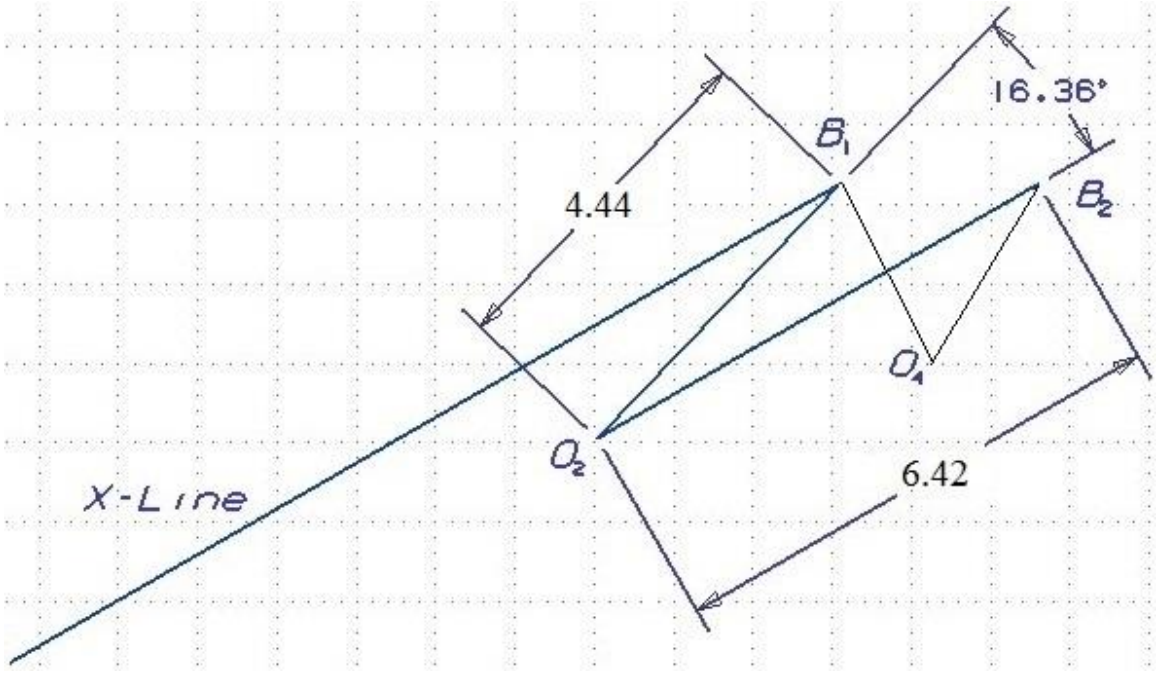
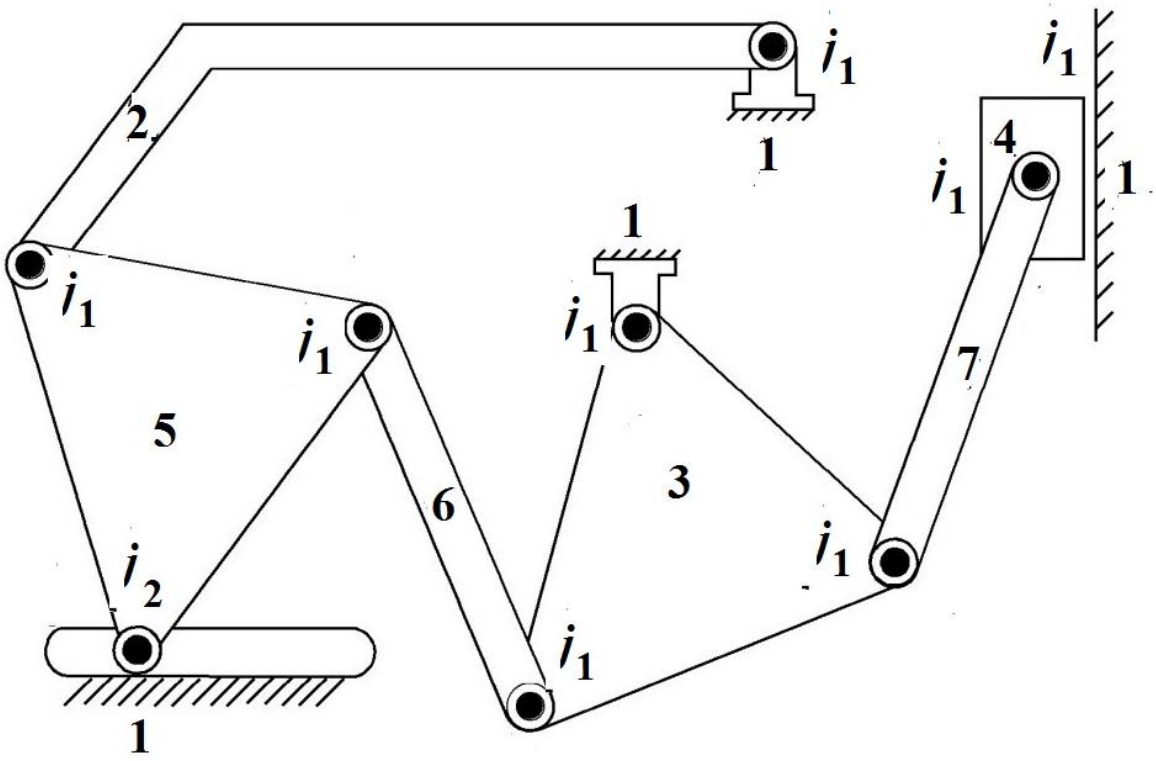
1.20 A crank-rocker four-bar linkage is required to have an advance-to-return ratio Q = 1.2. The rocker is to have a length of 2.5 in and oscillate through a total angle of 60°. Determine a suitable set of link lengths for the remaining three links of the four-bar linkage.

