2/10

$$v^2 - v_0^2 = 2a(s - s_0)$$

$$0 - \left(\frac{96}{3.6}\right)^2 = 2a(36)$$

$$a = -9.88 \text{ m/s}^2$$

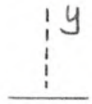
From 130 km/h:

$$0 - \left(\frac{130}{3.6}\right)^2 = 2(-9.88)s$$

$$\underline{s = 66.0 \text{ m}}$$

(Note: Braking tests support the notion of decelerations whose magnitudes are greater than 1g!)

2/11



① $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ 90 m:

$$90 = 0 + 50t - \frac{9.81}{2} t^2$$

$$t = 2.33,\ 7.86 \text{ s}$$

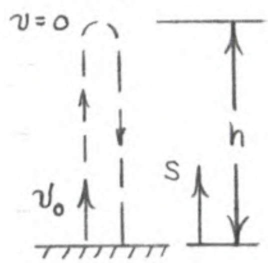
So impact must be when ball 1 is descending.

②: $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ 90 m:

$$90 = 0 + v_2 (7.86 - 3) - \frac{9.81}{2} (7.86 - 3)^2$$

$$\underline{v_2 = 42.4 \text{ m/s}}$$

2/12



$$v^2 = v_0^2 + 2a(s - s_0)$$

Apex: $0^2 = 200^2 + 2(-9.81)\,h$

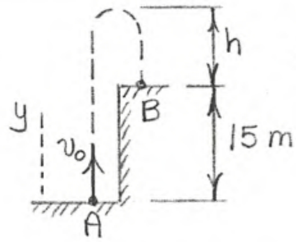
$$\underline{h = 2040 \text{ m}}$$

$$v = v_0 + at$$

Impact: $-200 = 200 - 9.81t$

$$\underline{t = 40.8 \text{ s}}$$

2/13



Evaluate $v^2 = v_0^2 - 2g(y - y_0)$ at apex:

$$0 = 25^2 - 2(9.81)(15 + h - 0)$$

$$\underline{h = 16.86 \text{ m}}$$

Evaluate $y = y_0 + v_0 t - \frac{1}{2} g t^2$ at B:

$$15 = 0 + 25t - \tfrac{1}{2}(9.81)t^2, \text{ or } 9.81t^2 - 50t + 30 = 0$$

$$t = \frac{50 \pm \sqrt{50^2 - 4(9.81)(30)}}{2(9.81)} = 0.695, \; 4.40 \text{ s}$$

$\underline{t = 4.40 \text{ s}}$ represents the <u>second</u> time at which $y = 15$ m.

$$v_B = v_0 - gt = 25 - 9.81(4.40) = -18.19 \text{ m/s}$$

$$\underline{(\text{or } 18.19 \text{ m/s downward})}$$

2/14 $v_B^2 - v_A^2 = 2a(s_B - s_A)$

$v_B^2 - 0 = 2(2.75)(3)$, $v_B = 4.06$ m/s

$s_B = s_A + v_0 t + \frac{1}{2}at^2$: $3 = \frac{1}{2}(2.75)t_{AB}^2$

$t_{AB} = 1.477$ s

$s_C = s_B + v_B t + \frac{1}{2}(0)t^2$: $4 = 4.06\, t_{BC}$

$t_{BC} = 0.985$ s

$t_{AC} = t_{AB} + t_{BC} = \underline{2.46 \text{ s}}$

2/15

|— — — s

home plate

$v = v_0 + at$ at end of acceleration:

$$v_{cr} = a t_{acc} \quad (1)$$

$v^2 = v_0^2 + 2a(s - s_0)$ at end of acceleration:

$$v_{cr}^2 = 2a(3) \quad (2)$$

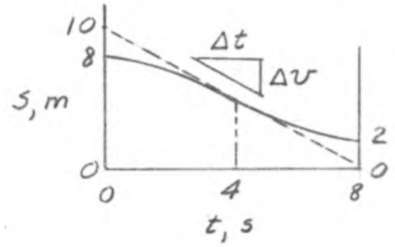
$s = s_0 + v_0 t + \frac{1}{2} a t^2$ at first base:

$$24 = v_{cr}(4 - t_{acc}) \quad (3)$$

Solve Eqs. (1)-(3) for v_{cr}, a, and t_{acc}:

$$\begin{cases} v_{cr} = 7.5 \text{ m/s} \\ a = 9.38 \text{ m/s}^2 \\ t_{acc} = 0.8 \text{ s} \end{cases}$$

