Chapter 2

1. The speed (assumed constant) is v = (90 km/h)(1000 m/km)/(3600 s/h) = 25 m/s. Thus, in 0.50 s, the car travels a distance $d = vt$ = (25 m/s)(0.50 s) ≈ 13 m.

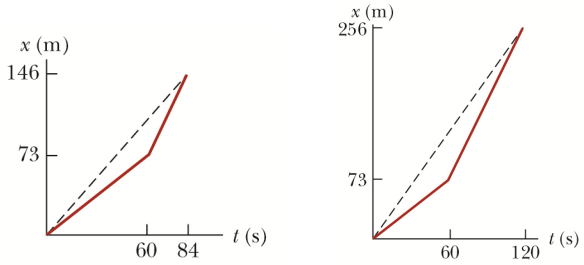
2. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2\ \text{m} + 73.2\ \text{m}}{\frac{73.2\ \text{m}}{1.22\ \text{m/s}} + \frac{73.2\ \text{m}}{3.05\ \text{m}}} = 1.74\ \text{m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22\ \text{m/s})(60\ \text{s}) + (3.05\ \text{m/s})(60\ \text{s})}{120\ \text{s}} = 2.14\ \text{m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



3. **THINK** This one-dimensional kinematics problem consists of two parts, and we are asked to solve for the average velocity and average speed of the car.

EXPRESS Since the trip consists of two parts, let the displacements during the first and second parts of the motion be Δx_1 and Δx_2 and the corresponding time intervals be Δt_1 and Δt_2. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is simply $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2.1.2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time Δt if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\,\Delta t$.

ANALYZE (a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time taken is

$$t_1 = \frac{40 \text{ km}}{30 \text{ km/h}} = 1.33 \text{ h}.$$

Similarly, during the second part of the trip, the displacement is $\Delta x_2 = 40$ km and the time interval is

$$t_2 = \frac{40 \text{ km}}{60 \text{ km/h}} = 0.67 \text{ h}.$$

The total displacement is

$$\Delta x = \Delta x_1 + \Delta x_2 = 40 \text{ km} + 40 \text{ km} = 80 \text{ km},$$

and the total time elapsed is

$$\Delta t = \Delta t_1 + \Delta t_2 = 2.00 \text{ h}.$$

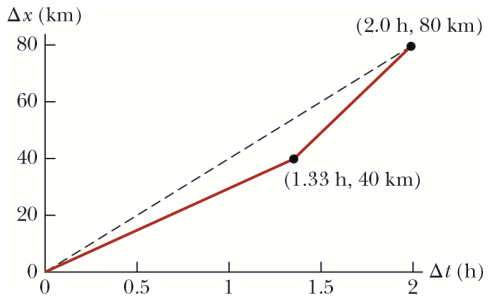
Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{80 \text{ km}}{2.0 \text{ h}} = 40 \text{ km/h}.$$

(b) In this case, the average speed is the same as the magnitude of the average velocity:

$$s_{\text{avg}} = 40 \text{ km/h}.$$

(c) The graph of the entire trip, shown below, consists of two connected line segments, the first having a slope of 30 km/h and connecting the origin to $(\Delta t_1, \Delta x_1) = (1.33 \text{ h}, 40 \text{ km})$ and the second having a slope of 60 km/h and connecting $(\Delta t_1, \Delta x_1)$ to $(\Delta t, \Delta x) = (2.00 \text{ h}, 80 \text{ km})$.



The slope of the dashed line drawn from the origin to $(\Delta t, \Delta x)$ represents the average velocity.

LEARN The average velocity is a vector quantity that depends only on the net displacement (also a vector) between the starting and ending points.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion), we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}},$$

which, after canceling D and plugging in $v_{\text{up}} = 40$ km/h and $v_{\text{down}} = 60$ km/h, yields 48 km/h for the average speed.

5. **THINK** In this one-dimensional kinematics problem, we're given the position function $x(t)$ and asked to calculate the position and velocity of the object at a later time.

EXPRESS The position function is

$$x(t) = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3.$$

The position of the object at some instant t_0 is simply given by $x(t_0)$. For the time interval $t_1 \le t \le t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2.1.2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

ANALYZE (a) Plugging $t = 1$ s into $x(t)$ yields

$$x(1 \text{ s}) = (3 \text{ m/s})(1 \text{ s}) - (4 \text{ m/s}^2)(1 \text{ s})^2 + (1 \text{ m/s}^3)(1 \text{ s})^3 = 0.$$

(b) With $t = 2$ s we get

$$x(2\text{ s}) = (3\text{ m/s})(2\text{ s}) - (4\text{ m/s}^2)(2\text{ s})^2 + (1\text{ m/s}^3)(2\text{ s})^3 = -2\text{ m}.$$

(c) With $t = 3$ s we have

$$x(3\text{ s}) = (3\text{ m/s})(3\text{ s}) - (4\text{ m/s}^2)(3\text{ s})^2 + (1\text{ m/s}^3)(3\text{ s})^3 = 0\text{ m}.$$

(d) Similarly, plugging in $t = 4$ s gives

$$x(4\text{ s}) = (3\text{ m/s})(4\text{ s}) - (4\text{ m/s}^2)(4\text{ s})^2 + (1\text{ m/s}^3)(4\text{ s})^3 = 12\text{ m}.$$

(e) The position at $t = 0$ is $x = 0$. Thus, the displacement between $t = 0$ and $t = 4$ s is

$$\Delta x = x(4\text{ s}) - x(0) = 12\text{ m} - 0 = 12\text{ m}.$$

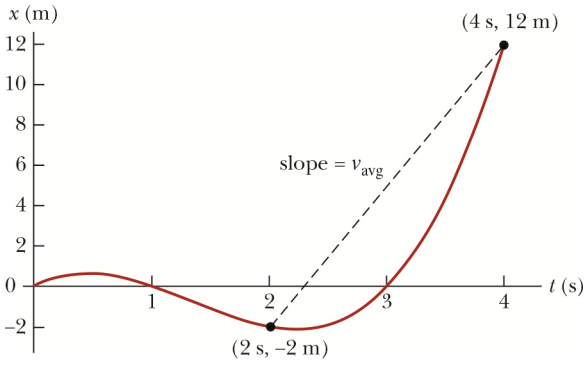
(f) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement:

$$\Delta x = x(4\text{ s}) - x(2\text{ s}) = 12\text{ m} - (-2\text{ m}) = 14\text{ m}.$$

Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m/s}.$$

(g) The position of the object for the interval $0 \le t \le 4$ is plotted below. The slope of the straight line drawn from the point at $(t, x) = (2\text{ s}, -2\text{ m})$ to the point at (4 s, 12 m) is the average velocity, the answer for part (f).



LEARN Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

6. Huber's speed is

$$v_0 = (200 \text{ m})/(6.509 \text{ s}) = 30.72 \text{ m/s} = 110.6 \text{ km/h},$$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 = (110.6 km/h + 19.0 km/h) = 129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, using Eq. 2.1.2, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200 \text{ m}}{36 \text{ m/s}} = 5.554 \text{ s}.$$

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time that elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

8. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L / v_s$. With each additional person, the depth increases by one body depth d.

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L / v_s} = \frac{dv_s}{L} = \frac{(0.25 \text{ m})(3.50 \text{ m/s})}{1.75 \text{ m}} = 0.50 \text{ m/s}.$$

(b) The amount of time required to reach a depth of $D = 5.0$ m is

$$t = \frac{D}{R} = \frac{5.0 \text{ m}}{0.50 \text{ m/s}} = 10 \text{ s}.$$

9. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$s_{\text{avg1}} = s_{\text{avg2}} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m},$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

10. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1, the car moves in the same direction as the wind. Then the effective speed of the car is given by $v_{eff,1} = v_c + v_w$, and the distance traveled is $d = v_{eff,1}t_1 = (v_c + v_w)t_1$. On the other hand, for the return trip during time interval t_2, the car moves in the opposite direction of the wind and the effective speed would be $v_{eff,2} = v_c - v_w$. The distance traveled is $d = v_{eff,2}t_2 = (v_c - v_w)t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}.$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2}\right)$. Thus, method 1 gives the car's speed v_c in a windless situation.

(b) If method 2 is used, the result would be

$$v_c' = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{v_c + v_w} + \frac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c\left[1 - \left(\frac{v_w}{v_c}\right)^2\right].$$

The fractional difference is

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c}\right)^2 = (0.0240)^2 = 5.76\times10^{-4}.$$

11. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed v = (160 km)/(1.25 h) = 128 km/h is needed.

12. (a) Let the fast and the slow cars be separated by a distance d at $t = 0$. If during the time interval $t = L/v_s = (12.0\text{ m})/(5.0\text{ m/s}) = 2.40\text{ s}$ in which the slow car has moved a distance of $L = 12.0$ m, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition indicates a separation of

$$d = vt - L = (25\text{ m/s})(2.4\text{ s}) - 12.0\text{ m} = 48.0\text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0$ m. At a later time t, the slow and the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d+x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00 \text{ m/s}}{25.0 \text{ m/s} - 5.00 \text{ m/s}} (96.0 \text{ m}) = 24.0 \text{ m},$$

which in turn gives $t = (24.0 \text{ m}) / (5.00 \text{ m/s}) = 4.80 \text{ s}$. Since the rear of the slow-car pack has moved a distance of $\Delta x = x - L = 24.0 \text{ m} - 12.0 \text{ m} = 12.0 \text{ m}$ downstream, the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0 \text{ m}}{4.80 \text{ s}} = 2.50 \text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, we find that the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55 \text{ km/h})(T/2) + (90 \text{ km/h})(T/2)}{T} = 72.5 \text{ km/h},$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h},$$

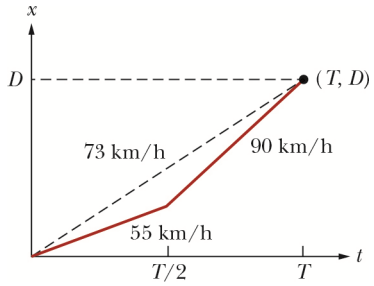
which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km/h}} + \frac{D}{68.3 \text{ km/h}}} = 70 \text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. We briefly describe the graph (with kilometers per hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D), where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D). The graph (not drawn to scale) is depicted below:



14. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right)e^{-t} + (19t)\left(\frac{de^{-t}}{dt}\right).$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where $T = 1$ second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when $t = 1$ s. Now that we know *when* it stops, we find out *where* it stops by plugging our result $t = 1$ into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find $x = 5.9$ m.

15. We use Eq. 2.2.1 to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(4 - 12t + 3t^2\right) = -12 + 6t.$$

Thus, at $t = 1$ s, the velocity is $v = -12 + (6)(1) = -6$ m/s.

(b) Since $v < 0$, it is moving in the $-x$ direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that $v = 0$ when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

16. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

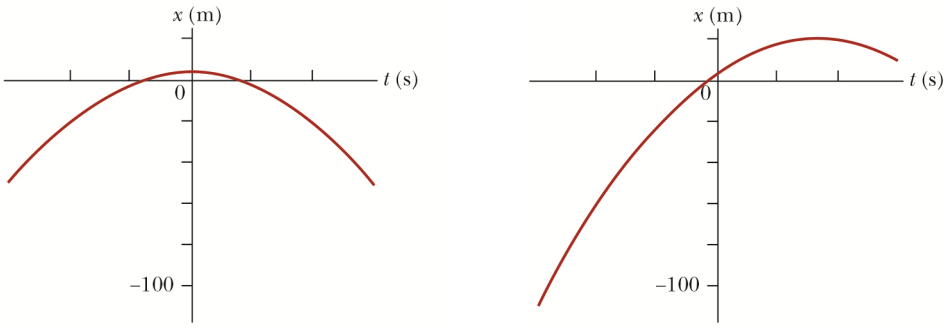
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m.

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the "shifted" graph that is relevant to part (f). In both cases, the time axis is given by $-3 \leq t \leq 3$.



