Chapter 1 • Introduction

P1.1 A gas at 20°C may be *rarefied* if it contains less than 10^{12} molecules per mm^3. If Avogadro's number is 6.023E23 molecules per mole, what air pressure does this represent?

Solution: The mass of one molecule of air may be computed as

$$m = \frac{\text{Molecular weight}}{\text{Avogadro's number}} = \frac{28.97\ \text{mol}^{-1}}{6.023\text{E}23\ \text{molecules/g}\cdot\text{mol}} = 4.81\text{E–}23\ \text{g}$$

Then the density of air containing 10^{12} molecules per mm^3 is, in SI units,

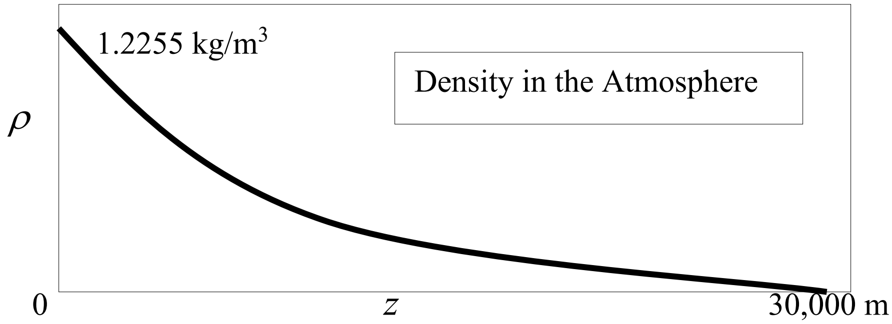
$$\rho = \left(10^{12}\ \frac{\text{molecules}}{\text{mm}^3}\right)\left(4.81\text{E–}23\ \frac{\text{g}}{\text{molecule}}\right)$$
$$= 4.81\text{E–}11\ \frac{\text{g}}{\text{mm}^3} = 4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}$$

Finally, from the perfect gas law, Eq. (1.13), at 20°C = 293 K, we obtain the pressure:

$$p = \rho RT = \left(4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}\right)\left(287\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}\right)(293\ \text{K}) = \mathbf{4.0\ Pa} \quad \textit{Ans.}$$

P1.2 Table A.6 lists the density of the standard atmosphere as a function of altitude. Use these values to estimate, crudely, say, within a factor of 2, the number of molecules of air in the entire atmosphere of the earth.

Solution: Make a plot of density ρ versus altitude z in the atmosphere, from Table A.6:



This writer's approximation: The curve is approximately an exponential, $\rho \approx \rho_o \exp(-bz)$, with b approximately equal to 0.00011 per meter. Integrate this over the entire atmosphere, with the radius of the earth equal to 6377 km:

$$m_{atmosphere} = \int \rho\, d(vol) \approx \int_0^{\infty} [\rho_o e^{-bz}](4\pi R_{earth}^2\, dz) =$$

$$= \frac{\rho_o 4\pi R_{earth}^2}{b} = \frac{(1.2255\, kg/m^3)4\pi(6.377E6\, m)^2}{0.00011/m} \approx 5.7E18\ kg$$

Dividing by the mass of one molecule ≈ 4.8E−23 g (see Prob. 1.1 above), we obtain the total number of molecules in the earth's atmosphere:

$$N_{molecules} = \frac{m(\text{atmosphere})}{m(\text{one molecule})} = \frac{5.7E21 \text{ grams}}{4.8E{-}23 \text{ gm/molecule}} \approx \mathbf{1.2E44}\ \textit{molecules} \quad \textit{Ans.}$$

This estimate, though crude, is within 10 per cent of the exact mass of the atmosphere.

P1.3 For the triangular element in Fig. P1.3, show that a tilted free liquid surface, in contact with an atmosphere at pressure pa, must undergo shear stress and hence begin to flow.