2/16



$$v_{AV} = \frac{s_2 - s_1}{\Delta t} = \frac{2-8}{8} = \underline{-0.75 \text{ m/s}}$$

$$v = v_4 = \frac{ds}{dt} = \frac{-10}{8} = \underline{-1.25 \text{ m/s}}$$

2/17 $v_0 = 100/3.6 = 27.8 \text{ m/s}$

$a = -g \sin\theta = -9.81 \sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588 \text{ m/s}^2$

(a) $v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$

(b) $v^2 = v_0^2 + 2a(s - s_0) = 27.8^2 + 2(-0.588)(100)$

$v = \underline{25.6 \text{ m/s}}$

2/18

$$v_C = v_B + a\Delta t_{B-C}, \quad a = \frac{(60-100)/3.6}{4} = -2.78 \text{ m/s}^2$$

$$\Delta s_{B-C} = v_B \Delta t_{B-C} + \frac{1}{2} a \Delta t_{B-C}^2$$

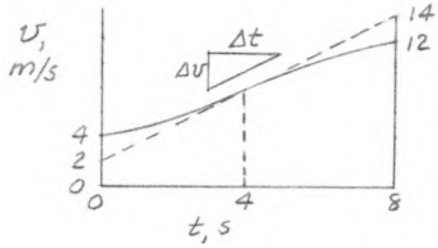
$$= \frac{100}{3.6} 4 + \frac{1}{2}(-2.78)4^2 = 88.9 \text{ m}$$

$$\Delta s_{A-D} = \Delta s_{A-B} + \Delta s_{B-C} + \Delta s_{C-D}$$

$$3000 = \frac{100}{3.6} t + 88.9 + \frac{60}{3.6} t, \quad \underline{t = 65.5 \text{ s}}$$

$$s = \Delta s_{A-B} = \frac{100}{3.6}(65.5) = 1819 \text{ m or } \underline{s = 1.819 \text{ km}}$$

2/19



$$a_{AV} = \frac{\Delta v}{\Delta t} = \frac{12-4}{8} = 1.00 \text{ m/s}^2$$

$$a_4 = \frac{dv}{dt} = \frac{14-2}{8} = 1.50 \text{ m/s}^2$$

$$\Delta a = a_4 - a_{AV} = 1.50 - 1.00 = 0.5 \ \frac{m}{s^2}$$

$$\Delta s = \int v\,dt \approx \frac{14+2}{2}(8) = 64 \text{ m}$$

2/20 $\int_{0.4}^{v} v\,dv = \int a_x\,dx$; $\frac{1}{2}(v^2 - 0.4^2)$ = area under a_x-x curve

Area $= \int a_x\,dx = (a_x)_{av}\Delta x = 3(120-40)10^{-3} = 0.240\ (m/s)^2$

Thus $v^2 = 0.4^2 + 2(0.240) = 0.16 + 0.48 = 0.64$

$v = \sqrt{0.64} = \underline{0.8\ m/s}$

2/21

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$0 = 4^2 + 2\left(-\frac{9.81}{4}\right)(s), \quad \underline{s = 3.26 \text{ m}}$$

$$v = v_0 + at: \quad 0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}, \quad t_{up} = 1.631 \text{ s}$$

$$t = 2t_{up} = 2(1.631) = \underline{3.26 \text{ s}}$$

2/22 Train : $v^2 = v_1^2 + 2as : \left(\frac{96}{3.6}\right)^2 = \left(\frac{130}{3.6}\right)^2 + 2a(800)$