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

17. We use Eq. 2.1.2 for average velocity and Eq. 2.2.1 for instantaneous velocity and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25 \text{ cm} - 21.75 \text{ cm}}{3.00 \text{ s} - 2.00 \text{ s}},$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time $t = 2.00$ s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At $t = 3.00$ s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

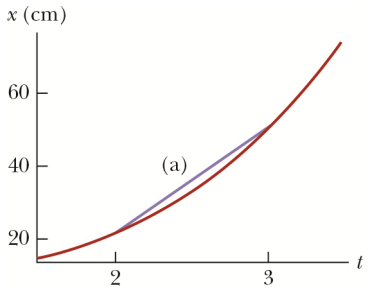
(d) At $t = 2.50$ s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (i.e., when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596,$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x versus t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$, we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t,$$

with length in meters and time in seconds. Plugging in the value $t = 3$ yields $x(3) = 54$ m.

(b) Similarly, plugging in the value $t = 3$ yields $v(3) = 18$ m/s.

(c) For $t = 3$, $a(3) = -12$ m/s^2.

(d) At the maximum x, we must have $v = 0$; eliminating the $t = 0$ root, the velocity equation reveals $t = 24/6 = 4$ s for the time of maximum x. Plugging $t = 4$ into the equation for x leads to $x = 64$ m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at $t = 4.0$ s.

(f) A maximum v requires $a = 0$, which occurs when $t = 24/12 = 2.0$ s. This, inserted into the velocity equation, gives $v_{max} = 24$ m/s.

(g) From (f), we see that the maximum of v occurs at $t = 24/12 = 2.0$ s.

(h) In part (e), the particle was (momentarily) motionless at $t = 4$ s. The acceleration at that time is readily found to be $24 - 12(4) = -24$ m/s^2.

(i) The *average velocity* is defined by Eq. 2.1.2, so we see that the values of x at $t = 0$ and $t = 3$ s are needed; these are, respectively, $x = 0$ and $x = 54$ m (found in part (a)). Thus,

$$v_{avg} = \frac{54 - 0}{3 - 0} = 18 \text{ m/s}.$$

19. **THINK** In this one-dimensional kinematics problem, we're given the speed of a particle at two instants and asked to calculate its average acceleration.

EXPRESS We take the initial direction of motion as the $+x$ direction. The average acceleration over a time interval $t_1 \le t \le t_2$ is given by Eq. 2.3.1:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

ANALYZE Let $v_1 = +18$ m/s at $t_1 = 0$ and $v_2 = -30$ m/s at $t_2 = 2.4$ s. Using Eq. 2.3.1, we find

$$a_{\text{avg}} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30 \text{ m/s}) - (+18 \text{ m/s})}{2.4 \text{ s} - 0} = -20 \text{ m/s}^2.$$

LEARN The average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18 \text{ m/s}) - (20 \text{ m/s}^2)t.$$

20. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

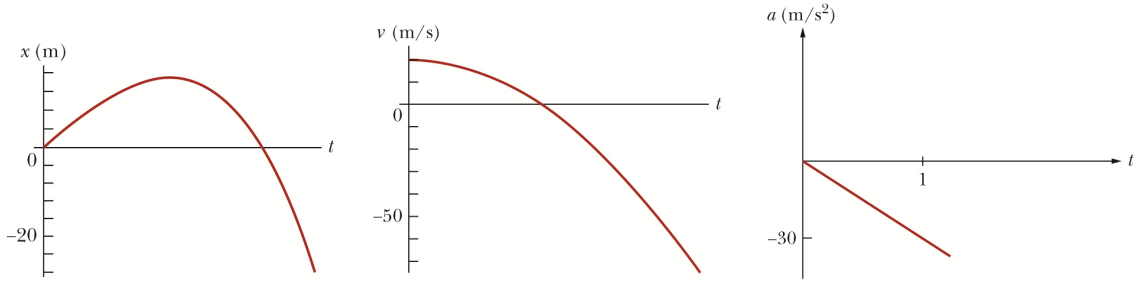
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (i.e., it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$.

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below.



21. We use Eq. 2.1.2 (average velocity) and Eq. 2.3.1 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min ≤ t ≤ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v\,\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) The entire interval considered is Δt = 8 – 2 = 6 min, which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3\,\text{min} = 180$ s. His position at t = 2 min is x = 0 and his position at t = 8 min is $x = v\,\Delta t' = (2.2)(180) = 396$ m. Therefore,

$$v_{\text{avg}} = \frac{396\text{ m} - 0}{360\text{ s}} = 1.10\text{ m/s}.$$

(b) The man is at rest at t = 2 min and has velocity v = +2.2 m/s at t = 8 min. Thus, keeping the answer to 3 significant figures,

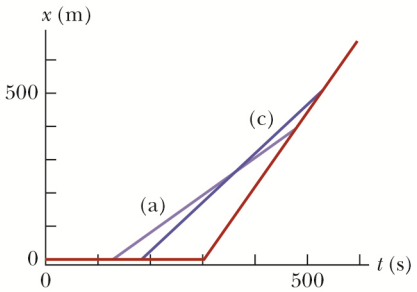
$$a_{\text{avg}} = \frac{2.2\text{ m/s} - 0}{360\text{ s}} = 0.00611\text{ m/s}^2.$$

(c) Now, the entire interval considered is Δt = 9 – 3 = 6 min (360 s again), whereas the subinterval in which he is moving is $\Delta t' = 9 - 5 = 4$ min = 240 s). His position at $t = 3$ min is x = 0, and his position at t = 9 min is $x = v\,\Delta t' = (2.2)(240) = 528$ m. Therefore,

$$v_{\text{avg}} = \frac{528\text{ m} - 0}{360\text{ s}} = 1.47\text{ m/s}.$$

(d) The man is at rest at t = 3 min and has velocity v = +2.2 m/s at t = 9 min. Consequently, a_{avg} = 2.2/360 = 0.00611 m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at x = 0 for 0 ≤ t < 300 s, and the linearly rising line for 300 ≤ t ≤ 600 s represents his constant-velocity motion. The lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here but would consist of two horizontal "steps" (one at v = 0 for 0 ≤ t < 300 s and the next at v = 2.2 m/s for 300 ≤ t ≤ 600 s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

22. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate, its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0 \text{ m/s}^2)}{3(2.0 \text{ m/s}^3)} = 1.0 \text{ s}.$$

For $t = 0$, $x = x_0 = 0$, and for $t = 1.0$ s, $x = 1.0$ m $> x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$ s).

(d) In the first 4 s, the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4 \text{ s}) = (3.0 \text{ m/s}^2)(4.0 \text{ s})^2 - (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 = -80 \text{ m}.$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(e) Its displacement is $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80$ m.

Thus, $\Delta x = -80$ m.

The velocity is given by $v = 2ct - 3bt^2 = (6.0 \text{ m/s}^2)t - (6.0 \text{ m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain

$$v(1 \text{ s}) = (6.0 \text{ m/s}^2)(1.0 \text{ s}) - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2 = 0.$$

(g) Similarly, $v(2 \text{ s}) = (6.0 \text{ m/s}^2)(2.0 \text{ s}) - (6.0 \text{ m/s}^3)(2.0 \text{ s})^2 = -12$ m/s,

(h) $v(3 \text{ s}) = (6.0 \text{ m/s}^2)(3.0 \text{ s}) - (6.0 \text{ m/s}^3)(3.0 \text{ s})^2 = -36$ m/s,

(i) $v(4 \text{ s}) = (6.0 \text{ m/s}^2)(4.0 \text{ s}) - (6.0 \text{ m/s}^3)(4.0 \text{ s})^2 = -72$ m/s.

The acceleration is given by $a = dv/dt = 2c - 6b = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain $a(1 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(1.0 \text{ s}) = -6.0 \text{ m/s}^2$,

(k) $a(2\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(2.0\text{ s}) = -18\text{ m/s}^2$,

(l) $a(3\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(3.0\text{ s}) = -30\text{ m/s}^2$,

(m) $a(4\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(4.0\text{ s}) = -42\text{ m/s}^2$.

23. **THINK** The electron undergoes a constant acceleration. Given the final speed of the electron and the distance it has traveled, we can calculate its acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the electron can be analyzed using the equations given in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

The acceleration can be found by solving Eq. 2.4.6.

ANALYZE With $v_0 = 1.50\times10^5$ m/s, $v = 5.70\times10^6$ m/s, $x_0 = 0$, and $x = 0.010$ m, we find the average acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times10^6\text{ m/s})^2 - (1.5\times10^5\text{ m/s})^2}{2(0.010\text{ m})} = 1.62\times10^{15}\text{ m/s}^2.$$

LEARN A good idea is to apply other equations in Table 2.4.1 not used for solving the problem as a consistency check. For example, since we now know the value of the acceleration, from Eq. 2.4.1, the time the electron takes to reach its final speed is

$$t = \frac{v - v_0}{a} = \frac{5.70\times10^6\text{ m/s} - 1.5\times10^5\text{ m/s}}{1.62\times10^{15}\text{ m/s}^2} = 3.426\times10^{-9}\text{ s}.$$

Substituting the value of t into Eq. 2.4.5, we find that the distance the electron travels is

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ &= 0 + (1.5\times10^5\text{ m/s})(3.426\times10^{-9}\text{ s}) + \frac{1}{2}(1.62\times10^{15}\text{ m/s}^2)(3.426\times10^{-9}\text{ s})^2 \\ &= 0.010\text{ m}. \end{aligned}$$

This is what was given in the problem statement. So we know the problem has been solved correctly.

24. In this problem we are given the initial and final speeds and the displacement and are asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2.4.6, $v^2 = v_0^2 + 2a(x - x_0)$.

(a) Given that $v_0 = 0$, $v = 1.6$ m/s, and $\Delta x = 5.0\ \mu$m, the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(1.6\text{ m/s})^2}{2(5.0\times10^{-6}\text{ m})} = 2.56\times10^5\text{ m/s}^2 = 2.6\times10^4\, g.$$

(b) During the speed-reduction stage, the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0 - (1.6\text{ m/s})^2}{2(1.0\times10^{-3}\text{ m})} = -1.28\times10^3\text{ m/s}^2 = -1.3\times10^2\, g.$$

The negative sign means that the spores are decelerating.

25. We separate the motion into two parts and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration is opposite the direction of motion).

(a) From Table 2.4.1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2.4.1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20\text{ m/s})^2 - (0)^2}{2(2.0\text{ m/s}^2)} = 100\text{ m}.$$

This position is then the *initial* position for part 2, so when the same equation is used in part 2, we obtain

$$x - 100\text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20\text{ m/s})^2}{2(-1.0\text{ m/s}^2)}.$$

Thus, the final position is x = 300 m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

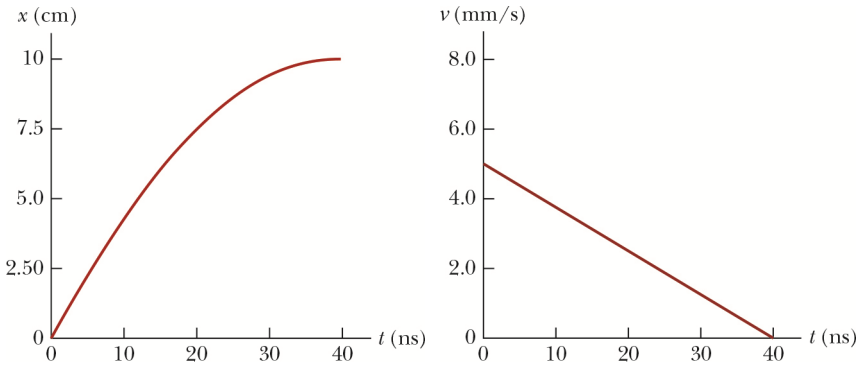
26. The constant-acceleration condition permits the use of Table 2.4.1.

(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{(5.00\times10^6)^2}{-1.25\times10^{14}} = 0.100 \text{ m}.$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so other equations from Table 2.4.1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.



27. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2.4.1 gives

$$v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, $t = +2.5$ s and we find $v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(2.5 \text{ s}) = 18 \text{ m/s}$.

28. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

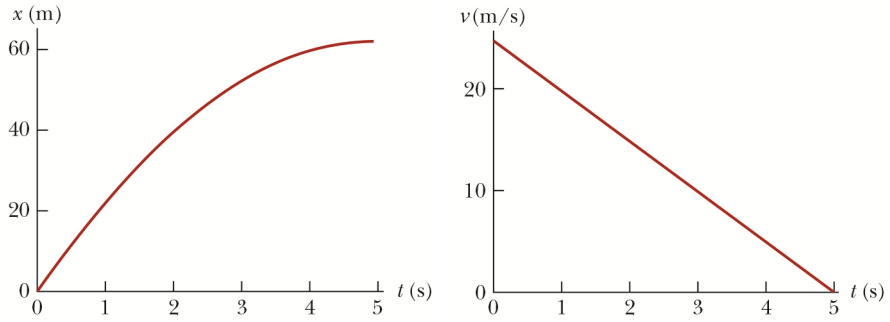
(a) The time to come to a halt is found using Eq. 2.4.1:

$$0 = v_0 + at \;\Rightarrow\; t = -\frac{24.6 \text{ m/s}}{-4.92 \text{ m/s}^2} = 5.00 \text{ s}.$$

(b) Although several of the equations in Table 2.4.1 will yield the result, we choose Eq. 2.4.6 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \Rightarrow x = -\frac{(24.6 \text{ m/s})^2}{2\left(-4.92 \text{ m/s}^2\right)} = 61.5 \text{ m}.$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s.



29. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a, so Table 2.4.1 can be used. Taking the direction of motion to be $+x$, then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

(a) We denote Δx as the distance moved during t_1 and use Eq. 2.4.6:

$$v^2 = v_0^2 + 2a_1 \Delta x \;\Rightarrow\; \Delta x = \frac{(5.08 \text{ m/s})^2}{2(1.22 \text{ m/s}^2)} = 10.59 \text{ m} \approx 10.6 \text{ m}.$$

(b) Using Eq. 2.4.1, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08 \text{ m/s}}{1.22 \text{ m/s}^2} = 4.17 \text{ s}.$$

The deceleration time t_2 turns out to be the same, so $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same, so they total to 2(10.59 m) = 21.18 m. This indicates that for a distance of 190 m – 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82 \text{ m}}{5.08 \text{ m/s}} = 33.21 \text{ s}.$$

Therefore, the total time is 8.33 s + 33.21 s ≈ 41.5 s.

30. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2.4.1.

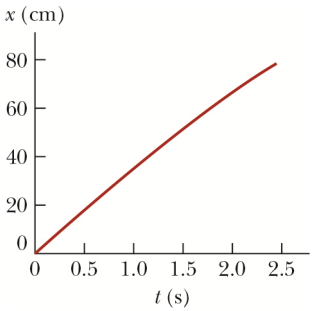
(a) Substituting v_0 = 137 km/h = 38.1 m/s, v = 90 km/h = 25 m/s, and a = –5.2 m/s^2 into $v = v_0 + at$, we obtain

$$t = \frac{25 \text{ m/s} - 38 \text{ m/s}}{-5.2 \text{ m/s}^2} = 2.5 \text{ s}.$$

(b) We take the car to be at x = 0 when the brakes are applied (at time t = 0). Thus, the coordinate of the car as a function of time is given by

$$x = (38 \text{ m/s})t + \frac{1}{2}(-5.2 \text{ m/s}^2)t^2$$

in SI units. This function is plotted from t = 0 to t = 2.5 s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



31. **THINK** The rocket ship undergoes a constant acceleration from rest, and we want to know the time elapsed and the distance traveled when the rocket reaches a certain speed.

EXPRESS Since the problem involves constant acceleration, the motion of the rocket can be readily analyzed using the equations in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

ANALYZE (a) Given $a = 9.8 \text{ m/s}^2$, $v_0 = 0$, and $v = 0.1c = 3.0\times10^7$ m/s, we solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{3.0\times10^7 \text{ m/s} - 0}{9.8 \text{ m/s}^2} = 3.1\times10^6 \text{ s},$$

which is about 1.2 months. So the rocket takes 1.2 months to reach a speed of $0.1c$ starting from rest with a constant acceleration of 9.8 m/s^2.

(b) To calculate the distance traveled during this time, we evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$ and $v_0 = 0$. The result is

$$x = \frac{1}{2}(9.8 \text{ m/s}^2)(3.1\times10^6 \text{s})^2 = 4.6\times10^{13} \text{ m}.$$

LEARN In solving parts (a) and (b), we did not use Eq. 2.4.6: $v^2 = v_0^2 + 2a(x - x_0)$. This equation can be used to check our answers. The final speed based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(9.8 \text{ m/s}^2)(4.6\times10^{13} \text{ m} - 0)} = 3.0\times10^7 \text{ m/s},$$

which is what is given in the problem statement. So we know the problem has been solved correctly.

32. The acceleration is found from Eq. 2.4.1 (or, suitably interpreted, Eq. 2.3.1):

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2.$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right)g = 21g.$$

33. **THINK** The car undergoes a constant negative acceleration to avoid impacting a barrier. Given its initial speed, we want to know the distance it has traveled and the time elapsed prior to the impact.

EXPRESS Since the problem involves constant acceleration, the motion of the car can be analyzed using the equations in Table 2.4.1:

$$v = v_0 + at \qquad (2.4.1)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 + 2a(x - x_0). \qquad (2.4.6)$$

We take $x_0 = 0$ and $v_0 = 56.0$ km/h = 15.55 m/s to be the initial position and speed of the car. Solving Eq. 2.4.5 with $t = 2.00$ s gives the acceleration a. Once a is known, the speed of the car upon impact can be found by using Eq. 2.4.1.

ANALYZE (a) Using Eq. 2.4.5, we find the acceleration to be

$$a = \frac{2(x - v_0 t)}{t^2} = \frac{2\left[(24.0\text{ m}) - (15.55\text{ m/s})(2.00\text{ s})\right]}{(2.00\text{ s})^2} = -3.56\text{ m/s}^2,$$

or $|a| = 3.56\text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car; the car is slowing down.

(b) The speed of the car at the instant of impact is

$$v = v_0 + at = 15.55\text{ m/s} + (-3.56\text{ m/s}^2)(2.00\text{ s}) = 8.43\text{ m/s},$$

which can also be converted to 30.3 km/h.

LEARN In solving parts (a) and (b), we did not use Eq. 2.4.6. This equation can be used as a consistency check. The final speed based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{(15.55\text{ m/s})^2 + 2(-3.56\text{ m/s}^2)(24\text{ m} - 0)} = 8.43\text{ m/s},$$

which is what was calculated in (b). This indicates that the problem has been solved correctly.

34. Let d be the 220 m distance between the cars at $t = 0$ and v_1 be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of $x_1 = 44.5$ m) and v_2 be the 40 km/h =100/9 m/s speed (corresponding to a passing point of $x_2 = 76.6$ m) of the red car. We have two equations (based on Eq. 2.4.7):

$$d - x_1 = v_0 t_1 + \frac{1}{2} a t_1^2, \qquad \text{where } t_1 = x_1 / v_1$$

$$d - x_2 = v_0 t_2 + \frac{1}{2} a t_2^2, \qquad \text{where } t_2 = x_2 / v_2.$$

We simultaneously solve these equations and obtain the following results:

(a) The initial velocity of the green car is $v_0 = -13.9$ m/s, or roughly −50 km/h (the negative sign means that it's in the $-x$ direction).

(b) The corresponding acceleration of the car is a = –2.0 m/s^2 (the negative sign means that it's in the –x direction).

35. The positions of the cars as a function of time are given by

$$x_r(t) = x_{r0} + \frac{1}{2}a_r t^2 = (-35.0\text{ m}) + \frac{1}{2}a_r t^2$$
$$x_g(t) = x_{g0} + v_g t = (270\text{ m}) - (20\text{ m/s})t,$$

where we have substituted the velocity and not the speed for the green car. The two cars pass each other at $t = 12.0$ s when the graphed lines cross. This indicates that

$$(270\text{ m}) - (20\text{ m/s})(12.0\text{ s}) = 30\text{ m} = (-35.0\text{ m}) + \frac{1}{2}a_r(12.0\text{ s})^2,$$

which can be solved to give $a_r = 0.90$ m/s^2.

36. (a) Equation 2.4.5 is used for part 1 of the trip and Eq. 2.4.8 is used for part 2:

$$\Delta x_1 = v_{01}t_1 + \frac{1}{2}a_1 t_1^2, \qquad \text{where } a_1 = 2.25\text{ m/s}^2 \text{ and } \Delta x_1 = \frac{900}{4}\text{m}$$

$$\Delta x_2 = v_2 t_2 - \frac{1}{2}a_2 t_2^2, \qquad \text{where } a_2 = -0.75\text{ m/s}^2 \text{ and } \Delta x_2 = \frac{3(900)}{4}\text{m}.$$

In addition, $v_{01} = v_2 = 0$. Solving these equations for the times and adding the results give $t = t_1 + t_2 = 56.6$ s.

(b) Equation 2.4.6 is used for part 1 of the trip:

$$v^2 = (v_{01})^2 + 2a_1\,\Delta x_1 = 0 + 2(2.25)\left(\frac{900}{4}\right) = 1013\text{ m}^2/\text{s}^2,$$

which leads to v = 31.8 m/s for the maximum speed.

37. (a) From the figure, we see that x_0 = –2.0 m. From Table 2.4.1, we can apply

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

with t = 1.0 s, and then again with t = 2.0 s. This yields two equations for the two unknowns, v_0 and a:

$$0.0-(-2.0\text{ m})=v_0(1.0\text{ s})+\frac{1}{2}a(1.0\text{ s})^2$$
$$6.0\text{ m}-(-2.0\text{ m})=v_0(2.0\text{ s})+\frac{1}{2}a(2.0\text{ s})^2.$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0$ m/s^2.

(b) The fact that the answer is positive tells us that the acceleration is in the $+x$ direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34$ m/s^2 until the midway point and then decelerates at $a_2 = -1.34$ m/s^2 until it comes to a stop ($v_2 = 0$) at the next station. The velocity at the midpoint is v_1, which occurs at x_1 = 806/2 = 403 m.

(a) Equation 2.4.6 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \quad \Rightarrow \quad v_1 = \sqrt{2(1.34\text{ m/s}^2)(403\text{ m})} = 32.9\text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2.4.5)

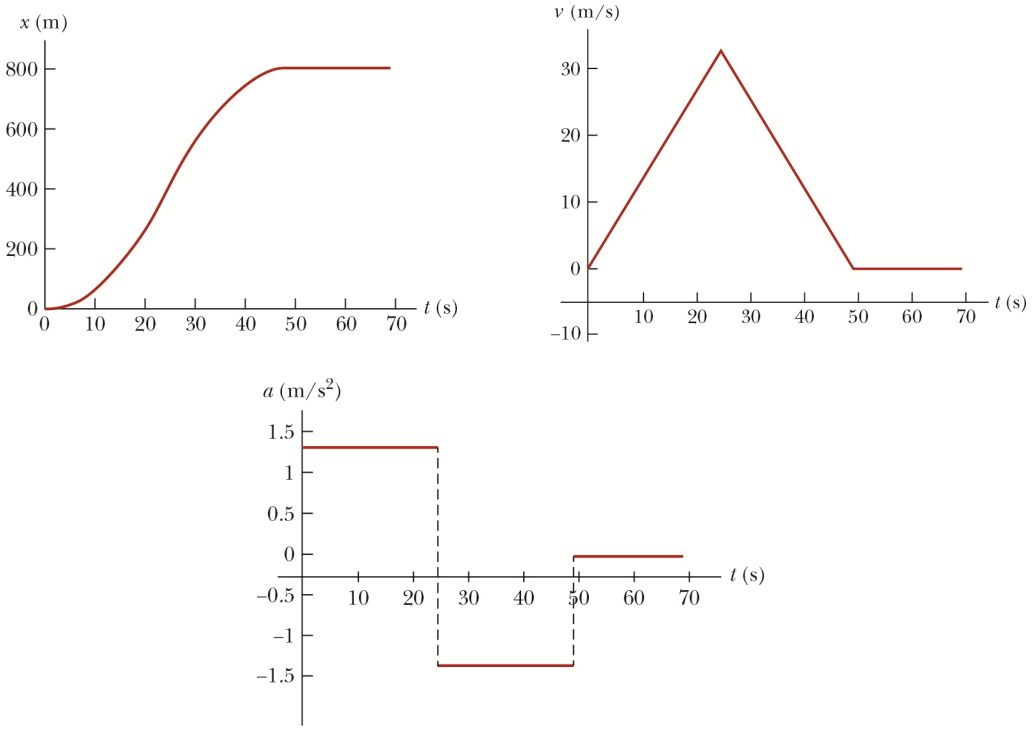
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{2(403\text{ m})}{1.34\text{ m/s}^2}} = 24.53\text{ s}.$$

Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain t = 49.1 s for the travel time between stations.