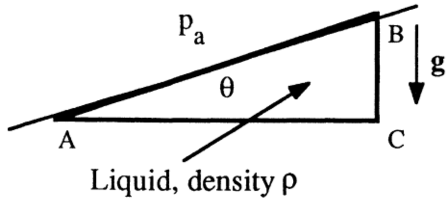
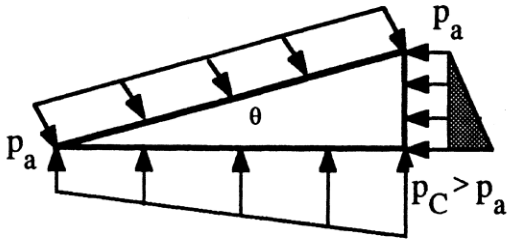


Fig. P1.3

Solution: Assume zero shear. Due to element weight, the pressure along the lower and right sides must vary linearly as shown, to a higher value at point C. Vertical forces are presumably in balance with ele-ment weight included. But horizontal forces are out of balance, with the unbalanced force being to the left, due to the shaded excess-pressure triangle on the right side BC. Thus hydrostatic pressures cannot keep the element in balance, and shear and flow result.



P1.4 Sand, and other granular materials, definitely *flow*, that is, you can pour them from a container or a hopper. There are whole textbooks on the "transport" of granular materials [54]. Therefore, is sand a *fluid*? Explain.

Solution: Granular materials do indeed *flow*, at a rate that can be measured by "flowmeters". But they are *not* true fluids, because they can support a small shear stress without flowing. They may rest at a finite angle without flowing, which is not possible for liquids (see Prob. P1.3). The maximum such angle, above which sand begins to flow, is called the *angle of repose*. A familiar example is sugar, which pours easily but forms a significant angle of repose on a heaping spoonful. The physics of granular materials are complicated by effects such as particle cohesion, clumping, vibration, and size segregation. See Ref. 48 to learn more.

P1.5 A formula for estimating the mean free path of a perfect gas is:

$$\ell = 1.26\frac{\mu}{\rho\sqrt{RT}} = 1.26\frac{\mu}{p}\sqrt{RT} \qquad (1)$$

where the latter form follows from the ideal-gas law, $\rho = p/RT$. What are the dimensions of the constant "1.26"? Estimate the mean free path of air at 20°C and 7 kPa. Is air *rarefied* at this condition?

Solution: We know the dimensions of every term except "1.26":

$$\{\ell\} = \{L\} \quad \{\mu\} = \left\{\frac{M}{LT}\right\} \quad \{\rho\} = \left\{\frac{M}{L^3}\right\} \quad \{R\} = \left\{\frac{L^2}{T^2\Theta}\right\} \quad \{T\} = \{\Theta\}$$

Therefore the above formula (first form) may be written dimensionally as

$$\{L\} = \{1.26?\}\frac{\{M/L\cdot T\}}{\{M/L^3\}\sqrt{[\{L^2/T^2\cdot\Theta\}\{\Theta\}]}} = \{1.26?\}\{L\}$$

Since we have {L} on both sides, {1.26} = {unity}, that is, the constant is dimensionless. The formula is therefore dimensionally homogeneous and should hold for any unit system.

For air at 20°C = 293 K and 7000 Pa, the density is $\rho = p/RT = (7000)/[(287)(293)] = 0.0832$ kg/m^3. From Table A-2, its viscosity is 1.80E–5 N·s/m^2. Then the formula predicts a mean free path of

$$\ell = 1.26\frac{1.80\text{E–}5}{(0.0832)[(287)(293)]^{1/2}} \approx \mathbf{9.4E\text{–}7\ m} \quad \textit{Ans.}$$

This is quite small. We would judge this gas to approximate a continuum if the physical scales in the flow are greater than about 100 ℓ, that is, greater than about 94 μm.