$a = -0.371 \text{ m/s}^2$

$s = v_1 t + \frac{1}{2}at^2 : 1600 = \frac{130}{3.6}t - \frac{0.371}{2}t^2$

$t = 68.1 \text{ s}$ or $t = 126.8 \text{ s}$ (disregard)

Car: $t = 68.1 - 4 = 64.1 \text{ s}$

$s = v_1 t + \frac{1}{2}at^2 : 2000 = \frac{80}{3.6}(64.1) + \frac{a}{2}(64.1)^2$

$\underline{a = 0.280 \text{ m/s}^2}$

$v = v_1 + at = \left(\frac{80}{3.6}\right) + 0.280(64.1) = 40.2 \text{ m/s}$

$\underline{(144.6 \text{ km/h})}$

2/23 $v_A = \frac{130}{3.6} = 36.1 \text{ m/s} = \text{constant}$

$s_A = v_A t = 36.1t$

$(s_P)_{acc} = \frac{1}{2} a_P t^2 = \frac{1}{2}(6)t^2 = 3t^2$

$(v_P)_{acc} = a_P t \; ; \quad t_{acc} = \frac{(v_P)_{acc}}{a_P} = \frac{160/3.6}{6} = 7.41 \text{ s}$

So $(s_P)_{acc} = 3(7.41)^2 = 164.6 \text{ m}$

$s_P = (s_P)_{acc} + (s_P)_{cr} = 164.6 + \frac{160}{3.6}(t - 7.41)$

$= 44.4t - 164.6$

Now set $s_A = s_P$: $36.1t = 44.4t - 164.6$

$t = 19.75 \text{ s}$

Then $s = 36.1(19.75) = \underline{713 \text{ m}}$

2/24 $(a_A)_{dec} = \dfrac{(100-130)/3.6}{5} = -1.667 \text{ m/s}^2$ 5)

$(s_A)_{dec} = (v_A)_0 t_{dec} + \frac{1}{2}(a_A)_{dec} t_{dec}^2$

$= \dfrac{130}{3.6}(5) + \frac{1}{2}(-1.667)5^2 = 159.7 \text{ m}$ 6)

$s_A = (s_A)_{dec} + (s_A)_{cr} = 159.7 + \dfrac{100}{3.6}(t-5)$

$= 20.8 + 27.8t$ 7), 8)

From solution to previous problem,

$s_P = 44.4t - 164.6$

Set $s_A = s_P$: $20.8 + 27.8t = 44.4t - 164.6$

$t = 11.13 \text{ s}$ 9)

Then $s_P = 44.4(11.13) - 164.6$

$= \underline{330 \text{ m}}$

2/25 $(a_A)_{acc} = \dfrac{(150-130)/3.6}{5} = 1.111 \text{ m/s}^2$

$(s_A)_{acc} = (v_A)_0 t_{acc} + \frac{1}{2}(a_A)_{acc} t_{acc}^2$

$= \frac{130}{3.6}(5) + \frac{1}{2}(1.111)(5)^2 = 194.4 \text{ m}$

$s_A = (s_A)_{acc} + (s_A)_{cr} = 194.4 + \frac{150}{3.6}(t-5)$

$= 41.7t - 13.89$

From solution to Prob. 2/

$s_P = 44.4t - 164.6$

Set $s_A = s_P$: $41.7t - 13.89 = 44.4t - 164.6$

$t = 54.3 \text{ s}$

Then $s_P = 44.4(54.3) - 164.6$

$= \underline{2250 \text{ m}}$ (!)