Following the procedure of Example 1.4, the required advance-to-return ratio gives $Q=(180°+\phi)/(180°-\phi)=1.2$ and, therefore, we must have $\phi=16.36°$. Then, with the X-line chosen at 30°, the drawing shown below gives measured distances of $R_{O_4O_2}=r_1=4.34$ in, $R_{B_2O_2}=r_3+r_2=6.42$ in, and $R_{B_1O_2}=r_3-r_2=4.44$ in. From these we get one possible solution, which has link lengths of $R_{O_4O_2}=r_1=4.34$ in, $R_{AO_2}=r_2=0.99$ in, $R_{BA}=r_3=5.43$ in, and $R_{BO_4}=r_4=2.50$ in. <u>*Ans.*</u>



Distances shown are in inches

1.21 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$$n = 7,\ j_1 = 8, \text{ and } j_2 = 1.$$

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

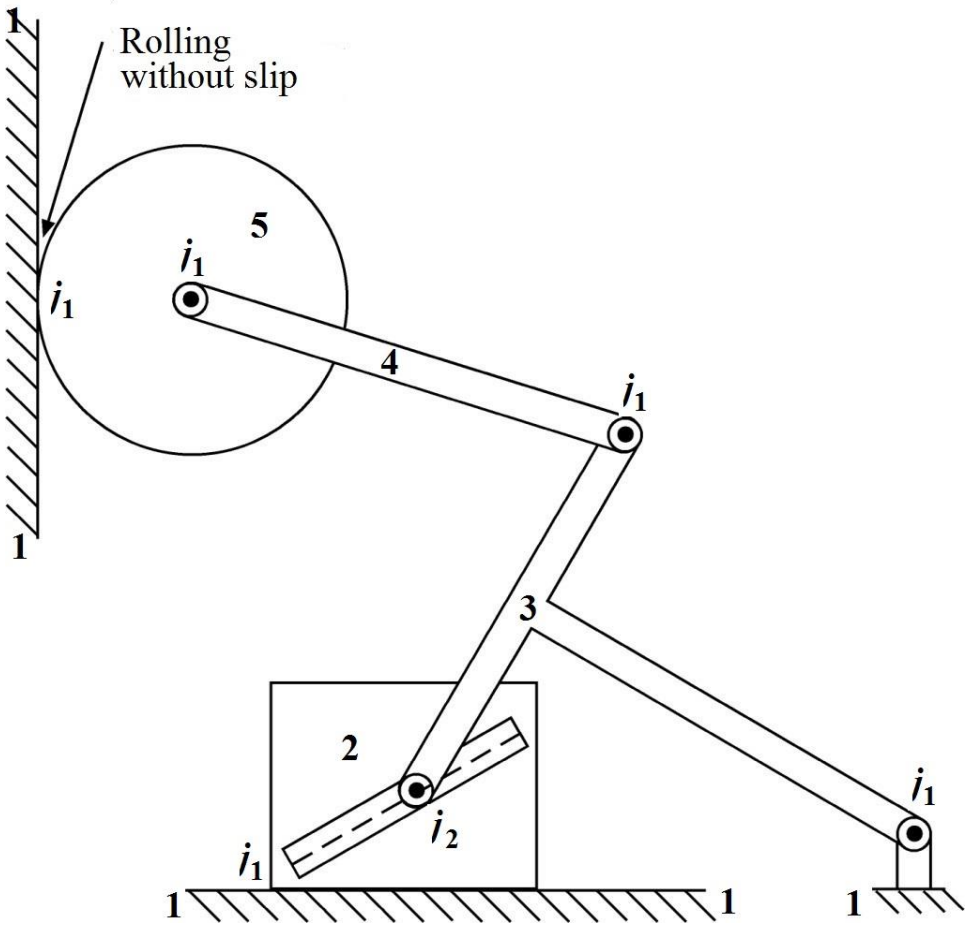
$$m = 3(7-1) - 2(8) - 1(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 3 would be suitable as inputs since these are pinned to the ground link. Translation of the slider (link 4) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.22 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 5 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 5$, $j_1 = 5$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