P1.6 Henri Darcy, a French engineer, proposed that the pressure drop Δp for flow at velocity V through a tube of length L could be correlated in the form

$$\frac{\Delta p}{\rho} = \alpha L V^2$$

If Darcy's formulation is consistent, what are the dimensions of the coefficient α?
Solution: From Table 1.2, introduce the dimensions of each variable:

$$\left\{\frac{\Delta p}{\rho}\right\} = \left\{\frac{ML^{-1}T^{-2}}{ML^{-3}}\right\} = \left\{\frac{L^2}{T^2}\right\} = \left\{\alpha L V^2\right\} = \{\alpha\}\{L\}\left\{\frac{L^2}{T^2}\right\}$$

Solve for $\{\alpha\} = \{L^{-1}\}$ *Ans.*

[The complete Darcy correlation is $\alpha = f/(2D)$, where D is the tube diameter, and f is a dimensionless friction factor (Chap. 6).]

P1.7 Convert the following inappropriate quantities into SI units: (*a*) 2.283E7 U.S. gallons per day; (*b*) 4.48 furlongs per minute (racehorse speed); and (*c*) 72,800 avoirdupois ounces per acre.

Solution: (*a*) (2.283E7 gal/day) x (0.0037854 m^3/gal) ÷ (86,400 s/day) =
1.0 m^3/s *Ans.*(*a*)
(*b*) 1 furlong = (⅛)mile = 660 ft.
Then (4.48 furlongs/min)x(660 ft/furlong)x(0.3048 m/ft)÷(60 s/min) =
15 m/s *Ans.*(*b*)
(*c*) (72,800 oz/acre)÷(16 oz/lbf)x(4.4482 N/lbf)÷(4046.9 acre/m^2) =
5.0 N/m^2 = 5.0 Pa *Ans.*(*c*)

P1.8 Suppose that bending stress σ in a beam depends upon bending moment M and beam area moment of inertia I and is proportional to the beam half-thickness y. Suppose also that, for the particular case M = 2900 in·lbf, y = 1.5 in, and I = 0.4 in^4, the predicted stress is 75 MPa. Find the only possible dimensionally homogeneous formula for σ.

Solution: We are given that σ = y fcn(M,I) and we are *not* to study up on strength of materials but only to use dimensional reasoning. For homogeneity, the right hand side must have dimensions of stress, that is,

$$\{\sigma\} = \{y\}\,\{\text{fcn(M,I)}\}, \quad \text{or:} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\,\{\text{fcn(M,I)}\}$$

$$\text{or:} \quad \text{the function must have dimensions } \{\text{fcn(M,I)}\} = \left\{\frac{M}{L^2T^2}\right\}$$

Therefore, to achieve dimensional homogeneity, we somehow must combine bending moment, whose dimensions are $\{ML^2T^{-2}\}$, with area moment of inertia, $\{I\} = \{L^4\}$, and end up with $\{ML^{-2}T^{-2}\}$. Well, it is clear that {I} contains neither mass {M} nor time {T} dimensions, but the bending moment contains both mass and time and in exactly the combination we need, $\{MT^{-2}\}$. Thus it must be that σ *is proportional to M also*. Now we have reduced the problem to:

$$\sigma = yM\,\text{fcn(I)}, \quad \text{or} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\left\{\frac{ML^2}{T^2}\right\}\{\text{fcn(I)}\}, \quad \text{or:} \quad \{\text{fcn(I)}\} = \mathbf{\{L^{-4}\}}$$

We need just enough *I*'s to give dimensions of $\{L^{-4}\}$: we need the formula to be exactly *inverse* in *I*. The correct dimensionally homogeneous beam bending formula is thus:

$$\boldsymbol{\sigma} = \mathbf{C}\frac{\mathbf{My}}{\mathbf{I}}, \quad \text{where } \{C\} = \{\text{unity}\} \quad \textit{Ans.}$$

The formula admits to an arbitrary dimensionless constant C whose value can only be obtained from known data. Convert stress into English units: σ = (75 MPa)/(6894.8) = 10880 lbf/in^2. Substitute the given data into the proposed formula:

$$\sigma = 10880\ \frac{\text{lbf}}{\text{in}^2} = C\frac{My}{I} = C\frac{(2900\ \text{lbf·in})(1.5\ \text{in})}{0.4\ \text{in}^4}, \quad \text{or:} \quad \mathbf{C \approx 1.00} \quad \textit{Ans.}$$

The data show that C = 1, or $\boldsymbol{\sigma}$ **= My/I**, our old friend from strength of materials.