(c) With a "dead time" of 20 s, we have $T = t + 20$ = 69.1 s for the total time between start-ups. Thus, Eq. 2.1.2 gives

$$v_{\text{avg}} = \frac{806\text{ m}}{69.1\text{ s}} = 11.7\text{ m/s}.$$

(d) The graphs for x, v, and a as a function of t are shown below. The third graph, $a(t)$, consists of three horizontal "steps" — one at 1.34 m/s^2 during $0 < t < 24.53$ s, the next at –1.34 m/s^2 during 24.53 s $< t <$ 49.1 s, and the last at zero during the "dead time" 49.1 s $< t <$ 69.1 s).



39. (a) We note that v_A = 12/6 = 2 m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at x = 28 m when t = 4 s. This must be the value of x for car B at that time; we use Eq. 2.4.5:

$$28 \text{ m} = (12 \text{ m/s})t + \frac{1}{2}a_B t^2, \text{ where } t = 4.0 \text{ s}.$$

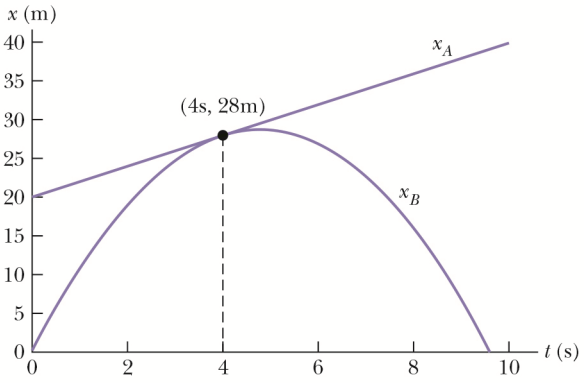
This yields a_B = –2.5 m/s^2.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides t = 4 s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \frac{1}{2}a_B t^2,$$

where $a_B = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_B)}$ is zero. In our case, it is zero — which means there is only one root. The cars are side by side only once at t = 4 s.

(c) A sketch is shown below. It consists of a straight line (x_A) tangent to a parabola (x_B) at t = 4.



(d) We only care about real roots, which means $10^2 - 2(-20)(a_B) \geq 0$. If $|a_B| > 5/2$, then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \Rightarrow$ two real roots. The cars are side by side at two different times.

40. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

(a) The velocity is constant during the reaction time T, so the distance traveled during it is

$$d_r = v_0 T - (15.28 \text{ m/s})\,(0.75 \text{ s}) = 11.46 \text{ m}.$$

We use Eq. 2.4.6 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \Rightarrow d_b = -\frac{(15.28 \text{ m/s})^2}{2\left(-5.18 \text{ m/s}^2\right)},$$

which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40 \text{ m})/(15.28 \text{ m/s}) = 2.6$ s, which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is 32/15.28 = 2.1 s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

41. The displacement (Δx) for each train is the "area" in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is 1/2(base) × (height). Thus, the (absolute value of the) displacement for

one train (1/2)(40 m/s)(5 s) = 100 m, and that of the other train is (1/2)(30 m/s)(4 s) = 60 m. The initial "gap" between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is 200 – 160 = 40 m.

42. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two-second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2.4.5) 51.11 m. In light of the fact that the initial "gap" between cars was 25 m, this means the gap has narrowed by 10.0 m – that is, to a distance of 15.0 m between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2.4.5) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it t_0), the gap between the cars is 10.6 m. The speed of the police car at t_0 is 30.56 – 5(2.4) = 18.56 m/s. Collision occurs at time t when $x_{\text{you}} = x_{\text{police}}$ (we choose coordinates such that your position is $x = 0$ and the police car's position is $x = 10.6$ m at t_0). Eq. 2.4.5 becomes, for each car:

$$x_{\text{police}} - 10.6 = 18.56(t - t_0) - \frac{1}{2}(5)(t - t_0)^2$$

$$x_{\text{you}} = 30.56(t - t_0) - \frac{1}{2}(5)(t - t_0)^2.$$

Subtracting equations, we find

$$10.6 = (30.56 - 18.56)(t - t_0) \quad \Rightarrow \quad 0.883 \text{ s} = t - t_0.$$

At that time your speed is $30.56 + a(t - t_0) = 30.56 - 5(0.883) \approx 26$ m/s (or 94 km/h).

43. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2.4.1 is permitted. We start with Eq. 2.4.7 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them, D, as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell.$$

We now use Eq. 2.4.1 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell,$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell \right) \left(\frac{v_\ell - v_t}{D} \right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

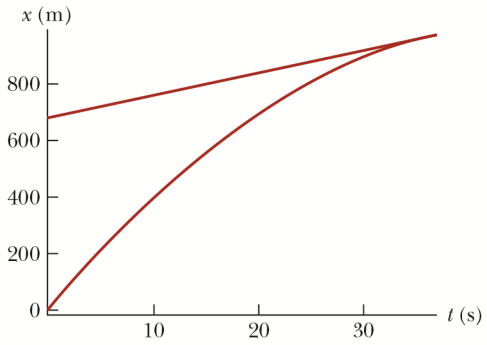
Hence,

$$a = -\frac{1}{2(0.676 \text{ km})}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}} \right)^2 = -12\ 888 \text{ km/h}^2,$$

which we convert as follows:

$$a = \left(-12\ 888 \text{ km/h}^2\right)\left(\frac{1000 \text{ m}}{1 \text{ km}} \right)\left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = -0.994 \text{ m/s}^2,$$

so its *magnitude* is $|a| = 0.994$ m/s^2. A graph is shown here for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

44. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2.4.1 (with Δy replacing Δx) because this is constant-acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2} g t^2$, with $y = 0.544$ m and $t = 0.200$ s, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.544 \text{ m} + (9.8 \text{ m/s}^2)(0.200 \text{ s})^2/2}{0.200 \text{ s}} = 3.70 \text{ m/s}.$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70 \text{ m/s} - (9.8 \text{ m/s}^2)(0.200 \text{ s}) = 1.74 \text{ m/s}.$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{(3.7 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.698 \text{ m}.$$

Thus, the armadillo goes 0.698 – 0.544 = 0.154 m higher.

45. **THINK** As the ball travels vertically upward, it has gravitational acceleration. The kinematics is one-dimensional.

EXPRESS We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2.4.1 (with Δy replacing Δx) because this is constant-acceleration motion:

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

We set $y_0 = 0$. Upon reaching the maximum height y, the ball momentarily has zero speed ($v = 0$). Therefore, we can relate its initial speed v_0 to y via the equation $0 = v^2 = v_0^2 - 2gy$. The time the ball takes to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0/g$. Therefore, for the entire trip (from the time it leaves the ground until the time it returns to the ground), the total flight time is $T = 2t = 2v_0/g$.

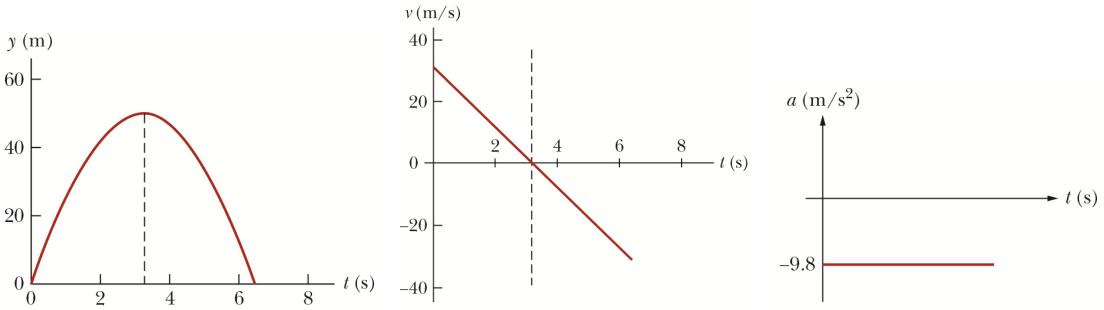
ANALYZE (a) At the highest point $v = 0$ and $v_0 = \sqrt{2gy}$. With $y = 50$ m, we find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m})} = 31.3 \text{ m/s} \approx 31 \text{ m/s}.$$

(b) Using the result from (a) for v_0, the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2(31.3 \text{ m/s})}{9.8 \text{ m/s}^2} = 6.4 \text{ s}.$$

(c) The plots of y, v, and a as a function of time are shown in the figure. The acceleration graph is a horizontal line at –9.8 m/s^2. At $t = 3.19$ s, $y = 50$ m.



LEARN In calculating the total flight time of the ball, we could have used Eq. 2.4.5. At $t = T > 0$, the ball returns to its original position $(y = 0)$. Therefore,

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \;\Rightarrow\; T = \frac{2v_0}{g}.$$

46. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s² (where *down* is our $-y$ direction) for the duration of the fall. This is constant-acceleration motion, and we may use Table 2.4.1 (with Δy replacing Δx).

(a) Using Eq. 2.4.6 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\,\Delta y} = -\sqrt{0 - 2(9.8\text{ m/s}^2)(-1700\text{ m})} = -183\text{ m/s}.$$

Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

47. **THINK** The wrench is in free fall with an acceleration of $a = -g = -9.8$ m/s².

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s² (taking *down* as the $-y$ direction) for the duration of the fall. This is constant-acceleration motion, which justifies the use of Table 2.4.1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

Since the wrench had initial speed $v_0 = 0$, knowing its speed of impact allows us to apply Eq. 2.4.6 to calculate the height from which it was dropped.

ANALYZE (a) Using $v^2 = v_0^2 + 2a\,\Delta y$, we find the initial height to be

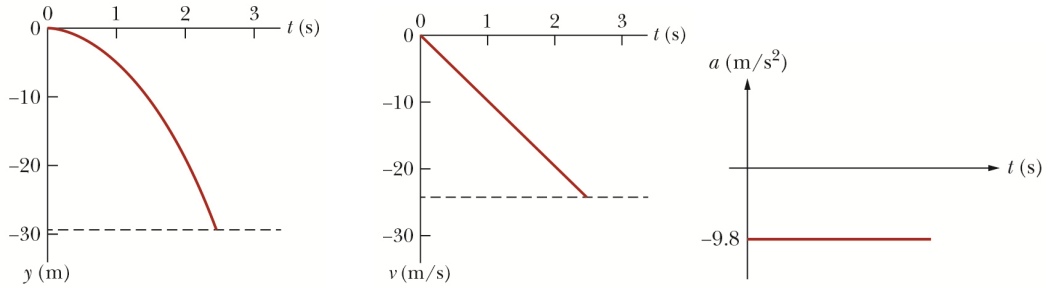
$$\Delta y = \frac{v_0^2 - v^2}{-2a} = \frac{0 - (-24\ \text{m/s})^2}{-2(-9.8\ \text{m/s}^2)} = -29.4\ \text{m}.$$

So it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24\ \text{m/s})}{9.8\ \text{m/s}^2} = 2.45\ \text{s}.$$

(c) SI units are used in the graphs shown here, and the initial position is taken as the coordinate origin. The acceleration graph is a horizontal line at –9.8 m/s².



LEARN As the wrench falls, with $a = -g < 0$, its speed increases, but its velocity becomes more negative, as indicated by the second graph.

48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s² (taking *down* as the $-y$ direction) for the duration of the fall. This is constant-acceleration motion, which justifies the use of Table 2.4.1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2.4.5 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\,\Delta y}}{g},$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12 \text{ m/s} + \sqrt{(-12 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(-30 \text{ m})}}{9.8 \text{ m/s}^2} = 1.54 \text{ s}.$$

(b) Enough information is now known that any of the equations in Table 2.4.1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2.4.6:

$$v = \sqrt{v_0^2 - 2g\,\Delta y} = 27.1 \text{ m/s},$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity).

49. **THINK** In this problem a package is dropped from a hot-air balloon that is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. This allows us to use Table 2.4.1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2.4.1)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2.4.5)$$

$$v^2 = v_0^2 - 2g(y - y_0). \qquad (2.4.6)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s, and that its initial coordinate is $y_0 = +80$ m. The time the package takes to hit the ground can be found by solving Eq. 2.4.5 with $y = 0$.