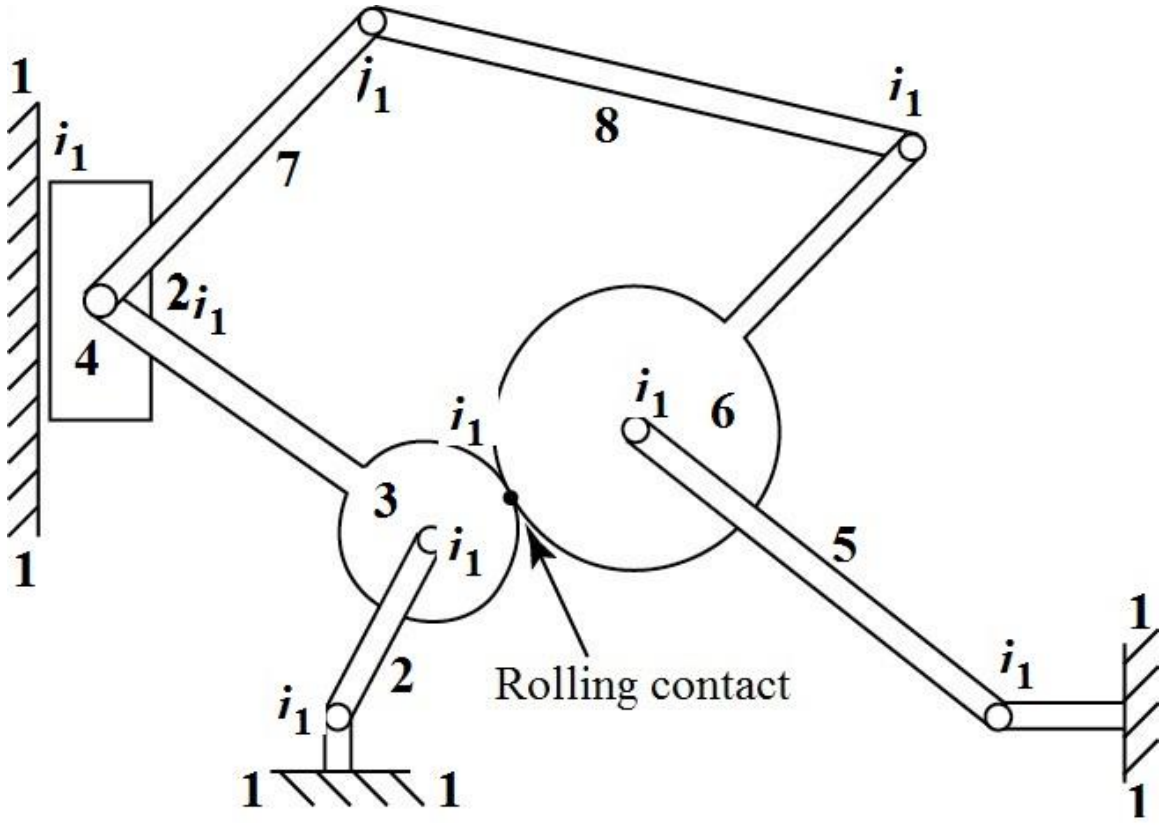
$$m = 3(5-1) - 2(5) - 1(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism, that is, for a single input value there is a unique posture.