P1.9 A hemispherical container, 26 inches in diameter, is filled with a liquid at 20°C and weighed. The liquid weight is found to be 1617 ounces. (*a*) What is the density of the fluid, in kg/m^3? (*b*) What fluid might this be? Assume standard gravity, g = 9.807 m/s^2.

Solution: First find the volume of the liquid in m^3:

$$\text{Hemisphere volume} = \frac{1}{2}\left(\frac{\pi}{6}\right)D^3 = \frac{1}{2}\left(\frac{\pi}{6}\right)(26\ in)^3 = 4601\, in^3 \div \left(61024\frac{in^3}{m^3}\right) = 0.0754\, m^3$$

$$\text{Liquid mass} = 1617\ oz \div 16 = 101\, lbm\left(0.45359\frac{kg}{lbm}\right) = 45.84\ kg$$

$$\text{Then the liquid density} = \frac{45.84\ kg}{0.0754\ m^3} = \mathbf{607}\frac{\boldsymbol{kg}}{\boldsymbol{m^3}} \quad \textit{Ans.}(a)$$

From Appendix Table A.3, this could very well be **ammonia**. *Ans.*(*b*)

P1.10 The Stokes-Oseen formula [10] for drag on a sphere at low velocity V is:

$$\mathrm{F} = 3\pi\mu\mathrm{DV} + \frac{9\pi}{16}\rho\mathrm{V}^2\mathrm{D}^2$$

where D = sphere diameter, μ = viscosity, and ρ = density. Is the formula homogeneous?

Solution: Write this formula in dimensional form, using Table 1-2:

$$\{\mathrm{F}\} = \{3\pi\}\{\mu\}\{\mathrm{D}\}\{\mathrm{V}\} + \left\{\frac{9\pi}{16}\right\}\{\rho\}\{\mathrm{V}\}^2\{\mathrm{D}\}^2\ ?$$

$$\text{or:}\quad \left\{\frac{\mathrm{ML}}{\mathrm{T}^2}\right\} = \{1\}\left\{\frac{\mathrm{M}}{\mathrm{LT}}\right\}\{\mathrm{L}\}\left\{\frac{\mathrm{L}}{\mathrm{T}}\right\} + \{1\}\left\{\frac{\mathrm{M}}{\mathrm{L}^3}\right\}\left\{\frac{\mathrm{L}^2}{\mathrm{T}^2}\right\}\{\mathrm{L}^2\}\ ?$$

where, hoping for homogeneity, we have assumed that all constants (3,π,9,16) are *pure*, i.e., {unity}. Well, yes indeed, all terms have dimensions {ML/T^2}! Therefore the Stokes-Oseen formula (derived in fact from a theory) is ***dimensionally homogeneous.***

P1.11 In English Engineering units, the specific heat c_p of air at room temperature is approximately 0.24 Btu/(lbm-°F). When working with kinetic energy relations, it is more appropriate to express c_p as a velocity-squared per absolute degree. Give the numerical value, in this form, of c_p for air in (*a*) SI units, and (*b*) BG units.