ANALYZE (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for t):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 \text{ m/s} + \sqrt{(12 \text{ m/s})^2 + 2\left(9.8 \text{ m/s}^2\right)(80 \text{ m})}}{9.8 \text{ m/s}^2} = 5.447 \approx 5.4 \text{ s}.$$

(b) The velocity of the package when it hits the ground can be calculated using Eq. 2.4.1:

$$v = v_0 - gt = 12 \text{ m/s} - (9.8 \text{ m/s}^2)(5.447 \text{ s}) = -41.38 \text{ m/s}.$$

So its final *speed* is about 41 m/s.

LEARN Our answers can be verified by using Eq. 2.4.6, which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(0 - 80 \text{ m})} = -41.38 \text{ m/s},$$

which agrees with that calculated in (b).

50. The y coordinate of Apple 1 obeys $y - y_{01} = -\frac{1}{2} gt^2$ where $y = 0$ when $t = 2.0$ s. This allows us to solve for y_{01}, and we find $y_{01} = 19.6$ m.

The graph for the coordinate of Apple 2 (which is thrown apparently at $t = 1.0$ s with velocity v_2) is

$$y - y_{02} = v_2(t - 1.0) - \frac{1}{2} g\,(t - 1.0)^2,$$

where $y_{02} = y_{01} = 19.6$ m and where $y = 0$ when $t = 2.25$ s. Thus, we obtain $|v_2| = 9.6$ m/s, approximately.

51. (a) With upward chosen as the $+y$ direction, we use Eq. 2.4.1 to find the initial velocity of the package:

$$v = v_0 + at \quad \Rightarrow \quad 0 = v_0 - (9.8 \text{ m/s}^2)(2.0 \text{ s}),$$

which leads to $v_0 = 19.6$ m/s. Now we use Eq. 2.4.5:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m}.$$

We note that the "2.0 s" in this second computation refers to the time interval $2 < t < 4$ in the graph (whereas the "2.0 s" in the first computation referred to the $0 < t < 2$ time interval shown in the graph).

(b) In our computation for part (b), the time interval ("6.0 s") refers to the $2 < t < 8$ portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m},$$

or $|\Delta y| = 59$ m.

52. The full extent of the bolt's fall is given by

$$y - y_0 = -\frac{1}{2} g t^2,$$

where $y - y_0 = -90$ m (if upward is chosen as the positive y direction). Thus the time for the full fall is found to be $t = 4.29$ s. The first 80% of its free-fall distance is given by $-72 = -g\tau^2/2$, which requires time $\tau = 3.83$ s.

(a) Thus, the final 20% of its fall takes $t - \tau = 0.45$ s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v| = 38$ m/s, approximately.

(c) Similarly, $v_{final} = -gt \quad \Rightarrow \quad |v_{final}| = 42$ m/s.

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

EXPRESS The speed of the boat is constant, given by $v_b = d/t$, where d is the distance (12 m) of the boat from the bridge when the key is dropped and t is the time the key takes in falling. To calculate t, we take the time to be zero at the instant the key is dropped and compute the time t when $y = 0$ by using $y = y_0 + v_0 t - \frac{1}{2} g t^2$ with $y_0 = 45$ m. Once t is known, the speed of the boat can be calculated.

ANALYZE Since the initial velocity of the key is zero, we have $y_0 = \frac{1}{2} g t^2$. Thus, the time the key takes to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0$ m/s.

LEARN From the general expression

$$v_b = \frac{d}{t} = \frac{d}{\sqrt{2y_0/g}} = d\sqrt{\frac{g}{2y_0}},$$

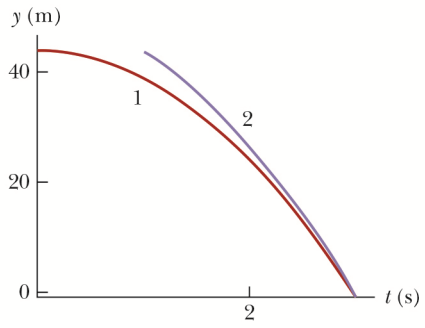
we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

54. (a) We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2.4.5 (with Δy replacing Δx) because this is constant-acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so v_0 is being used for the initial *speed*). SI units are used throughout:

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2 .$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



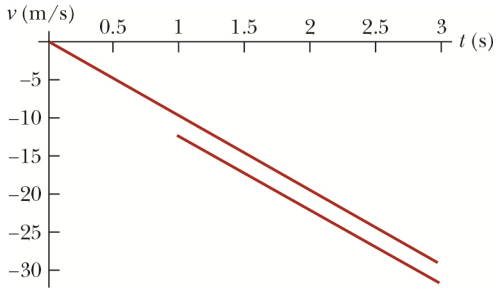
$$-43.9 \text{ m} = (-v_0)(1.99 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.99 \text{ s})^2 ,$$

which leads to $v_0 = 12.3$ m/s.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1).$$

The plot is shown below:



55. **THINK** The free-falling moist-clay ball strikes the ground with a nonzero speed, and it undergoes deceleration before coming to rest.

EXPRESS During contact with the ground, its average acceleration is given by $a_{\text{avg}} = \dfrac{\Delta v}{\Delta t}$, where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0\times10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground ($y = 0$).

ANALYZE (a) Now, to find the velocity just *before* contact, we take $t = 0$ to be when the ball is dropped. Using Eq. 2.4.6 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{0 - 2(9.8\ \text{m/s}^2)(0 - 15\ \text{m})} = -17.15\ \text{m/s},$$

where the negative option of the square root is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0 - (-17.15\ \text{m/s})}{20.0 \times 10^{-3}\ \text{s}} = 857\ \text{m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration is upward.

LEARN Since Δt is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

56. We use Eq. 2.4.6,

$$v_B^2 = v_A^2 + 2a(y_B - y_A),$$

with $a = -9.8$ m/s^2, $y_B - y_A = 0.40$ m, and $v_B = \dfrac{1}{3} v_A$. It is then straightforward to solve: $v_A = 3.0$ m/s, approximately.

57. The average acceleration during contact with the floor is $a_{\text{avg}} = (v_2 - v_1) / \Delta t$, where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8\ \text{m/s}^2)(-4.00\ \text{m})} = -8.85\ \text{m/s},$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8 \text{ m/s}^2)(-2.00 \text{ m} + 4.00 \text{ m})} = 6.26 \text{ m/s}.$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 \text{ m/s} - (-8.85 \text{ m/s})}{12.0 \times 10^{-3} \text{ s}} = 1.26 \times 10^3 \text{ m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration is upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.
(a) We find t' and t from Eq. 2.4.5 (with $v_0 = 0$):

$$y' = \frac{1}{2} g t'^2 \;\Rightarrow\; t' = \sqrt{\frac{2y'}{g}}$$

$$y = \frac{1}{2} g t^2 \;\Rightarrow\; t = \sqrt{\frac{2y}{g}}.$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50}.$$

Letting $t' = t - 1.00$ (SI units understood) and cross multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \;\Rightarrow\; t = \frac{1.00}{1 - \sqrt{0.50}},$$

which yields $t = 3.41$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$, we find h = 57 m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50}$ = +0.707 instead of –0.707. If we had instead let $\sqrt{0.50}$ = –0.707, then our answer for t would have been roughly 0.6 s, which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped), which certainly does not fit with the physical situation described in the problem.

59. We neglect air resistance, which justifies setting $a = -g$ = –9.8 m/s² (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2.4.1 (with Δy replacing Δx) because this is constant-acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) The time drop 1 leaves the nozzle is taken as t = 0, and its time of landing on the floor t_1 can be computed from Eq. 2.4.5, with v_0 = 0 and y_1 = –2.00 m:

$$y_1 = -\frac{1}{2}gt_1^2 \quad \Rightarrow \quad t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00\text{ m})}{9.8\text{ m/s}^2}} = 0.639\text{ s}.$$

At that moment the fourth drop begins to fall, and from the regularity of the dripping, we conclude that drop 2 leaves the nozzle at t = 0.639/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213$ s = 0.426 s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.426\text{ s})^2 = -0.889\text{ m},$$

or about 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426$ s = 0.213 s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.213\text{ s})^2 = -0.222\text{ m},$$

or about 22 cm below the nozzle.

60. To find the "launch" velocity of the rock, we apply Eq. 2.4.1 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8\text{ m/s}^2\right)\left(2.5\text{ s}\right),$$

so v_0 = 24.5 m/s (with +y up). Now we use Eq. 2.4.5 to find the height of the tower (taking y_0 = 0 at the ground level):

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = (24.5 \text{ m/s})(1.5 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.5 \text{ s})^2 .$$

Thus, we obtain y = 26 m.

61. We choose *down* as the +y direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1 and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1$ = 1.20 m and $t_2 - t_1$ = 0.125 s. Using Eq. 2.4.5 we have

$$y_2 - y_1 = v_1 (t_2 - t_1) + \frac{1}{2} g (t_2 - t_1)^2 ,$$

which immediately yields

$$v_1 = \frac{1.20 \text{ m} - \frac{1}{2}(9.8 \text{ m/s}^2)(0.125 \text{ s})^2}{0.125 \text{ s}} = 8.99 \text{ m/s}.$$

From this, Eq. 2.4.6 (with v_0 = 0) reveals the value of y_1:

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{(8.99 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 4.12 \text{ m}.$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall"), we know that $t_3 - t_2$ = 2.00/2 = 1.00 s. And this means $t_3 - t_1$ = 1.00 s + 0.125 s = 1.125 s. Now Eq. 2.4.5 produces

$$y_3 - y_1 = v_1 (t_3 - t_1) + \frac{1}{2} g (t_3 - t_1)^2$$

$$y_3 - 4.12 \text{ m} = (8.99 \text{ m/s})(1.125 \text{ s}) + \frac{1}{2}(9.8 \text{ m/s}^2)(1.125 \text{ s})^2 ,$$

which yields $y_3 = H$ = 20.4 m.

62. The height reached by the player is y = 0.76 m (where we have taken the origin of the y axis at the floor and +y to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8 \text{ m/s}^2)(0.76 \text{ m})} = 3.86 \text{ m/s}.$$

This is a consequence of Eq. 2.4.6 where velocity v vanishes. As the player reaches y_1 = 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(0.61\ \text{m})} = 1.71\ \text{m/s}.$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2.4.7:

$$\Delta y_1 = \frac{1}{2}(v_1 + v)t_1 \quad \Rightarrow \quad t_1 = \frac{2(0.15\ \text{m})}{1.71\ \text{m/s} + 0} = 0.175\ \text{s},$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2.4.5:

$$0.15\ \text{m} = v_0 t_2 - \frac{1}{2}gt_2^2 = (3.86\ \text{m/s})t_2 - \frac{1}{2}(9.8\ \text{m/s}^2)t_2^2,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.041 s = 41 ms, which means that the total time spent in that bottom 15 cm (both ascending and descending) is 2(41 ms) = 82 ms.

63. The time t the pot spends passing in front of the window of length L = 2.0 m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8\ \text{m/s}^2$ (taking *down* to be the $-y$ direction), Eq. 2.4.8 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Rightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2.4.6 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00\ \text{m}/0.25\ \text{s} - (9.80\ \text{m/s}^2)(0.25\ \text{s})/2)^2}{2(9.80\ \text{m/s}^2)} = 2.34\ \text{m}.$$

64. The graph shows $y = 25$ m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2.4.5 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2} g_p t^2 \quad \Rightarrow \quad 25 \text{ m} - 0 = (0)(2.5 \text{ s}) + \frac{1}{2} g_p (2.5 \text{ s})^2,$$

so $g_p = 8.0 \text{ m/s}^2$.

(b) That same (max) point on the graph can be used to find the initial velocity:

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad \Rightarrow \quad 25 \text{ m} - 0 = \frac{1}{2}(v_0 + 0)(2.5 \text{ s}).$$

Therefore, $v_0 = 20 \text{ m/s}$.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2.4.10:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

(a) From Fig. 2.6.2*a*, we see that the head begins to accelerate from rest ($v_0 = 0$) at t_0 = 110 ms and reaches a maximum value of 90 m/s^2 at t_1 = 160 ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110) \times 10^{-3} \text{ s} \cdot (90 \text{ m/s}^2) = 2.25 \text{ m/s},$$

which is equal to v_1, the speed at t_1.
(b) To compute the speed of the torso at t_1=160 ms, we divide the area into four regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_B = \frac{1}{2}(0.0600 \text{ s})(50.0 \text{ m/s}^2) = 1.50 \text{ m/s}.$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_C = (0.0200 \text{ s})\ (50.0 \text{ m/s}^2) = 1.00 \text{ m/s}.$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.0400 \text{ s})\ (50.0 + 20.0) \text{ m/s}^2 = 1.40 \text{ m/s}.$$

Substituting these values into Eq. 2.4.10, with $v_0 = 0$ then gives

$$v_1 - 0 = 0 + 1.50 \text{ m/s} + 1.00 \text{ m/s} + 1.40 \text{ m/s} = 3.90 \text{ m/s},$$

or $v_1 = 3.90$ m/s.

66. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2.6.4:

$$x_1 - x_0 = \begin{pmatrix} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2.22 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010 \text{ s})\,(2 \text{ m/s}) = 0.01 \text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040 \text{ s})\,(2 + 4) \text{ m/s} = 0.12 \text{ m}.$$

Substituting these values into Eq. 2.6.4 with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at t_1 = 120 ms. From 50 to 90 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040 \text{ s})\,(4 + 5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030 \text{ s})\,(5 + 7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2.6.4, with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or $x_1 = 0.50$ m.

67. The problem is solved using Eq. 2.6.5:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

To compute the speed of the unhelmeted, bare head at $t_1 = 7.0$ ms, we divide the area under the a vs. t graph into four regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s})\,(120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s})\,(120 + 140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})\,(140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s})\,(200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2.6.5, with $v_0 = 0$, then gives

$$v_{\text{unhelmeted}} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0030 \text{ s})\,(40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_B = (0.0010 \text{ s})\,(40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_\text{C} = \frac{1}{2}(0.0020\text{ s})\,(40 + 80)\text{ m/s}^2 = 0.12\text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_\text{D} = \frac{1}{2}(0.0010\text{ s})\,(80\text{ m/s}^2) = 0.040\text{ m/s}.$$

Substituting these values into Eq. 2.6.5, with $v_0 = 0$, then gives

$$v_\text{helmeted} = 0.060\text{ m/s} + 0.040\text{ m/s} + 0.12\text{ m/s} + 0.040\text{ m/s} = 0.26\text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_\text{unhelmeted} - v_\text{helmeted} = 0.82\text{ m/s} - 0.26\text{ m/s} = 0.56\text{ m/s}.$$

68. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$v = \text{area} = \frac{1}{2}(10^{-2}\text{ s})(100\text{ m/s}^2) + \frac{1}{2}(10^{-2}\text{ s})(100\text{ m/s}^2 + 400\text{ m/s}^2) + \frac{1}{2}(10^{-2}\text{ s})(400\text{ m/s}^2)$$
$$= 5.0\text{ m/s}.$$

69. Since $v = dx/dt$ (Eq. 2.2.1), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\text{base}\times\text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00.$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t) dt = 20.0 - 4.00t^2.$$

The condition that $v_1 = v_2$ means

$$12.0t + 3.00 = 20.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 17.0 = 0,$$

which can be solved to give (taking positive root) $t = (-3 + \sqrt{26})/2 = 1.05$ s. Thus, the velocity at this time is $v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6$ m/s.

71. (a) The derivative (with respect to time) of the given expression for x yields the "velocity" of the spot:

$$v(t) = 9 - \frac{9}{4} t^2,$$

with 3 significant figures understood. It is easy to see that $v = 0$ when $t = 2.00$ s.

(b) At $t = 2$ s, $x = 9(2) - \frac{3}{4}(2)^3 = 12$. Thus, the location of the spot when $v = 0$ is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is $a = -\frac{9}{2} t$, which gives an acceleration of -9.00 cm/s^2 (negative sign indicating leftward) when the spot is 12 cm from the left edge of screen.

(d) Since $v > 0$ for times less than $t = 2$ s, then the spot had been moving rightward.

(e) As implied by our answer to part (c), it moves leftward for times immediately after $t = 2$ s. In fact, the expression found in part (a) guarantees that for all $t > 2$, $v < 0$ (i.e., until the clock is "reset" by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some $t > 2$ s cannot be the right edge; it is the left edge ($x = 0$). Solving the expression given in the problem statement (with $x = 0$) for positive t yields the answer: The spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

72. We adopt the convention frequently used in the text, that "up" is the positive y direction.

(a) At the highest point in the trajectory $v = 0$. Thus, with $t = 1.60$ s, the equation $v = v_0 - gt$ yields $v_0 = 15.7$ m/s.

(b) One equation that is not dependent on our result from part (a) is $y - y_0 = vt + \frac{1}{2}gt^2$; this readily gives $y_{max} - y_0 = 12.5$ m for the highest ("max") point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into $y - y_0 = v_0t + \frac{1}{2}gt^2$ with $t = 6.00$ s and $y = 0$ (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(6.00 \text{ s})^2.$$

Therefore, y_0 (the height of the building) is equal to 82.3 m.

73. We denote the required time as t, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v, we write

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = (vt)_{\text{truck}},$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5 \text{ m/s})}{2.2 \text{ m/s}^2} = 8.6 \text{ s}.$$

Therefore,

$$\Delta x = vt = (9.5 \text{ m/s})(8.6 \text{ s}) = 82 \text{ m}.$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = (2.2 \text{ m/s}^2)(8.6 \text{ s}) = 19 \text{ m/s}.$$

74. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35 \text{ m}}{\tan 4.3^\circ} = 465.5 \text{ m} \approx 0.465 \text{ km}.$$

This corresponds to a time of flight found from Eq. 2.1.2 (with $v = v_{avg}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465 \text{ km}}{1300 \text{ km/h}} = 0.000\ 358 \text{ h} \approx 1.3 \text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

75. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2,$$

where v_0 is the initial velocity and a is the acceleration (which we expect to have a negative value since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied, the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}, \qquad x_2 = v_{02} t_r - \frac{1}{2}\frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for t_r and a, we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} \left(v_{02} - v_{01}\right)}$$

and

$$a = -\frac{1}{2}\frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

(a) Substituting $x_1 = 56.7$ m, $v_{01} = 80.5$ km/h = 22.4 m/s, $x_2 = 24.4$ m, and $v_{02} = 48.3$ km/h = 13.4 m/s, we find

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})} = \frac{(13.4 \text{ m/s})^2 (56.7 \text{ m}) - (22.4 \text{ m/s})^2 (24.4 \text{ m})}{(22.4 \text{ m/s})(13.4 \text{ m/s})(13.4 \text{ m/s} - 22.4 \text{ m/s})}$$
$$= 0.74 \text{ s}.$$

(b) Similarly, substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m, and v_{02} = 48.3 km/h = 13.4 m/s gives

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2}\frac{(13.4\text{ m/s})(22.4\text{ m/s})^2 - (22.4\text{ m/s})(13.4\text{ m/s})^2}{(13.4\text{ m/s})(56.7\text{ m}) - (22.4\text{ m/s})(24.4\text{ m})}$$
$$= -6.2\text{ m/s}^2.$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded-off values are displayed in the above substitutions, what we have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m/s).

76. (a) A constant velocity is equal to the ratio of displacement to elapsed time. Thus, for the vehicle to be traveling at a constant speed v_p over a distance D_{23}, the time delay should be $t = D_{23}/v_p$.

(b) The time required for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p/a$. During this time interval, the distance traveled is $\Delta x_0 = at_0^2/2 = v_p^2/2a$. The car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2, and the time elapsed is $t_1 = (D_{12} - \Delta x_0 - d)/v_p$. Thus, the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - (v_p^2/2a) - d}{v_p}$$
$$= t_r + \frac{1}{2}\frac{v_p}{a} + \frac{D_{12} - d}{v_p}.$$

77. **THINK** The speed of the hot rod changes due to a nonzero acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the hot rod can be analyzed using the equations given in Table 2.4.1. We take the $+x$ direction to be in direction of motion, so

$$v = (60\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = +16.7\text{ m/s}$$

and $a > 0$. The location where the hot rod starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

ANALYZE (a) Using Eq. 2.3.1, we find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7\text{ m/s} - 0}{5.4\text{ s} - 0} = 3.09\text{ m/s}^2.$$

(b) Assuming constant acceleration $a = a_{\text{avg}} = 3.09\ \text{m/s}^2$, the total distance traveled during the 5.4 s time interval is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(3.09\ \text{m/s}^2)(5.4\ \text{s})^2 = 45\ \text{m}.$$

(c) Using Eq. 2.4.5, we find the time required to travel a distance of x = 250 m:

$$x = \frac{1}{2}at^2 \ \Rightarrow\ t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\ \text{m})}{3.09\ \text{m/s}^2}} = 12.73\ \text{s}.$$

LEARN The displacement of the hot rod as a function of time can be written as $x(t) = \frac{1}{2}(3.09\ \text{m/s}^2)t^2$. Note that we could have chosen Eq. 2.4.7 to solve for (b):

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7\ \text{m/s})(5.4\ \text{s}) = 45\ \text{m}.$$

78. We take the moment of applying brakes to be t = 0. The deceleration is constant, so Table 2.4.1 can be used. Our primed variables (such as $v_0' = 72\ \text{km/h} = 20\ \text{m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when t = 0) and unprimed variables refer to the other (moving in the $-x$ direction and located at x_0 = +950 m when t = 0). We note that the acceleration of the unprimed train is in the *positive* direction, even though the train is slowing down; its initial velocity is v_0 = –144 km/h = –40 m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2.4.6, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v_0')^2}{2a'} = \frac{0 - (20\ \text{m/s})^2}{-2\ \text{m/s}^2} = 200\ \text{m}.$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_0^2 + 2a\,\Delta x} = \sqrt{(-40\ \text{m/s})^2 + 2(1.0\ \text{m/s}^2)(200\ \text{m} - 950\ \text{m})}$$
$$= 10\ \text{m/s}$$

using Eq 2.4.6 again. Specifically, its velocity at that moment would be –10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root), then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the

time it stops (Eq. 2.4.1 yields $t = 20$ s) and seeing where the unprimed train is at that moment (Eq. 2.4.8 yields $x = 350$ m, still a good distance away from contact).

79. The y coordinate of Piton 1 obeys $y - y_{01} = -\frac{1}{2}gt^2$ where $y = 0$ when $t = 3.0$ s. This allows us to solve for y_{01}, and we find $y_{01} = 44.1$ m. The graph for the coordinate of Piton 2 (which is thrown apparently at $t = 1.0$ s with velocity v_1) gives us

$$y - y_{02} = v_1(t-1.0) - \frac{1}{2}g\,(t - 1.0)^2,$$

where $y_{02} = y_{01} + 10 = 54.1$ m and where (again) $y = 0$ when $t = 3.0$ s. Thus we obtain $|v_1| = 17$ m/s, approximately.

80. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s, and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2.4.6 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{(50\text{ m/s})^2 - (30\text{ m/s})^2}{2(160\text{ m})} = 5.0\text{ m/s}^2.$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2.4.7:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160\text{ m})}{30\text{ m/s} + 50\text{ m/s}} = 4.0\text{ s}.$$

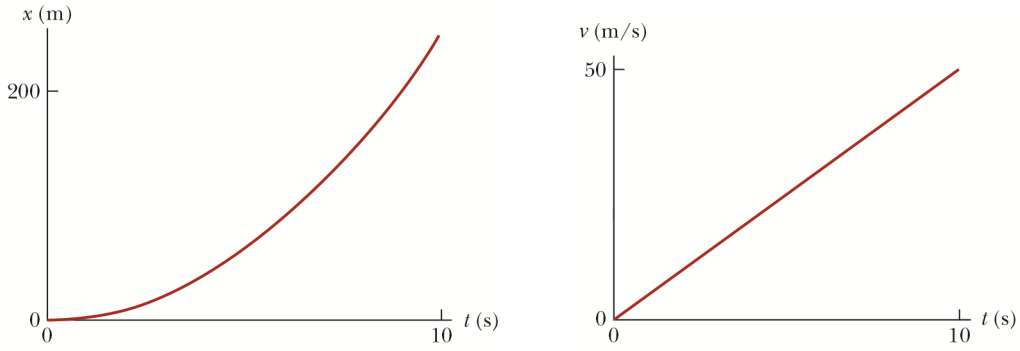
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2.4.1:

$$v_1 = v_0 + at_1 \quad \Rightarrow \quad t_1 = \frac{30\text{ m/s}}{5.0\text{ m/s}^2} = 6.0\text{ s}.$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2.4.7:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30\text{ m/s})(6.0\text{ s}) = 90\text{ m}.$$

(e) The graphs are shown below.