Rotation of link 3 would be a suitable input since it is pinned to the ground link. Translation of the slider (link 2) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.23 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 5 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 8$, $j_1 = 10$, and $j_2 = 0$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

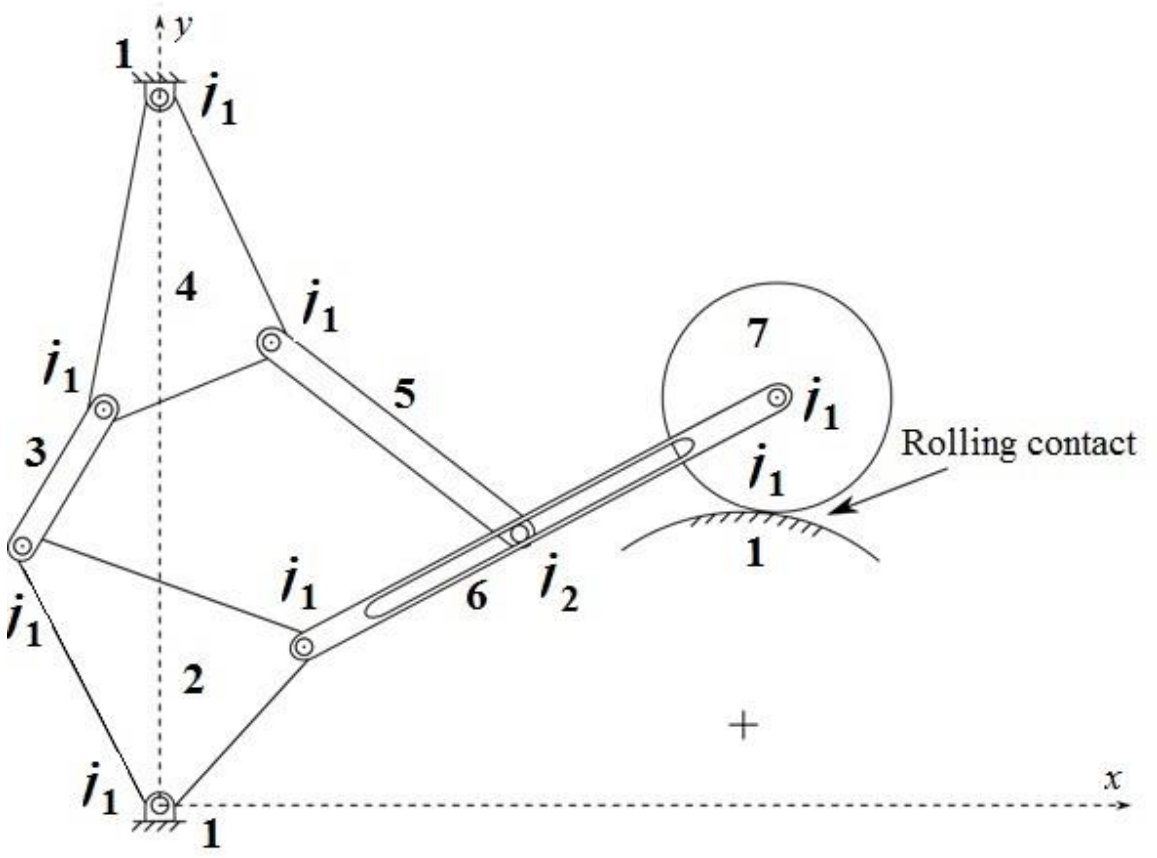
$$m = 3(8-1) - 2(10) - 1(0) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 5 would be suitable inputs since they are pinned to the ground link. Translation of the slider (link 4) would also be a suitable input. Other choices for the input are not particularly practical. *Ans.*

1.24 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 7$, $j_1 = 8$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

