pa}$, respectively. **(a)** Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed to be equivalent to that of carbon dioxide. **(b)** Compare the answer from part (a) with the density of the Earth's atmosphere during a spring day when the temperature is $18\ ^\circ\text{C}$ and the pressure $101.6\ \text{kPa}$ (abs).

SOLUTION:

(a) $$\rho_{\text{Mars}} = \frac{p}{RT} = \frac{900\ \frac{\text{N}}{\text{m}^2}}{\left(188.9\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(1\ \frac{\frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[(-50\ ^\circ\text{C}+273)\ \text{K}\right]} = \boxed{0.0214\ \frac{\text{kg}}{\text{m}^3}}$$

(b) $$\rho_{\text{Earth}} = \frac{p}{RT} = \frac{101.6\times10^3\ \frac{\text{N}}{\text{m}^2}}{\left(286.9\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(1\ \frac{\frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[(18\ ^\circ\text{C}+273)\ \text{K}\right]} = \boxed{1.22\ \frac{\text{kg}}{\text{m}^3}}$$

Thus, $$\frac{\rho_{\text{Mars}}}{\rho_{\text{Earth}}} = \frac{0.0214\ \frac{\text{kg}}{\text{m}^3}}{1.22\ \frac{\text{kg}}{\text{m}^3}} = 0.0175 = \boxed{1.75\%}$$

Problem 1.5.4

A closed tank having a volume of $2\ \text{ft}^3$ is filled with $0.30\ \text{lb}$ of a gas. A pressure gage attached to the tank reads $12\ \text{psi}$ when the gas temperature is $80\ °\text{F}$. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

SOLUTION:

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30\ \text{lb}}{\left(32.2\ \dfrac{\text{ft}}{\text{s}^2}\right)\left(2\ \text{ft}^3\right)} = 4.66 \times 10^{-3}\ \dfrac{\text{slugs}}{\text{ft}^3}$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)\ \text{psia}$ (atmospheric pressure assumed to be $\approx 14.7\ \text{psia}$)
and with $T = (80\ °\text{F} + 460)\ °\text{R}$ it follows that

$$\rho = \frac{\left(26.7\ \dfrac{\text{lb}}{\text{in.}^2}\right)\left(144\ \dfrac{\text{in.}^2}{\text{ft}^2}\right)}{R\,(540)\,°\text{R}} = \frac{7.12}{R}\ \frac{\text{slugs}}{\text{ft}^3} \qquad \textbf{(1)}$$

From Table 1.7 Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (BG Units)

$R = 1.554 \times 10^3\ \dfrac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °\text{R}}$ for oxygen … and … $R = 1.242 \times 10^4\ \dfrac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °\text{R}}$ for helium.

Thus, from Eq. 1 if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3}\ \frac{\text{slugs}}{\text{ft}^3} = 4.58 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4}\ \frac{\text{slugs}}{\text{ft}^3} = 5.73 \times 10^{-4}\ \frac{\text{slugs}}{\text{ft}^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be $\boxed{\text{oxygen}}$.

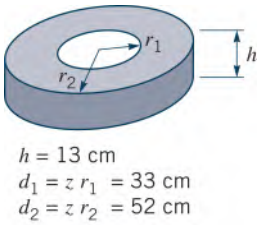
Problem 1.5.5

Assume that the air volume in a small automobile tire is constant and equal to the volume between two concentric cylinders 13 cm high with diameters of 33 cm and 52 cm. The air in the tire is initially at 25 °C and 202 kPa. Immediately after air is pumped into the tire, the temperature is 30 °C and the pressure is 303 kPa. What mass of air was added to the tire? What would be the air pressure after the air has cooled to a temperature of 0 °C?

SOLUTION:

The mass of air added to the tire is the difference of the final mass of air m_f and the initial mass m_i. Assuming air is an ideal gas,

$$m_f - m_i = \left(\frac{p}{RT}\cancel{V}\right)_f - \left(\frac{p}{RT}\cancel{V}\right)_i = \frac{\cancel{V}}{R}\left(\frac{p_f}{T_f} - \frac{p_i}{T_i}\right)$$