81. **THINK** Because the particle undergoes a *variable* acceleration, an integration is required to calculate velocity.

EXPRESS With a variable acceleration $a(t) = dv/dt$, the velocity of the particle at time t_1 is given by Eq. 2.6.1:

$$v_1 = v_0 + \int_{t_0}^{t_1} a(t)\, dt,$$

where v_0 is the velocity at time t_0. In our situation, we have $a = 5.0t$. We also know that $v_0 = 17$ m/s at $t_0 = 2.0$ s.

ANALYZE Integrating (from t = 2 s to t = 4 s) the acceleration to get the velocity and using the values given in the problem lead to

$$v = v_0 + \int_{t_0}^{t} a\, dt = v_0 + \int_{t_0}^{t} (5.0t)\, dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(4^2 - 2^2) = 47 \text{ m/s}.$$

LEARN The velocity of the particle as a function of t is

$$v(t) = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(t^2 - 4) = 7 + 2.5t^2$$

in SI units (m/s). Since the acceleration is linear in t, we expect the velocity to be quadratic in t and the displacement to be cubic in t.

82. The velocity v at t = 6 (SI units and-with two significant figures understood) is $v_{\text{given}} + \int_{-2}^{6} a dt$. A quick way to implement this is to recall the area of a triangle ($\frac{1}{2}$ base × height). The result is v = 7 m/s + 32 m/s = 39 m/s.

83. The object, once it is dropped ($v_0 = 0$) is in free fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2.4.5 repeatedly.

(a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus, for $t_1 = 50.0$ ms,

$$D_1 = \frac{\left(9.8\ \text{m/s}^2\right)\left(50.0\times10^{-3}\ \text{s}\right)^2}{2} = 0.0123\ \text{m} = 1.23\ \text{cm}.$$

(b) For $t_2 = 100$ ms, $D_2 = \dfrac{\left(9.8\ \text{m/s}^2\right)\left(100\times10^{-3}\ \text{s}\right)^2}{2} = 0.049\ \text{m} = 4D_1$.

(c) For $t_3 = 150$ ms, $D_3 = \dfrac{\left(9.8\ \text{m/s}^2\right)\left(150\times10^{-3}\ \text{s}\right)^2}{2} = 0.11\ \text{m} = 9D_1$.

(d) For $t_4 = 200$ ms, $D_4 = \dfrac{\left(9.8\ \text{m/s}^2\right)\left(200\times10^{-3}\ \text{s}\right)^2}{2} = 0.196\ \text{m} = 16D_1$.

(e) For $t_4 = 250$ ms, $D_5 = \dfrac{\left(9.8\ \text{m/s}^2\right)\left(250\times10^{-3}\ \text{s}\right)^2}{2} = 0.306\ \text{m} = 25D_1$.

84. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so $v = 1600(1000/3600) = 444$ m/s.

(a) Equation 2.4.1 gives $444 = a(1.8)$ or $a = 247$ m/s^2. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) g = 25g.$$

(b) Equation 2.4.7 readily yields

$$x = \frac{1}{2}\left(v_0 + v\right) t = \frac{1}{2}\left(444\ \text{m/s}\right)\left(1.8\ \text{s}\right) = 400\ \text{m}.$$

85. (a) The time interval is the ratio of the distance to the ball's speed:

$$\Delta t = \frac{\Delta x}{v} = \frac{60.5 \text{ ft}}{95 \text{ mi/h}} = \frac{60.5 \text{ ft}}{(95 \text{ mi/h})\dfrac{1.466 \text{ ft/s}}{1 \text{ mi/h}}} = 0.434 \text{ s} = 434 \text{ ms}.$$

(b) The distance is

$$\Delta x = v\,\Delta t = (95.0 \text{ mi/h})\left(\frac{1.466 \text{ ft/s}}{1 \text{ mi/h}}\right)(20\times10^{-3} \text{ s}) = 2.79 \text{ ft}.$$

86. For the two passages through the lower level, we write

$$\Delta y_L = v_L \Delta T_L - \tfrac{1}{2} g\,\Delta T_L^2 = 0.$$

For the two passages through the upper level, we write

$$\Delta y_U = v_U \Delta T_U - \tfrac{1}{2} g\,\Delta T_U^2 = 0.$$

From the graph, the time between the instants when the upward moving ball has velocities v_U and v_L is $\frac{1}{2}(\Delta T_L - \Delta T_U)$, and the vertical distance traveled is H. So, we rewrite

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$

as

$$H = \left(\frac{v_L + v_U}{2}\right)\tfrac{1}{2}\left(\Delta T_L - \Delta T_U\right) = \tfrac{1}{4}\left(v_L\,\Delta T_L - v_U\,\Delta T_U\right),$$

where we used $v_L / \Delta T_L = v_U / \Delta T_U$ by combining the first two equations. Combining all three equations, we find

$$g = \frac{8H}{\Delta T_L^2 - \Delta T_H^2}.$$

87. (a) The position x is given by the integral of the velocity. So the position at $t = 0$ is related to the position at $t = t_A$ by

$$x_0 + \int_0^A v\,dt = x_A,$$

where $x_A = 14$ m, and the integration gives us −20 m. Thus, $x_0 = 34$ m.

(b) Now we have

$$x_0 + \int_0^B v\,dt = x_B,$$

where the integration gives us (−20 m) + (+20 m) = 0. So, x_B = 34 m.

88. The puck's displacement Δx is equal to the graphical integration of the velocity. The graph consists of two triangles, the first with a positive area and the second with a negative area. We write

$$\Delta x = \int v\,dt = \tfrac{1}{2}(5\text{ s})(10\text{ m/s}) - \tfrac{1}{2}(9\text{ s})(18\text{ m/s}) = -56\text{ m}.$$

89. We invert the slope of the graph to get 20 km/My. Then we write

$$\begin{aligned} v &= (20\text{ km/My})(1\times10^5\text{ cm/km})(1\times10^{-6}\text{ My/y}) \\ &= 2\text{ cm/y}. \end{aligned}$$

90. The distance traveled by a car at speed v = 31.3 m/s during the reaction time of t = 0.75 s is

$$d = vt_r = (31.3\text{ m/s})(0.750\text{ s}) = 23.5\text{ m}.$$

91. In general, the average velocity is the ratio of the displacement to the time interval for the displacement: $v = \Delta x / \Delta t$. We have (a) 9.8 m/s, (b) 12 m/s, and (c) 11 m/s.

92. (a) We can apply the constant-acceleration equations to both car and motorcycle, but for the motorcycle we must consider the motion in two stages: (1) First it travels through distance x_{m1} with zero initial velocity and acceleration a_m = 8.40 m/s^2, reaching speed v_m = 58.8 m/s. (2) Then it travels through distance x_{m2} with constant speed v_m = 58.8 m/s and zero acceleration. We want the time t when the midpoints have the same coordinate: x_c for the car and the sum $x_{m1} + x_{m2}$ for the motorcycle:

$$x_c = x_{m1} + x_{m2}.$$

For the car, we set x_0 = 0 and v_0 = 0 and write

$$x_c = \tfrac{1}{2}a_c t^2.$$

For the motorcycle, we first find time t_m to maximum speed v_m. We set v_0 = 0, $v = v_m$ = 58.8 m/s, and $a = a_m$ =8.40 m/s^2. We find

$$t_m = \frac{v_m}{a_m} = \frac{58.8\text{ m/s}}{8.40\text{ m/s}^2} = 7.00\text{ s}.$$

To find the distance x_{m1} traveled during that first stage, we set $x_0 = 0$ and $v_0 = 0$ and use our result for t_m:

$$x_{m1} = \tfrac{1}{2} a_m t_m^2 = \tfrac{1}{2} a_m \left(\frac{v_m}{a_m} \right)^2 = \frac{v_m^2}{2a_m}.$$

For the second stage with time $t - t_m$, we write

$$x_{m2} = v_m (t - t_m) = v_m (t - 7.00 \text{ s}).$$

Next, we return to $x_c = x_{m1} + x_{m2}$, putting in our results:

$$\tfrac{1}{2} a_c t^2 = \frac{v_m^2}{2a_m} + v_m (t - 7.00 \text{ s}).$$

This quadratic equation yields two answers, $t = 4.44378$ s and $t = 16.562$ s, but only the second answer is meaningful because we know that the car passes the motorcycle *after* the motorcycle reaches its top speed at $t = 7.00$ s. Thus, the passing occurs at

$$t = 16.562 \text{ s} \approx 16.6 \text{ s}.$$

(b) We can now find the position of the car (and motorcycle) when the passing occurs:

$$x_c = \tfrac{1}{2} a_c t^2 = \tfrac{1}{2} (5.60 \text{ m/s}^2)(16.562 \text{ s})^2 = 768 \text{ m}.$$

93. We convert meters per second to body lengths per second:

$$\text{body lengths per second} = \frac{\text{speed in meters per second}}{\text{body length in meters}} = \frac{0.855 \text{ m/s}}{7.92 \times 10^{-3} \text{ m}} = 108.$$

94. The distance you travel during your reaction time is

$$d_r = v_0 t_r.$$

For a trailing distance of nL, the braking distance d is

$$d = nL - d_r = nL - v_0 t_r.$$

We can find the speed v at the end of the braking from one of the constant-acceleration equations:

$$v^2 = v_0^2 + 2ad$$
$$v = \left[v_0^2 + 2ad \right]^{0.5} = \left[v_0^2 + 2a(nL - v_0 t_r) \right]^{0.5}.$$

(a) Substituting v_0 = 31.3 m/s, a = −8.50 m/s^2, n = 10, L = 4.50 m, and t_r = 0.750 s, we find that your speed just before hitting the car is v = 24.774 m/s ≈ 24.8 m/s (≈ 55.4 mi/h).

(b) If your control system activates the braking with no delay (t_r = 0), v = 14.7 m/s (32.8 mi/h), which is obviously much safer.

(c) We start with

$$v = \left[v_0^2 + 2a(nL - v_0 t_r) \right]^{0.5},$$

set v = 0, and solve for n:

$$n = \frac{2av_0 t_r - v_0^2}{2aL}.$$

For t_r = 0.750 s, n = 18.0, that is, you need a trailing distance of 18.0 car lengths.

(d) For no response time, n = 12.8.

95. At constant velocity v, the time interval t required to travel through displacement Δx is

$$t = \frac{\Delta x}{v}.$$

(a) For the two speed values on the Texas highway, the time difference is

$$t_{60} - t_{85} = \frac{\Delta x}{v_{60}} - \frac{\Delta x}{v_{85}} = (41 \text{ mi})\left(\frac{1}{60 \text{ mi/h}} - \frac{1}{85 \text{ mi/h}} \right) = 0.20 \text{ h} = 12 \text{ min}.$$

Some would consider the time saved to be small compared to the added risk at the speed limit.

(b) The calculation is similar for residential driving:

$$t_{25} - t_{45} = \frac{\Delta x}{v_{25}} - \frac{\Delta x}{v_{45}} = (5.5 \text{ mi})\left(\frac{1}{25 \text{ mi/h}} - \frac{1}{45 \text{ mi/h}}\right) = 0.098 \text{ h} = 5.9 \text{ min.}$$

96. During the interval $\Delta t = 15.0$ s, the rear of car A moves from $x_{A1} = 0$ to position

$$x_{A2} = v_A \Delta t.$$

At $t = 0$, the front of car C is at position $x_{C1} = -d - 4L$. At the end of the maneuver, we want the front to be at

$$x_{C2} = x_{A2} - 3L = v_A \Delta t - 3L.$$

Thus, C moves through displacement

$$\Delta x_C = x_{C2} - x_{C1} = (v_A \Delta t - 3L) - (-d - 4L) = v_A \Delta t + L + d.$$

The displacement is also given by $\Delta x_C = v_C \Delta t$. Equating the two expressions for the displacement and inserting the given data, we find

$$\begin{aligned} v_A \Delta t + L + d &= v_C \Delta t \\ d &= \Delta t (v_C - v_A) - L \\ &= (15.0 \text{ s})(27.0 \text{ m/s} - 22.0 \text{ m/s}) - 4.50 \text{ m} \\ &= 70.5 \text{ m.} \end{aligned}$$

Thus, the computer system will not allow the passing maneuver if the front of car C is initially closer to the rear of car B than 70.5 m.