Solution: From Appendix C, *Conversion Factors*, 1 Btu = 1055.056 J (or N-m) = 778.17 ft-lbf, and 1 lbm = 0.4536 kg = (1/32.174) slug. Thus the conversions are:

$$\text{SI units}: 0.24\frac{Btu}{lbm^\circ F} = 0.24\frac{1055.056\,N\cdot m}{(0.4536\,kg)(1K/1.8)} = 1005\frac{N\cdot m}{kg\cdot K} = \mathbf{1005}\frac{m^2}{s^2\,K} \quad Ans.(a)$$

$$\text{BG units}: 0.24\frac{Btu}{lbm^\circ F} = 0.24\frac{778.17\,ft\cdot lbf}{[(1/32.174)slug](1^\circ R)} = 6009\frac{ft\cdot lbf}{slug\cdot^\circ R} = \mathbf{6009}\frac{ft^2}{s^2\,^\circ R} \quad Ans.(b)$$

P1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad Ans.$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

P1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if Δp = 35 psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that QΔp has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s};\quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2};\quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*

P1.14 The volume flow Q over a dam is proportional to dam width B and also varies with gravity g and excess water height H upstream, as shown in Fig. P1.14. What is the only possible dimensionally homo-geneous relation for this flow rate?

Solution: So far we know that Q = B fcn(H,g). Write this in dimensional form:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} = \{B\}\{f(H,g)\} = \{L\}\{f(H,g)\},$$

$$\text{or:}\quad \{f(H,g)\} = \left\{\frac{L^2}{T}\right\}$$

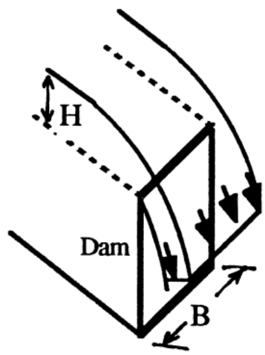


Fig. P1.14

So the function fcn(H,g) must provide dimensions of $\{L^2/T\}$, but only *g* contains *time*. Therefore g must enter in the form $g^{1/2}$ to accomplish this. The relation is now

$Q = Bg^{1/2}\text{fcn}(H)$, or: $\{L^3/T\} = \{L\}\{L^{1/2}/T\}\{\text{fcn}(H)\}$, or: $\{\text{fcn}(H)\} = \{\mathbf{L^{3/2}}\}$In order for fcn(H) to provide dimensions of $\{L^{3/2}\}$, the function must be a 3/2 power. Thus the final desired homogeneous relation for dam flow is:

$$\mathbf{Q = C\,B\,g^{1/2}\,H^{3/2}},\quad \text{where C is a dimensionless constant} \quad \textit{Ans.}$$

P1.15 The height *H* that fluid rises in a liquid barometer tube depends upon the liquid density *ρ*, the barometric pressure *p*, and the acceleration of gravity *g*. (a) Arrange these four variables into a single dimensionless group. (b) Can you deduce (or guess) the numerical value of your group?

Solution: This is a problem in *dimensional analysis*, covered in detail in Chapter 5. Use the symbols for dimensions suggested with Eq. (1.2): *M* for mass, *L* for length, *T* for time, *F* for force,

$$\{H\} = \{L\},\quad \{\rho\} = \{M/L^3\},\quad \{g\} = \{L/T^2\},\quad \{p\} = \{F/L^2\} = \{M/(LT^2)\}$$

where the change in pressure dimensions uses Newton's law, $\{F\} = \{ML/T^2\}$. We see that we can cancel mass by dividing density by pressure:

$$\left\{\frac{\rho}{p}\right\} = \left\{\frac{ML^{-3}}{ML^{-1}T^{-2}}\right\} = \left\{\frac{T^2}{L^2}\right\}$$

We can eliminate time by multiplying by $\{g\}$: $\{(\rho/p)(g)\} = \{(T^2/L^2)(L/T^2)\} = \{L^{-1}\}$. Finally, we can eliminate length by multiplying by the height $\{H\}$:

$$\left\{\left(\frac{\rho g}{p}\right)(H)\right\} = \{L^{-1}\}\{L\} = \{1\} \quad \text{dimensionless}$$

Thus the desired dimensionless group is $\boldsymbol{\rho gH/p}$, or its inverse, $\boldsymbol{p/\rho gH}$. *Answer* (a)

(b) You might remember from physics, or other study, that the barometer formula is $p \approx \rho gH$. Thus this dimensionless group has a value of approximately 1.0, or **unity**. *Answer* (b)