Now

$$\cancel{V} = \pi\left(r_2^2 - r_1^2\right)h = \pi\left[(26\text{ cm})^2 - (16.5\text{ cm})^2\right](13\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3$$

$$= 0.0165\text{ m}^3$$

$$m_f - m_i = \frac{(0.0165\text{ m}^3)}{\left(287.0\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{303\text{ kPa}}{(273+30)\text{ K}} - \frac{202\text{ kPa}}{(273+25)\text{ K}}\right]\left(\frac{1000\dfrac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)$$

$$\boxed{m_f - m_i = 0.0185\text{ kg}}$$

Now consider the cooling process. The initial state will be 30 °C and 303 kPa. The final state will be 0 °C and p_f. Applying the ideal gas law to both states gives

$$\left(\frac{p}{RT}\cancel{V}\right)_i = \left(\frac{p}{RT}\cancel{V}\right)_f$$

Since $\cancel{V}_f = \cancel{V}_i$

$$p_f = p_i\left(\frac{T_f}{T_i}\right) = (303\text{ kPa})\left(\frac{273+0}{273+30}\right) = \boxed{p_f = 273\text{ kPa}}$$

Problem 1.5.6

A compressed air tank contains $5\ \text{kg}$ of air at a temperature of $80\ °\text{C}$. A gage on the tank reads $300\ \text{kPa}$. Determine the volume of the tank.

SOLUTION:

$$\text{Volume} = \frac{\text{mass}}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{(300+101)\times 10^3\ \frac{\text{N}}{\text{m}^3}}{\left(286.9\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(1\ \frac{\text{N}\cdot\text{m}}{\text{s}}\right)\left[(80\ °\text{C}+273)\,\text{K}\right]} = 3.96\ \frac{\text{kg}}{\text{m}^3}$$

$$\text{Volume} = \frac{5\ \text{kg}}{3.96\ \frac{\text{kg}}{\text{m}^3}} = \boxed{1.26\ \text{m}^3}$$

Problem 1.5.7

A rigid tank contains air at pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

SOLUTION:

$p = \rho RT$

For a rigid closed tank, the air mass and volume are constant so $\rho = \text{constant}$. Thus, from the equation here (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad \textbf{(1)}$$

where $p_1 = 90$ psia, $T_1 = 60\ ^\circ\text{F} + 460 = 520\ ^\circ\text{R}$

and $T_2 = 110\ ^\circ\text{F} + 460 = 570\ ^\circ\text{R}$

From Eq. 1

$$p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{570\ ^\circ\text{R}}{520\ ^\circ\text{R}}\right)(90\ \text{psia}) = \boxed{98.7\ \text{psia}}$$

Problem 1.5.8

The density of oxygen contained in a tank is 2.0 kg/m^3 when the temperature is 25 °C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

SOLUTION:

$$p = \rho RT = \left(2.0\ \frac{\text{kg}}{\text{m}^3}\right)\left(259.8\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left[(25\ ^\circ\text{C} + 273)\ \text{K}\right] = 155\ \text{kPa (abs)}$$

$$p(\text{gage}) = P_{\text{abs}} - P_{\text{atm}} = 155\ \text{kPa} - 97\ \text{kPa} = \boxed{58\ \text{kPa}}$$

Problem 1.5.9

The helium-filled blimp shown in Fig. P1.5.9 is used at various athletic events. Determine the number of pounds of helium within it if its volume is $68{,}000\ \text{ft}^3$ and the temperature and pressure are $80\ °\text{F}$ and 14.2 psia, respectively.

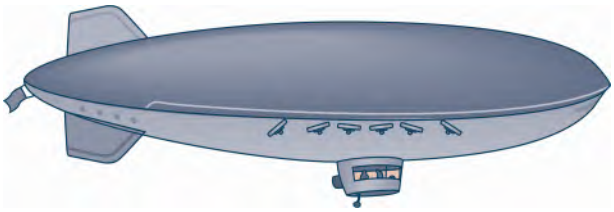


Figure P1.5.9

SOLUTION:

$W = \gamma V$ where $V = 68{,}000\ \text{ft}^3$ and $\gamma = \rho g = \left(\frac{p}{RT}\right) g$

Thus,

$$\gamma = \frac{14.2\ \frac{\text{lb}}{\text{in.}^2}\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(\left(1.242\times 10^4\ \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot °\text{R}}\right)(80+460)\ °\text{R}\right)}\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right) = 9.82\times 10^{-3}\ \frac{\text{slug}}{\text{ft}^2\cdot\text{s}^2}\left(\frac{1\ \text{lb}}{\left(\frac{\text{slug ft}}{\text{s}^2}\right)}\right)$$

$$= 9.82\times 10^{-3}\ \frac{\text{lb}}{\text{ft}^3}$$

Hence,

$$W = 9.82\times 10^{-3}\ \frac{\text{lb}}{\text{ft}^3}\left(68{,}000\ \text{ft}^3\right) = \boxed{668\ \text{lb}}$$

Problem 1.5.10

Develop a computer program for calculating the density of an ideal gas when the gas pressure in Pascals (abs), the temperature in degrees Celsius, and the gas constant in $\mathrm{J/kg \cdot K}$ are specified. Plot the density of helium as a function of temperature from $0\ ^\circ\mathrm{C}$ to $200\ ^\circ\mathrm{C}$ and pressures of 50, 100, 150, and 200 kPa (abs).