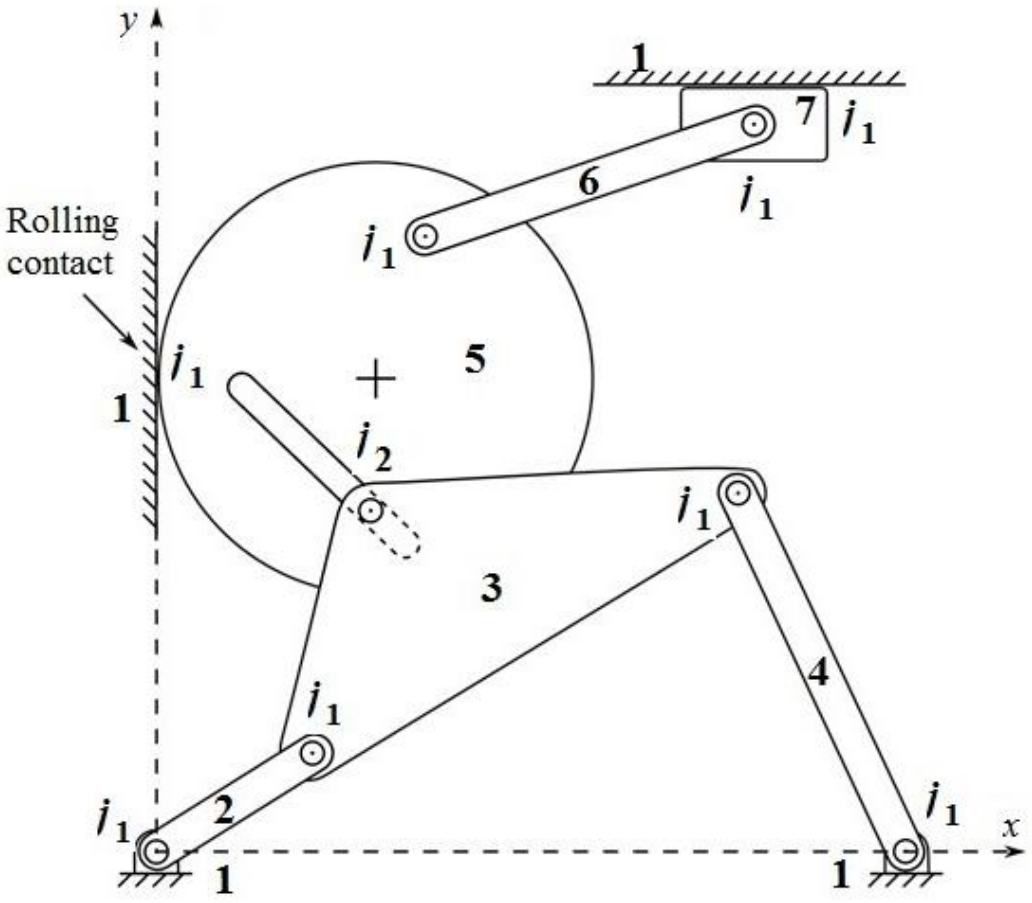
$$m = 3(7-1) - 2(8) - 1(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.25 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 7$, $j_1 = 8$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

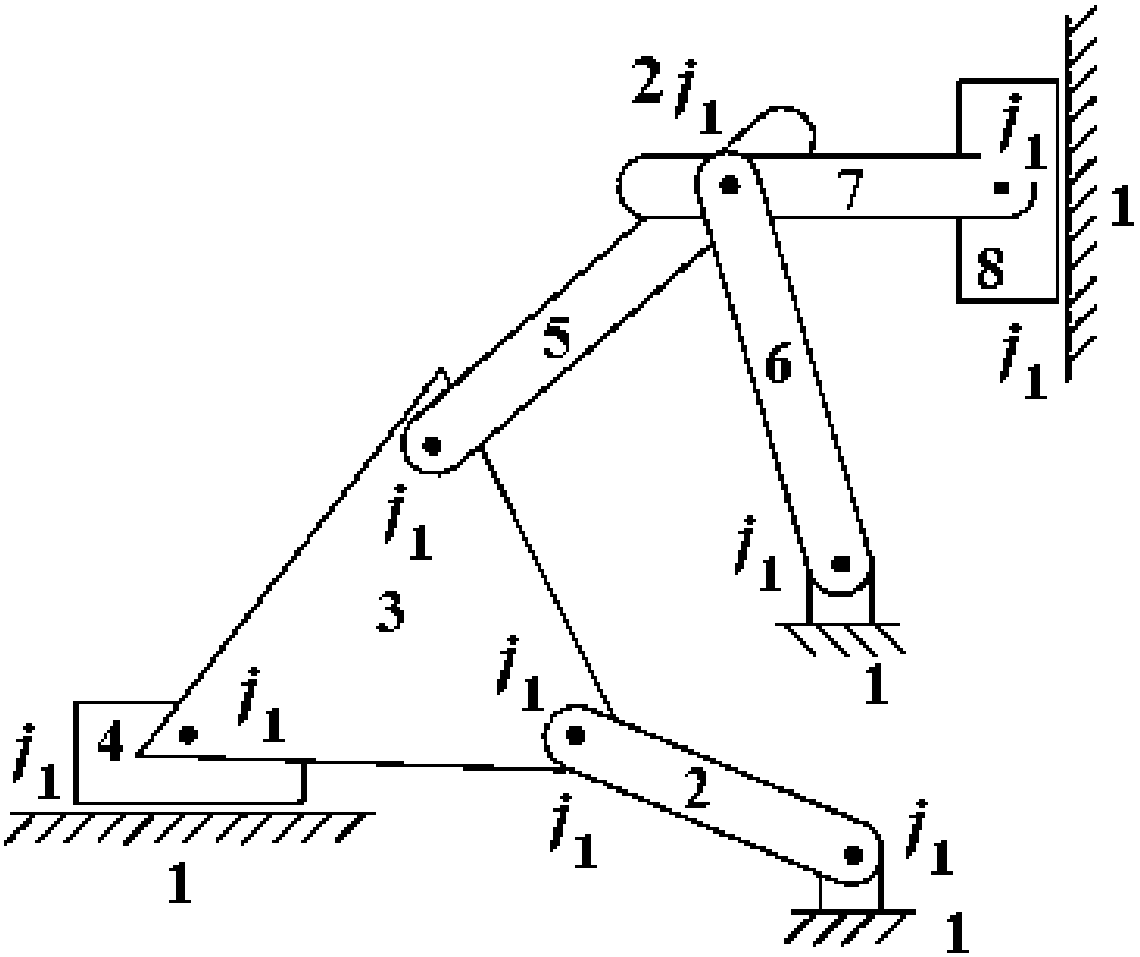
$$m = 3(7-1) - 2(8) - 1(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Translation of the slider (link 7) would also be suitable as an input. Other choices for the input are not particularly practical. *Ans.*