P1.16 Test the dimensional homogeneity of the boundary-layer *x*-momentum equation:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\rho g_x+\frac{\partial \tau}{\partial y}$$

Solution: This equation, like **all** theoretical partial differential equations in mechanics, is dimensionally homogeneous. Test each term in sequence:

$$\left\{\rho u\frac{\partial u}{\partial x}\right\}=\left\{\rho v\frac{\partial u}{\partial y}\right\}=\frac{M}{L^3}\frac{L}{T}\frac{L/T}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial p}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

$$\{\rho g_x\}=\frac{M}{L^3}\frac{L}{T^2}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\};\quad \left\{\frac{\partial \tau}{\partial x}\right\}=\frac{M/LT^2}{L}=\left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

All terms have dimension $\{ML^{-2}T^{-2}\}$. This equation may use *any* consistent units.

P1.17 Investigate the consistency of the Hazen-Williams formula from hydraulics:

$$Q=61.9D^{2.63}\left(\frac{\Delta p}{L}\right)^{0.54}$$

What are the dimensions of the constant "61.9"? Can this equation be used with confidence for a variety of liquids and gases?

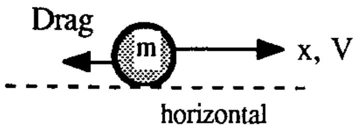
Solution: Write out the dimensions of each side of the equation:

$$\{Q\}=\left\{\frac{L^3}{T}\right\}\stackrel{?}{=}\{61.9\}\{D^{2.63}\}\left\{\frac{\Delta p}{L}\right\}^{0.54}=\{61.9\}\{L^{2.63}\}\left\{\frac{M/LT^2}{L}\right\}^{0.54}$$

The constant 61.9 has *fractional* dimensions: $\{61.9\}=\mathbf{\{L^{1.45}T^{0.08}M^{-0.54}\}}$ *Ans.*

Clearly, the formula is extremely inconsistent and cannot be used with confidence for any given fluid or condition or units. Actually, the Hazen-Williams formula, still in common use in the watersupply industry, is valid only for **water** flow in smooth pipes larger than 2-in. diameter and turbulent velocities less than 10 ft/s and (certain) English units. This formula should be held at arm's length and given a vote of "No Confidence."

***P1.18** ("*" means "difficult"—not just a plug-and-chug, that is) For small particles at low velocities, the first (linear) term in Stokes' drag law, Prob. 1.10, is dominant, hence F = KV, where K is a constant. Suppose a particle of mass m is constrained to move horizontally from the initial position x = 0 with initial velocity $V = V_0$. Show (a) that its velocity will decrease exponentially with time; and (b) that it will stop after travelling a distance $x = mV_0/K$.



Solution: Set up and solve the differential equation for forces in the *x*-direction:

$$\sum F_x = -\text{Drag} = ma_x, \quad \text{or:} \quad -KV = m\frac{dV}{dt}, \quad \text{integrate} \int_{V_o}^{V} \frac{dV}{V} = -\int_0^t \frac{m}{K}dt$$

$$\text{Solve} \quad \mathbf{V = V_o e^{-mt/K}} \quad \text{and} \quad \mathbf{x = \int_0^t V\,dt = \frac{mV_o}{K}\left(1 - e^{-mt/K}\right)} \quad \textit{Ans.}\text{ (a,b)}$$

Thus, as asked, V drops off exponentially with time, and, as $t \to \infty, x = K\dfrac{V_o}{m}$

P1.19 In his study of the circular hydraulic jump formed by a faucet flowing into a sink, Watson [47] proposes a parameter combining volume flow rate *Q*, density ρ and viscosity μ of the fluid, and depth *h* of the water in the sink. He claims that the grouping is dimensionless, with *Q* in the numerator. Can you verify this?