SOLUTION:

For an ideal gas

$$p = \rho \mathrm{R} T$$

so that

$$\rho = \frac{p}{\mathrm{R}T}$$

where p is the absolute pressure, R is the gas constant, and T is the absolute temperature. Thus, if the temperature is in $^\circ\mathrm{C}$ then

$$T = {}^\circ\mathrm{C} + 273.15\ \mathrm{K}$$

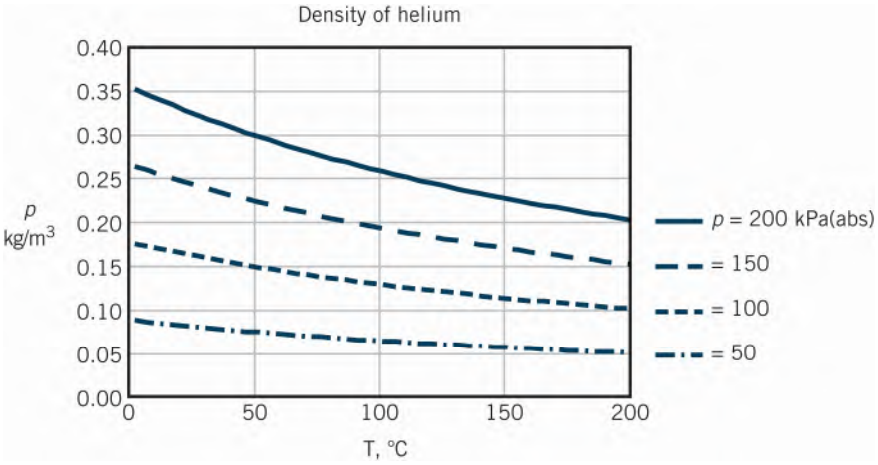
A spreadsheet (Excel) program for calculating ρ is given here.

This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees C, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.				
A	B	C	D	
Pressure	Temperature	Gas constant	Density	
Pa	°C	J/kg·K	kg/m³	
1.01E+05	15	286.9	1.22	
		Formula: =A10/((B10+273.15)*C10)		

Example: Calculate ρ for $p = 200$ kPa, temperature $= 20\ ^\circ\mathrm{C}$, and $R = 287\ \dfrac{\mathrm{J}}{\mathrm{kg \cdot K}}$

This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees Celsius, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.			
A	B	C	D
Pressure	Temperature	Gas constant	Density
Pa	°C	J/kg·K	kg/m^3
2.00E+05	20	287	2.38

The density of helium is plotted in the graph here.



Reserve Problem 1.5.1R

Acetylene (C_2H_2) is frequently used as a fuel for welding and cutting steel because it burns at the highest temperature of the commercially available fuel gasses (6300 °F / 3400 °C). If the gas in an acetylene tank can be modeled as an ideal gas, what is the density of acetylene at a temperature of 70 °F and a pressure of 20 psi?

SOLUTION:

$$\mathrm{R} = 1716 \frac{\mathrm{ft \cdot lb}}{\mathrm{slug} \cdot {}^{\circ}\mathrm{R}}$$

$$P = \rho \mathrm{R} T$$

$$\rho = \frac{P}{\mathrm{R}T} = \frac{20 \dfrac{\cancel{\mathrm{lb}}}{\cancel{\mathrm{in.}^2}} \times \dfrac{144 \cancel{\mathrm{in.}^2}}{\mathrm{ft}^2}}{\left(1716 \dfrac{\mathrm{ft \cdot \cancel{lb}}}{\mathrm{slug} \cdot \cancel{{}^{\circ}\mathrm{R}}}\right)\left((70+460)\cancel{{}^{\circ}\mathrm{R}}\right)} \quad \rightarrow \quad \boxed{\rho = 3.17 \times 10^{-3} \frac{\mathrm{slug}}{\mathrm{ft}^3}}$$

Reserve Problem 1.5.2R

Assuming that classroom size is determined using an average of 18 ft^2 per student, and a room height of 12 ft, determine the weight of air contained in a 40-student classroom assuming the air is at standard temperature and pressure and using the ideal gas model for the air, ideal gas.

