

# Chapter 3

## ENTROPY GENERATION, OR EXERGY DESTRUCTION

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### Problem 3.1

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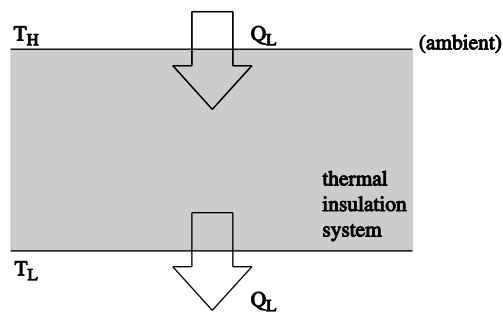
It was shown in Fig. 3.9 that the refrigerator deposits exergy in the  $(T_L)$  space at the rate

$$(-E_{Q_L}) = Q_L \left( \frac{T_H}{T_L} - 1 \right) \quad (1)$$

Looking now at the thermal insulation (the “system”) sandwiched between  $(T_H)$  and  $(T_L)$ , we see that  $Q_L$  enters from  $T_H$  and exits through  $T_L$ . The net exergy flow *into* this system is [see eq. (3.14)]

$$\begin{aligned} \text{net exergy inflow} &= Q_L \left( 1 - \frac{T_H}{T_H} \right) + (-Q_L) \left( 1 - \frac{T_H}{T_L} \right) \\ &= Q_L \left( \frac{T_H}{T_L} - 1 \right) > 0 \end{aligned} \quad (2)$$

The fact that this quantity is positive means that the thermal insulation system acts as a “sink” for exergy. Note that the quantities of eqs. (1) and (2) are equal.



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## Problem 3.2

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The nonflow exergy of a closed system is

$$\Xi = E - E_0 - T_0(S - S_0) + P_0(V - V_0) \quad (3.32)$$

If the system is isolated, then

- (i)  $E$  is fixed, eq. (2.49)
- (ii)  $V$  is fixed [see then discussion above eq. (2.49)]
- (iii)  $S$  increases or, in the limit, remains the same

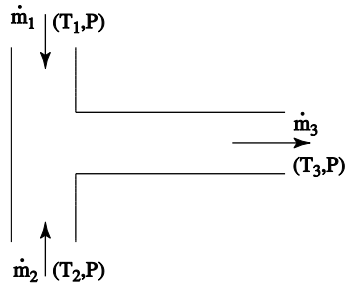
In conclusion, if the isolated system undergoes a change, its nonflow exergy  $\Xi$  decreases or, in the limit, remains the same.

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# Problem 3.3

The first law and the second law for the control volume (mixing chamber)

are



$$xh_1 + (1-x)h_2 = h_3 \quad (1)$$

$$\dot{S}_{\text{gen}} = \dot{m}s_3 - x\dot{m}s_1 - (1-x)\dot{m}s_2 > 0 \quad (2)$$

where  $\dot{m} = \dot{m}_1 + \dot{m}_2$ . When the two streams  $\dot{m}_1$  and  $\dot{m}_2$  carry the same ideal gas, eqs. (1)–(2) become

$$xT_1 + (1-x)T_2 = T_3 \quad (3)$$

$$\frac{\dot{S}_{\text{gen}}}{\dot{m}c_p} = x \ln \frac{T_3}{T_1} + (1-x) \ln \frac{T_3}{T_2} > 0 \quad (4)$$

Eliminating  $T_3$  between eqs. (3)–(4) yields

$$\frac{\dot{S}_{\text{gen}}}{\dot{m}c_p} = \ln \left[ \frac{x + \tau(1-x)}{\tau^{1-x}} \right] > 0 \quad (5)$$

in which  $\tau$  is the inlet temperature ratio,

$$\tau = \frac{T_2}{T_1}$$

Solving  $\partial \dot{S}_{\text{gen}} / \partial x = 0$ , we obtain the critical mass flow rate fraction for maximum irreversibility:

$$x = \frac{\tau - 1 - \tau \ln \tau}{(1 - \tau) \ln \tau}$$

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## Problem 3.4

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The maximum useful work is the nonflow exergy “stored” in the purchased bottle,

$$\Xi = m[u - u_0 - T_0(s - s_0) + P_0(v - v_0)]$$

or, using  $u = h - Pv$

$$\Xi = m[h - h_0 - T_0(s - s_0) + v(P_0 - P)]$$

In this problem,  $P_0 - P = 0$  and

$$m = \frac{0.05 \text{ m}^3}{1.24 \times 10^{-3} \text{ m}^3 / \text{kg}} = 40.3 \text{ kg}$$

Hence

$$\begin{aligned} \Xi &= 40.3 \text{ kg} [-121.5 - 172.1 - 300(2.85 - 6.25)] \frac{\text{kJ}}{\text{kg}} \\ &= 2.93 \times 10^4 \end{aligned}$$

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## Problem 3.5

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Consider the aggregate system indicated by the dashed line in the figure below: This system is closed and contains all the components that operate at temperatures higher than  $T_0$ . We apply the first law to this system and obtain

$$\dot{W} = \dot{Q}_H - \dot{Q}_0 = T_0 \left( \frac{\dot{Q}_H}{T_0} - \frac{\dot{Q}_0}{T_0} \right)$$

where the quantity in the parentheses is the total rate of entropy generation of the system:

$$\dot{S}_{\text{gen}} = \frac{\dot{Q}_H}{T_0} - \frac{\dot{Q}_0}{T_0}$$

In conclusion, the power required by the heat pump is proportional to the rate of entropy generation in the heat pump and the rest of the space situated at temperatures above  $T_0$ :

$$\dot{W} = T_0 \dot{S}_{\text{gen}}$$