2/26 $a = 130 - kx$, where $k = \dfrac{130}{0.150} = 867\ \text{s}^{-2}$

So $a = 130(1 - 6.67x)$ (x in meters)

$v\,dv = a\,dx$: $\displaystyle\int_0^v v\,dv = 130\int_0^x (1 - 6.67x)\,dx$

$v^2 = 260\,(x - 3.33x^2)$, $v = \dfrac{dx}{dt} = \sqrt{260\,(x - 3.33x^2)}$ (taking + sign)

$$\int_0^t dt = \int_0^x \frac{dx}{\sqrt{260\,(x - 3.33x^2)}}$$

$$t = \frac{1}{\sqrt{260}}\,\frac{1}{\sqrt{3.33}}\,\sin^{-1}\left[\frac{2(3.33)x - 1}{\sqrt{1}}\right]_0^x$$

$$= 0.0340\left[\sin^{-1}(6.67x - 1) + \frac{\pi}{2}\right]$$

(a) $x = 0.075$ m : $t = 0.0340\left[\sin^{-1}\left(6.67(0.075) - 1\right) + \frac{\pi}{2}\right]$

$= \underline{0.0356\ \text{s}}$

(b) $x = 0.150$ m : $t = 0.0340\left[\sin^{-1}\left(6.67(0.150) - 1\right) + \frac{\pi}{2}\right]$

$= \underline{0.0534\ \text{s}}$

2/27 $a = \frac{dv}{dt}$; $v_m = \int a\,dt = at = 6(20) = \underline{120 \text{ m/s}}$

Corresponding $h = \frac{1}{2}at^2 = \frac{1}{2}(6)(20)^2 = 1200 \text{ m}$

During upward coast, $\int_{v_m}^{0} v\,dv = \int_{0}^{\Delta h} -g\,dy$

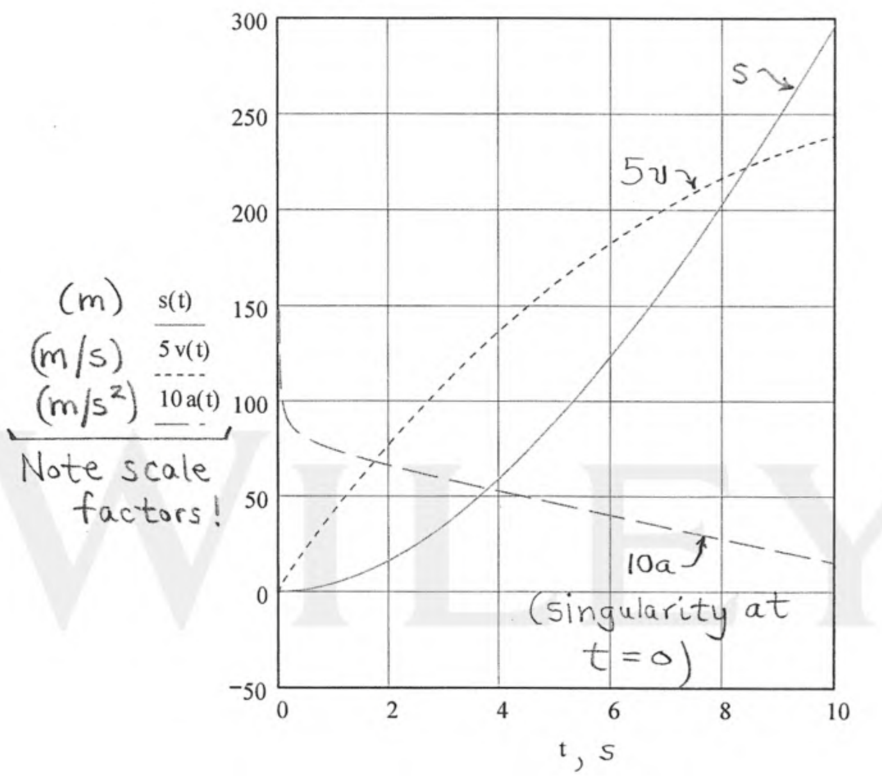
$v_m^2 = 2g\Delta h$, $\Delta h = \frac{\overline{120}^2}{2(9.81)} = 734 \text{ m}$