1.26 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 8 links and the joint types are illustrated in the figure below. *Ans*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 8$, $j_1 = 10$, and $j_2 = 0$.

Note that the double-pin joint between links 5, 6, and 7 is counted as 2 j_1 joints.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(8-1) - 2(10) - 1(0) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

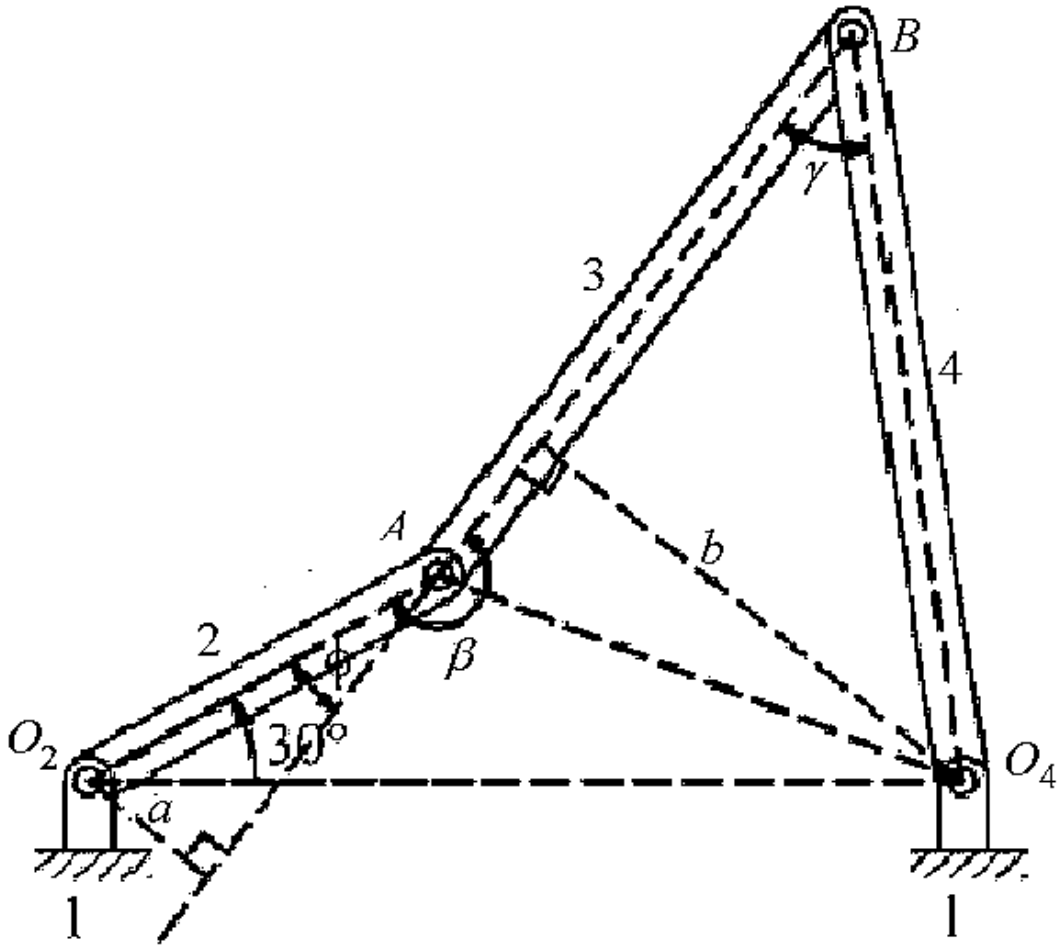
Rotation of either link 2 or link 6 would be suitable inputs since they are pinned to the ground link. Translation of either of the sliders (link 4 or link 8) would also be suitable as inputs. Other choices for the input are not particularly practical. *Ans.*