Solution: Check the dimensions of these four variables, from Table 1.2:

$$\{Q\} = \{L^3/T\} \ ; \ \{\rho\} = \{M/L^3\} \ ; \ \{\mu\} = \{M/LT\} \ ; \ \{h\} = \{L\}$$

Can we make this dimensionless? First eliminate mass $\{M\}$ by dividing density by viscosity, that is, ρ/μ has units $\{T/L^2\}$. (I am pretending that kinematic viscosity is unfamiliar to the students in this introductory chapter.) Then combine ρ/μ and *Q* to eliminate time: $(\rho/\mu)Q$ has units {L}. Finally, divide that by a single depth *h* to form a dimensionless group:

$$\left\{\frac{\rho Q}{\mu h}\right\} = \frac{\{M/L^3\}\{L^3/T\}}{\{M/LT\}\{L\}} = \{1\} = \text{dimensionless} \quad \textit{Ans.} \quad \text{Watson is correct.}$$

P1.20 Books on porous media and atomization claim that the viscosity μ and surface tension Υ of a fluid can be combined with a characteristic velocity U to form an important dimensionless parameter. (*a*) Verify that this is so. (*b*) Evaluate this parameter for water at 20°C and a velocity of 3.5 cm/s. NOTE: Extra credit if you know the name of this parameter.

Solution: We know from Table 1.2 that $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{U\} = \{LT^{-1}\}$, and $\{\Upsilon\} = \{FL^{-1}\} = \{MT^{-2}\}$. To eliminate mass {M}, we must divide μ by Υ, giving $\{\mu/\Upsilon\} = \{TL^{-1}\}$. Multiplying by the velocity will thus cancel all dimensions:

$$\frac{\mu U}{\Upsilon} \text{ is dimensionless, as is its inverse, } \frac{\Upsilon}{\mu U} \qquad \textit{Ans.(a)}$$

The grouping is called the *Capillary Number*. (*b*) For water at 20°C and a velocity of 3.5 cm/s, use Table A.3 to find μ = 0.001 kg/m-s and Υ = 0.0728 N/m. Evaluate

$$\frac{\mu U}{\Upsilon} = \frac{(0.001 kg/m-s)(0.035 m/s)}{(0.0728 kg/s^2)} = \mathbf{0.00048}\,, \quad \frac{\Upsilon}{\mu U} = \mathbf{2080} \qquad \textit{Ans.(b)}$$

P1.21 Aeronautical engineers measure the pitching moment M_o of a wing and then write it in the following form for use in other cases:

$$M_o = \beta V^2 A C \rho$$

where V is the wing velocity, A the wing area, C the wing chord length, and ρ the air density. What are the dimensions of the coefficient β?

Solution: Write out the dimensions of each term in the formula:

$$\{M_o\} = \{FL\} = \left\{\frac{ML^2}{T^2}\right\} = \{\beta V^2 A C \rho\} = \{\beta\}\left\{\frac{L^2}{T^2}\right\}\{L^2\}\{L\}\left\{\frac{M}{L^3}\right\} = \left\{\frac{ML^2}{T^2}\right\}$$

Thus {β} = {unity} or *dimensionless*. It is proportional to the *moment coefficient* in aerodynamics.

P1.22 The *Ekman number*, Ek, arises in geophysical fluid dynamics. It is a dimensionless parameter combining seawater density ρ, a characteristic length L, seawater viscosity μ, and the Coriolis frequency $\Omega \sin\phi$, where Ω is the rotation rate of the earth and ϕ is the latitude angle. Determine the correct form of Ek if the viscosity is in the numerator.