Max. $h = 1200 + 734 = 1934 \text{ m}$ or $\underline{h = 1.934 \text{ km}}$

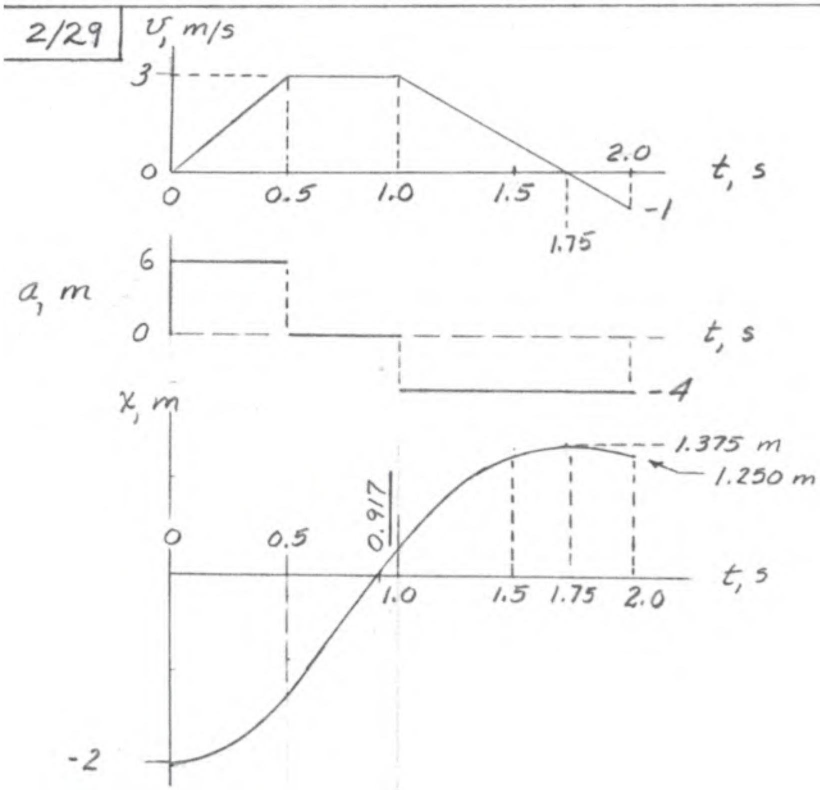
2/28 $\quad v = \frac{ds}{dt} = 7.3t - 0.3t^2 + 1.5\sqrt{t}$

$$\int_{s_0=0}^{s} ds = \int_0^t (7.3t - 0.3t^2 + 1.5\sqrt{t})\,dt$$

$\underline{s = 3.65t^2 - 0.1t^3 + t^{3/2}}$; $\underline{s(10) = 297 \text{ m}}$



2/29



v, m/s
3
0
0.5
1.0
1.5
2.0
1.75
-1
t, s
a, m
6
0
-4
t, s
x, m
1.375 m
1.250 m
0
0.5
0.917
1.0
1.5
1.75
2.0
t, s
-2

2/30 $v^2 = k/s$, $v = \dot{s} = \sqrt{k/s}$ where $k = (50)^2(225)$

$= 0.562(10^6)\ \frac{mm^3}{s^2}$

$v = \frac{ds}{dt}$ so $\sqrt{\frac{k}{s}} = \frac{ds}{dt}$, $\sqrt{k}\,dt = \sqrt{s}\,ds$

$\sqrt{0.562}(10^3)\int_0^t dt = \int_{225}^{s} \sqrt{s}\,ds$, $750t = \frac{2}{3}s^{3/2}\Big]_{225}^{s}$