SOLUTION:

$$P = \rho \mathrm{R} T$$

$$\rho = \frac{P}{\mathrm{R}T} = \frac{14.7 \dfrac{\cancel{\text{lb}}}{\cancel{\text{in.}^2}} \times \dfrac{144\ \cancel{\text{in.}^2}}{1\ \text{ft}^2}}{\left(1716 \dfrac{\text{ft} \cdot \cancel{\text{lb}}}{\text{lb}_\text{m} \cdot \cancel{^\circ\text{R}}}\right)\left(32 + 460\ \cancel{^\circ\text{R}}\right)} = 0.002507 \frac{\text{slug}}{\text{ft}^3}$$

$$W_{\text{AIR}} = \rho g \cancel{V} = \left(0.002507 \frac{\text{slug}}{\text{ft}^3}\right)\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)\left(18 \frac{\text{ft}^2}{\cancel{\text{student}}} \times 12\ \text{ft} \times 40\ \cancel{\text{students}}\right)$$

$$W_{\text{AIR}} = (0.002507)(32.2)(18 \times 12 \times 40) \frac{\text{slug} \cdot \text{ft}}{\text{s}^2} \times \frac{1\ \text{lb} \cdot \text{s}^2}{1\ \text{slug} \cdot \text{ft}} \quad \rightarrow \quad \boxed{W_{\text{AIR}} = 697\ \text{lb}}$$

Reserve Problem 1.5.3R

The pressure gage on a 7 ft^3 tank containing carbon dioxide at room temperature indicates a pressure of 275 psia. Use the ideal gas law to determine the mass of carbon dioxide in the tank using the ideal gas law.

SOLUTION:

Assume ideal gas model: $p = \rho \mathrm{R} T$

From text, R = 1130 ft-lb/slug-°R

$$\rho = \frac{p}{\mathrm{R}T} = \frac{\left(275 \dfrac{\mathrm{lb}}{\mathrm{in.}^2} \times \dfrac{144\ \mathrm{in.}^2}{1\ \mathrm{ft}^2}\right)}{\left(1130 \dfrac{\mathrm{ft \cdot lb}}{\mathrm{slug} \cdot {}^{\circ}\mathrm{R}} \times \dfrac{1\ \mathrm{slug}}{32.2\ \mathrm{lb_m}}\right)\left(72 + 460\ {}^{\circ}\mathrm{R}\right)} = 2.12 \frac{\mathrm{lb_m}}{\mathrm{ft}^3}$$

$$m = \rho \cancel{V} = \left(2.12 \frac{\mathrm{lb_m}}{\mathrm{ft}^3}\right)\left(7\ \mathrm{ft}^3\right) \qquad \rightarrow \qquad \boxed{m = 14.8\ \mathrm{lb_m}}$$

Reserve Problem 1.5.4R

A regulation basketball has a circumference of 29.5 in. Determine the mass of the air in the ball, expressed in slugs, when the air temperature is 70 °F and the ball is pumped up to the maximum allowable pressure of 8.5 psig.

SOLUTION:

$$m = \rho \cancel{V} = \left(\frac{p}{RT}\right)\left(\frac{4}{3}\pi R_{\text{BB}}{}^{3}\right)$$

$$\cancel{V} = \left(\frac{4}{3}\pi R_{\text{BB}}{}^{3}\right) = \left(\frac{4}{3}\pi\left(\frac{circum}{2\pi}\right)^{3}\right) = 433.5 \text{ in.}^{3}$$

$$\rho = \left(\frac{p}{RT}\right) = \left(\frac{(8.5+14.7)\dfrac{\text{lb}}{\text{in.}^{2}} \times \dfrac{144 \text{ in.}^{2}}{1 \text{ ft}^{2}}}{\left(1716\dfrac{\text{lb}\cdot\text{ft}}{\text{slug}\cdot{}^{\circ}\text{R}}T\right)\left(70+460\ {}^{\circ}\text{R}\right)}\right) = 0.003673\frac{\text{slug}}{\text{ft}^{3}}$$

$$m = \left(0.003673\frac{\text{slug}}{\text{ft}^{3}}\right)\left(433.5 \text{ in.}^{3} \times \frac{\text{ft}^{3}}{1728 \text{ in.}^{3}}\right) \quad \rightarrow \quad \boxed{m = 9.22\times10^{-4} \text{ slug}}$$