97. (a) Assume the entering car moves along the $+x$ direction. We can use the constant-acceleration equation

$$v^2 = v_0^2 + 2ad,$$

finding

$$\begin{aligned} v &= (v_0^2 + 2ad)^{0.5} \\ &= \left[(11.17 \text{ m/s})^2 + 2(4.0 \text{ m/s}^2)(36.58 \text{ m})\right]^{0.5} \\ &= 20.43 \text{ m/s} = 46 \text{ mi/h.} \end{aligned}$$

The car's speed is obviously dangerously lower than the speed of the other cars, and thus there were many rear-end collisions at the entrance ramps of the early freeways.

(b) We start with the same constant-acceleration equation but now write

$$d = \frac{v^2 - v_0^2}{2a} = \frac{(24.59 \text{ m/s})^2 - (11.17 \text{ m/s})^2}{2(4.0 \text{ m/s}^2)}$$
$$= 59.99 \text{ m} = 66 \text{ yd}.$$

98. During the interval $\Delta t = 15.0$ s, the rear of car A moves from $x_{A1} = 0$ to position

$$x_{A2} = v_A \Delta t.$$

At the end of the interval, we want the front of car B to be at position

$$x_{B2} = (x_{A2} + L) + 3L + L = x_{A2} + 5L = v_A \Delta t + 5L.$$

We also want the front of car C to be at

$$x_{C2} = x_{B2} + 10L = (v_A \Delta t + 5L) + 10L = v_A \Delta t + 15L.$$

At t = 0, the front of car C is at $x_{C1} = d - 3L$. Thus, during the maneuver, C moves through the displacement

$$\Delta x_C = x_{C2} - x_{C1} = (v_A \Delta t + 15L) - (d - 3L) = v_A \Delta t + 18L - d.$$

The displacement is also given by $\Delta x_C = -v_C \Delta t$, where $-v_C$ is the velocity of C along the x axis. Equating the two expressions for the displacement and inserting the given data, we find

$$v_A \Delta t + 18L - d = -v_C \Delta t$$
$$d = \Delta t(v_C + v_A) + 18L$$
$$= (15.0 \text{ s})(27.0 \text{ m/s} + 22.0 \text{ m/s}) + 18(4.50 \text{ m})$$
$$= 816 \text{ m}.$$

Thus, the control system will not allow the passing maneuver if the front of car C is initially closer to the front of car B than 816 m.

99. (a) O'Neil's average acceleration was

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{392.54 \text{ mi/h} - 0}{3.72 \text{ s} - 0} = 105.5 \ \frac{\text{mi}}{\text{h} \cdot \text{s}}$$
$$= \left(105.5 \ \frac{\text{mi}}{\text{h} \cdot \text{s}}\right)\left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 47.15 \text{ m/s}^2 \approx 47.2 \text{ m/s}^2.$$

(b) Dividing by 9.8 m/s^2, we find her acceleration in g units:

$$a_{\text{avg}} = \frac{47.15 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 4.81g.$$

(c) Beeding's average acceleration was

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{72.5 \text{ mi/h} - 0}{0.0400 \text{ s} - 0} = 1812 \ \frac{\text{mi}}{\text{h} \cdot \text{s}}$$
$$= \left(1812 \ \frac{\text{mi}}{\text{h} \cdot \text{s}}\right)\left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 810 \text{ m/s}^2.$$

(d) Dividing by 9.8 m/s^2, we find his acceleration in g units:

$$a_{\text{avg}} = \frac{810 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 82.7g.$$

Although Beeding's speed was much slower than O'Neil's, his ride was far more thrilling because of the large acceleration (which would have been lethal had it lasted much longer).

100. At the end of the acceleration period of t_a = 0.0450 y = 1.4200×10^6 s, the starship has the speed

$$v_c = at_a = (9.8 \text{ m/s}^2)(1.4200 \times 10^6 \text{ s}) = 1.3916 \times 10^7 \text{ m/s}$$

and has traveled the distance

$$d_a = \tfrac{1}{2}(9.8 \text{ m/s}^2)(1.4200 \times 10^6 \text{ s})^2 = 9.8803 \times 10^{12} \text{ m}.$$

It travels the same distance during the deceleration period. The star is at a distance of

$$L = 4.244 \text{ ly} = 4.0150 \times 10^{16} \text{ m}.$$

During the constant speed period, the starship travels the distance

$$d_c = L - 2d_a = (4.0150 \times 10^{16} \text{ m}) - 2(9.8803 \times 10^{12} \text{ m})$$
$$= 4.0130 \times 10^{16} \text{ m},$$

which takes a time of

$$t_c = \frac{d_c}{v_c} = \frac{4.0130\times10^{16}\ \text{m}}{1.3916\times10^{7}\ \text{m/s}} = 2.8837\times10^{9}\ \text{s} = 91.38\ \text{y}.$$

The total time is

$$t_{\text{total}} = (91.38\ \text{y}) + 2(0.0450\ \text{y}) = 91.5\ \text{y}.$$

101. (a) We get the speed at 9.000 s by taking a time derivative of $x(t)$ and then substituting $t = 9.000$ s:

$$v = \frac{dx}{dt} = \frac{d}{dt}(0.3305t^2 + 4.2060t) = 0.6610t + 4.2060$$
$$= 0.6610(9.000\ \text{s}) + 4.2060 = 10.16\ \text{m/s}.$$

(b) We take a time derivative of $v(t)$:

$$a = \frac{d}{dt}(0.6610t + 4.2060) = 0.6610\ \text{m/s}^2.$$

102. (a)–(b) The car−car separation given by the 2 s rule is the distance your car travels in 2 s. Take the time to be 2.00 s and let v_0 be your car's speed. Your acceleration a is zero. We write the distance you travel as

$$d_{2,\text{slow}} = v_0 t + \tfrac{1}{2}at^2 = (15.6\ \text{m/s})(2.00\ \text{s}) + \tfrac{1}{2}(0)(2.00\ \text{s})^2 = 31.2\ \text{m}$$
$$d_{2,\text{fast}} = v_0 t + \tfrac{1}{2}at^2 = (31.3\ \text{m/s})(2.00\ \text{s}) + \tfrac{1}{2}(0)(2.00\ \text{s})^2 = 62.6\ \text{m}.$$

(c)–(d) Similarly, the car−car separation given by the 3 s rule is the distance your car travels in 3.00 s. We now have

$$d_{3,\text{slow}} = v_0 t = (15.6\ \text{m/s})(3.00\ \text{s}) = 46.8\ \text{m}$$
$$d_{3,\text{fast}} = v_0 t = (31.3\ \text{m/s})(3.00\ \text{s}) = 93.9\ \text{m}.$$

(e) The stopping distance is the sum of the distance d_1 traveled during the reaction time and the distance d_2 traveled during the braking time. For d_1 we write $d_1 = v_0 t_r$. For d_2 we use,

$$v^2 = v_0^2 + 2ad_2,$$

substituting d_2 for $x - x_0$ and setting the final speed as $v = 0$. That gives us

$$d_2 = -\frac{v_0^2}{2a}.$$

The total stopping distance is then

$$d_{\text{tot}} = d_1 + d_2 = v_0 t_r - \frac{v_0^2}{2a},$$

where $t_r = 0.750$ s and $a = -8.50$ m/s^2.
For $v_0 = 15.6$ m/s, we find

$$d_{\text{tot,slow}} = (15.6 \text{ m/s})(0.750 \text{ s}) - \frac{(15.6 \text{ m/s})^2}{2(-8.50 \text{ m/s}^2)} = 26.0 \text{ m}.$$

(f) For $v_0 = 31.3$ m/s, we find

$$d_{\text{tot,fast}} = (31.3 \text{ m/s})(0.750 \text{ s}) - \frac{(31.3 \text{ m/s})^2}{2(-8.50 \text{ m/s}^2)} = 81.1 \text{ m}.$$

(g) The 2 s rule is fine at the slow speed but not at the fast speed.
(h) The 3 s rule is fine at both speeds.

103. In general, jerk = $\Delta a/\Delta t$, where the change in acceleration is divided by the time interval for the change. We find

(a) $$\text{jerk}_{(a)} = \frac{(1.8 \text{ m/s}^2) - 0}{2.0 \text{ s}} = +0.90 \text{ m/s}^3,$$

(b) $$\text{jerk}_{(b)} = \frac{(1.5 \text{ m/s}^2) - (1.8 \text{ m/s}^2)}{1.5 \text{ s}} = -0.20 \text{ m/s}^3,$$

(c) $$\text{jerk}_{(c)} = \frac{(-1.7 \text{ m/s}^2) - (1.5 \text{ m/s}^2)}{1.5 \text{ s}} = -2.1 \text{ m/s}^3,$$

(d) $$\text{jerk}_{(d)} = \frac{0 - (-1.7 \text{ m/s}^2)}{2.5 \text{ s}} = +0.68 \text{ m/s}^3.$$

104. The time for the ball to reach the pitcher is

$$\Delta t = \frac{\Delta x}{v}.$$

(a) For the wood bat, we find

$$\Delta t_w = \frac{(60.5 \text{ ft})(0.3048 \text{ m/ft})}{50.98 \text{ m/s}} = 0.3617 \text{ s} \approx 0.362 \text{ s}.$$

(b) Similarly, for the metal bat, we find

$$\Delta t_m = \frac{(60.5 \text{ ft})(0.3048 \text{ m/ft})}{61.50 \text{ m/s}} = 0.2998 \text{ s} \approx 0.300 \text{ s}.$$

(c) To find the percentage difference for the switch, we calculate

$$\frac{\Delta t_m - \Delta t_w}{\Delta t_w} = \frac{(0.2998 \text{ s}) - (0.3617 \text{ s})}{0.3617 \text{ s}}$$
$$= -0.171 = -17.1\%.$$

105. (a) Choose the release point to be the origin of a y axis. Setting $y_0 = 0$, $v_0 = 0$, and $t = 1.5$ s, we use one of the constant-acceleration equations to find the y coordinate of the wrench at that time:

$$y = v_0 t - \tfrac{1}{2} g t^2 = (0)(1.5 \text{ s}) - \tfrac{1}{2}(9.8 \text{ m/s}^2)(1.5 \text{ s})^2 = -11 \text{ m}.$$

(b) Of several ways we can find the velocity, here is an easy way:

$$v = v_0 - gt = 0 - (9.8 \text{ m/s}^2)(1.5 \text{ s}) = -15 \text{ m/s}.$$

So, the speed is 15 m/s.

106. Because we assume the acceleration is constant, we can use

$$v^2 = v_0^2 + 2ad$$
$$a = \frac{v^2 - v_0^2}{2d},$$

where $v = 0$ (the driver's final speed is zero), v_0 is the initial speed (either 15.65 m/s for 35.0 mi/h or 31.29 m/s for 70.0 mi/h), and $d = 0.500$ m is the stopping distance. This gives us $|a| = 25.0g$ for 35 mi/h and $|a| = 99.9g$ for 70 mi/h. In short, doubling the initial speed results in multiplying the acceleration by 4, for a far more dangerous stopping.

107. The travel distance is given by $d = vt$, where v is your speed and t is the duration of your distraction: (a) 6.3 m, 6.8 yd; (b) 25 m, 27 yd; (c) 63 m, 68 yd (more than half of a football playing field!).

108. The acceleration is given by the slope on the v versus t graph. (a) The slope for the fall is

$$\frac{-7.0 \text{ m/s} - 0}{(2.5 \text{ s}) - (1.0 \text{ s})} = -4.7 \text{ m/s}^2,$$

and (b) the slope for the impact is

$$\frac{0 - (-7.0 \text{ m/s})}{(4.0 \text{ s}) - (3.0 \text{ s})} = +7.0 \text{ m/s}^2.$$

109. (a) 17 kn

(b) The conversion can be found on many hand-held calculators, on the Web, or by converting units:

$$17 \text{ kn} = \frac{(17 \text{ knots passing})(47.25 \text{ ft between knots})(1.894\times10^{-4} \text{ mi/ft})(3600 \text{ s/h})}{28 \text{ s}}$$
$$= 20 \text{ mi/h}.$$

(c) Similarly,

$$17 \text{ kn} = \frac{(17 \text{ knots passing})(47.25 \text{ ft between knots})(3.048\times10^{-4} \text{ km/ft})(3600 \text{ s/h})}{28 \text{ s}}$$
$$= 31 \text{ km/h}.$$