$s^{3/2} = 3375 + 1125t$ but $v = \sqrt{k/s}$ so

$v = 750\,(3375 + 1125t)^{-1/3}$ & at $t = 3\text{ s}$,

$v = 750(6750)^{-1/3} = \underline{39.7\text{ mm/s}}$

2/31 From $a = v\frac{dv}{dx}$, $\int a\,dx = \int_v^0 v\,dv$

$\Rightarrow \frac{1}{2}v^2 = -\int a\,dx$

Interval, m	$a\Delta x$, m^2/s^2
0 - 0.1	$-6g(0.1)$
0.1 - 0.2	$-6.1g(0.1)$
0.2 - 0.3	$-6.4g(0.1)$
0.3 - 0.4	$-6.9g(0.1)$
0.4 - 0.5	$-7.6g(0.1)$
0.5 - 0.6	$-8.1g(0.1)$
0.6 - 0.7	$-8.9g(0.1)$
0.7 - 0.8	$-9.5g(0.1)$
	$\Sigma = -58.4\ m/s^2$

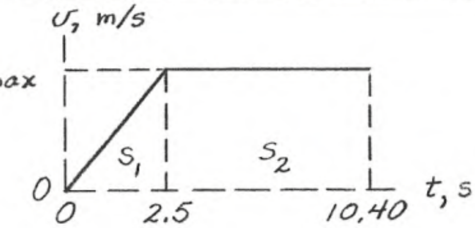
$\frac{1}{2}v^2 \cong -\Sigma a\Delta x = -(-58.4)$

$v = 10.80$ m/s or $10.80(3.6) = \underline{38.9\ \frac{km}{h}}$

2/32

$s_1 = \frac{1}{2}(2.5)v_{max}$

$s_2 = (10.40 - 2.5)v_{max}$



$s_1 + s_2 = (1.25 + 7.90)v_{max} = 100$

$$\underline{v_{max} = 10.93 \text{ m/s}}$$

2/33 $v = 20 - ks$ where $k = \frac{2}{3}\,s^{-1}$

So $v = 20 - \frac{2}{3}s$, $\frac{dv}{ds} = -\frac{2}{3}\,s^{-1}$

Note that $v = 10$ m/s when $s = 15$ m

$\therefore\ a = v\frac{dv}{ds} = 10\left(-\frac{2}{3}\right) = \underline{-6.67\ \text{m/s}^2}$

$v = \frac{ds}{dt} \Rightarrow \int_0^t dt = \int_0^{30}\frac{ds}{v} = \int_0^{30}\frac{ds}{20-\frac{2}{3}s}$

$t = -\frac{3}{2}\ln\left(20-\frac{2}{3}s\right)\Big|_0^{30} = -\frac{3}{2}\ln\frac{0}{20} \rightarrow \underline{\infty}$

2/34 $0 < t < 10\text{ s}$: $a = 6 - kt$, $k = \frac{6}{10}\text{ m/s}^3$

$a = \frac{dv}{dt} = 6\left(1 - \frac{t}{10}\right)$, t in s, a in m/s^2

$\int_0^v dv = \int_0^t 6\left(1 - \frac{t}{10}\right)dt$, $v = 6t - \frac{3}{10}t^2$

$v_{10} = 6(10) - \frac{3}{10}(10)^2 = 30\text{ m/s}$

$v = \frac{ds}{dt}$, $s_{10} = \int_0^{10}\left(6t - \frac{3}{10}t^2\right)dt = 200\text{ m}$

$t > 10\text{ s}$: $\Delta s = v_{10}\,\Delta t$, $\Delta t = \frac{400 - 200}{30} = 6.67\text{ s}$

$t = 10 + \Delta t = \underline{16.67\text{ s}}$

2/35 A to B; $v_B^2 = v_A^2 + 2a\Delta s$

$v_B^2 = (1.2)^2 + 2(0.3)(9.81)(3) = 19.10 \ (m/s)^2$

$v_B = 4.37 \ m/s$

$v_B = v_A + at$; $t_B = (4.37 - 1.2)/0.3(9.81) = 1.077 \ s$

$\Delta t = t_C - t_B = 2.8 - 1.077 = \underline{1.723 \ s}$

$v_C^2 = v_B^2 + 2a\Delta s$, $0 = 19.10 + 2a(3.6)$, $\underline{a = -2.65 \ m/s^2}$