Solution : First list the dimensions of the various quantities:

$$\{\rho\} = \{ML^{-3}\}; \quad \{L\} = \{L\}; \quad \{\mu\} = \{ML^{-1}T^{-1}\}; \quad \{\Omega \sin\phi\} = \{T^{-1}\}$$

Note that $\sin\phi$ is itself dimensionless, so the Coriolis frequency has the dimensions of Ω. Only ρ and μ contain mass {M}, so if μ is in the numerator, ρ must be in the denominator. That combination μ/ρ we know to be the kinematic viscosity, with units $\{L^2T^{-1}\}$. Of the two remaining variables, only $\Omega \sin\phi$ contains time $\{T^{-1}\}$, so it must be in the denominator. So far, we have the grouping $\mu/(\rho\,\Omega \sin\phi)$, which has the dimensions $\{L^2\}$. So we put the length-squared into the denominator and we are finished:

$$\text{Dimensionless Ekman number:} \quad \text{Ek} = \frac{\mu}{\rho L^2 \Omega \sin\phi} \qquad \textit{Ans.}$$

P1.23 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the energy released by an atomic bomb explosion. He assumed that the energy released, E, was a function of blast wave radius R, air density ρ, and time t. Arrange these variables into a single dimensionless group, which we may term the *blast wave number*.

Solution: These variables have the dimensions $\{E\} = \{ML^2/T^2\}$, $\{R\} = \{L\}$, $\{\rho\} = \{M/L^3\}$, and $\{t\} = \{T\}$. Multiplying E by t^2 eliminates time, then dividing by ρ eliminates mass, leaving $\{L^5\}$ in the numerator. It becomes dimensionless when we divide by R^5. Thus

$$\text{Blast wavenumber} = \frac{E t^2}{\rho R^5}$$

P1.24 Air, assumed to be an ideal gas with $k = 1.40$, flows isentropically through a nozzle. At section 1, conditions are sea level standard (see Table A.6). At section 2, the temperature is –50°C. Estimate (*a*) the pressure, and (*b*) the density of the air at section 2.

Solution: From Table A.6, $p_1 = 101350$ Pa, $T_1 = 288.16$ K, and $\rho_1 = 1.2255$ kg/m^3. Convert to absolute temperature, $T_2 = -50°C = 223.26$ K. Then, for a perfect gas with constant k,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1.4/(1.4-1)} = (0.7744)^{3.5} = 0.4087$$

$$\text{Thus} \quad p_2 = (0.4087)(101350\,Pa) = \mathbf{41,400}\ Pa \quad Ans.(a)$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/(k-1)} = \left(\frac{223.16}{288.16}\right)^{1/(1.4-1)} = (0.7744)^{2.5} = 0.5278$$

$$\text{Thus} \quad \rho_2 = (0.5278)(1.2255\,kg/m^3) = \mathbf{0.647}\ kg/m^3 \quad Ans.(b)$$

Alternately, once p_2 was known, we could have simply computed ρ_2 from the ideal-gas law.

$$\rho_2 = p_2/RT_2 = (41400)/[287(223.16)] = 0.647 \text{ kg/m}^3$$

P1.25 On a summer day in Narragansett, Rhode Island, the air temperature is 74ºF and the barometric pressure is 14.5 lbf/in^2. Estimate the air density in kg/m^3.

Solution: This is a problem in handling awkward units. Even if we use the BG system, we have to convert. But, since the problem calls for a metric result, better we should convert to SI units:

$$T = 74°F + 460 = 534°R \times 0.5556 \text{ (inside front cover)} = 297 \text{ K}$$

$$p = 14.5 \text{ lbf/in}^2 \times 6894.8 \text{ (inside front cover)} = 100{,}700 \text{ Pa}$$

The SI gas constant, from Eq. (1.12), is 287 m^2/(s^2·K). Thus, from the ideal gas law, Eq. (1.10),

$$\rho = \frac{p}{RT} = \frac{100{,}700 \text{ N/m}^2}{\left(287\dfrac{\text{m}^2}{\text{s}^2\cdot\text{K}}\right)(297\text{K})} = 1.18\frac{\text{N}\cdot\text{s}^2}{\text{m}^4}$$

This doesn't look like a density unit, until we realize that 1 N ≡ 1 kg·m/s^2. Making this substitution, we find that

$$\rho = \mathbf{1.18\ kg/m^3} \qquad Answer$$

Notice that faithful use of SI units will lead to faithful SI results, without further conversion.