1.27 Determine the mechanical advantage of the four-bar linkage in the posture illustrated.

The mechanical advantage, Eq. (3.41), can be written as

$$MA = \frac{b}{a} = \frac{R_{BO_4}\sin\gamma}{R_{AO_2}\sin\phi} = -\frac{R_{BO_4}\sin\gamma}{R_{AO_2}\sin\beta} \tag{1}$$

where the angles β, γ, and ϕ are as shown in Fig. P1.27.



$O_2O_4 = 120$ mm, $O_2A = 60$ mm, $AB = 100$ mm, and $O_4B = 130$ mm.

.

To determine the angles β, ϕ, and γ, the law of cosines for the triangle O_2AO_4 can be written as

$$\overline{AO_4}^2 = R_{O_4O_2}^2 + R_{AO_2}^2 - 2R_{O_4O_2}R_{AO_2}\cos(\measuredangle O_4O_2A)$$

$$= (120\text{ mm})^2 + (60\text{ mm})^2 - 2(120\text{ mm})(60\text{ mm})\cos 30° = 5529.234\ \mu\text{m}^2$$

Therefore,

$$\overline{AO_4} = 74.359\text{ mm}$$

The law of sines for the triangle O_2AO_4 can be written as

$$\frac{\overline{AO_4}}{\sin(\measuredangle AO_2O_4)} = \frac{R_{O_2O_4}}{\sin(\measuredangle O_2AO_4)}$$

Rearranging this equation gives

$$\sin(\measuredangle O_2AO_4) = \frac{R_{O_2O_4}}{AO_4}\sin(\measuredangle AO_2O_4) = \frac{120\text{ mm}}{74.359\text{ mm}}\sin 30° = 0.8069$$

Therefore, the angle is either

$\measuredangle O_2AO_4 = 53.79°$ or $\measuredangle O_2AO_4 = 126.21°$

Note that $\measuredangle O_2AO_4 = 53.79°$ can be eliminated since it is not a physically possible result for the open configuration of the four-bar linkage. Therefore, the correct result for the angle is

$$\measuredangle O_2AO_4 = 126.21° \tag{2}$$

The law of cosines for the triangle ABO_4 can be written as

$$R_{BO_4}^2 = R_{BA}^2 + \overline{AO_4}^2 - 2R_{BA}\overline{AO_4}\cos(\measuredangle O_4AB)$$

Rearanging this equation gives

$$\cos(\measuredangle O_4AB) = \frac{(100\text{ mm})^2 + (74.359\text{ mm})^2 - (130\text{ mm})^2}{2(100\text{ mm})(74.359\text{ mm})} = -0.092\,17$$

Therefore, the angle is either

$\measuredangle O_4AB = 95.29°$ or $\measuredangle O_4AB = -84.71°$

Note that the value $\measuredangle O_4AB = -84.71°$ can be eliminated since it is not physically possible for the open configuration of the four-bar linkage. Therefore, the correct result is

$$\measuredangle O_4AB = 95.29° \tag{3}$$

The angle

$$\beta = \measuredangle O_2AO_4 + \measuredangle O_4AB \tag{4}$$

Substituting Eqs. (2) and (3) into Eq. (4) gives

$$\beta = 126.21° + 95.29° = 221.50° \tag{5}$$

The law of cosines for the triangle ABO_4 can be written as

$$\overline{AO_4}^2 = R_{BA}^2 + R_{BO_4}^2 - 2R_{BA}R_{BO_4}\cos\gamma$$

Rearanging this equation gives

$$\cos\gamma = \frac{(100\text{ mm})^2 + (130\text{ mm})^2 - (74.359\text{ mm})^2}{2(100\text{ mm})(130\text{ mm})} = 0.821\,95$$

Therefore, the transmission angle is

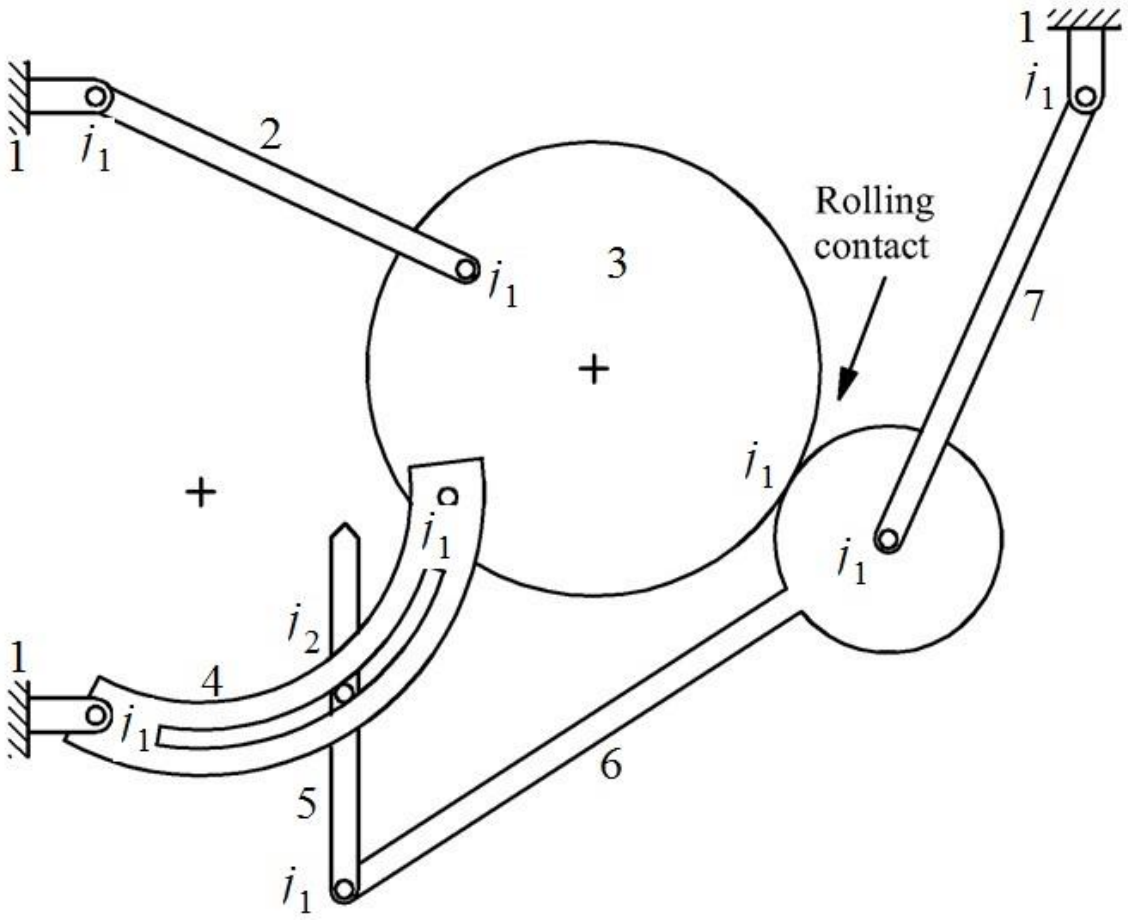
$$\gamma = 34.72° \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (1), the mechanical advantage is

$$MA = -\frac{(130\text{ mm})\sin 34.72^o}{(60\text{ mm})\sin 221.50^o} = -\frac{74.044\text{ mm}}{-39.757\text{ mm}} = 1.86 \qquad \underline{\textit{Ans.}}$$

1.28 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are indicated in the figure below *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$$n = 7,\ j_1 = 8, \text{ and } j_2 = 1.$$

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

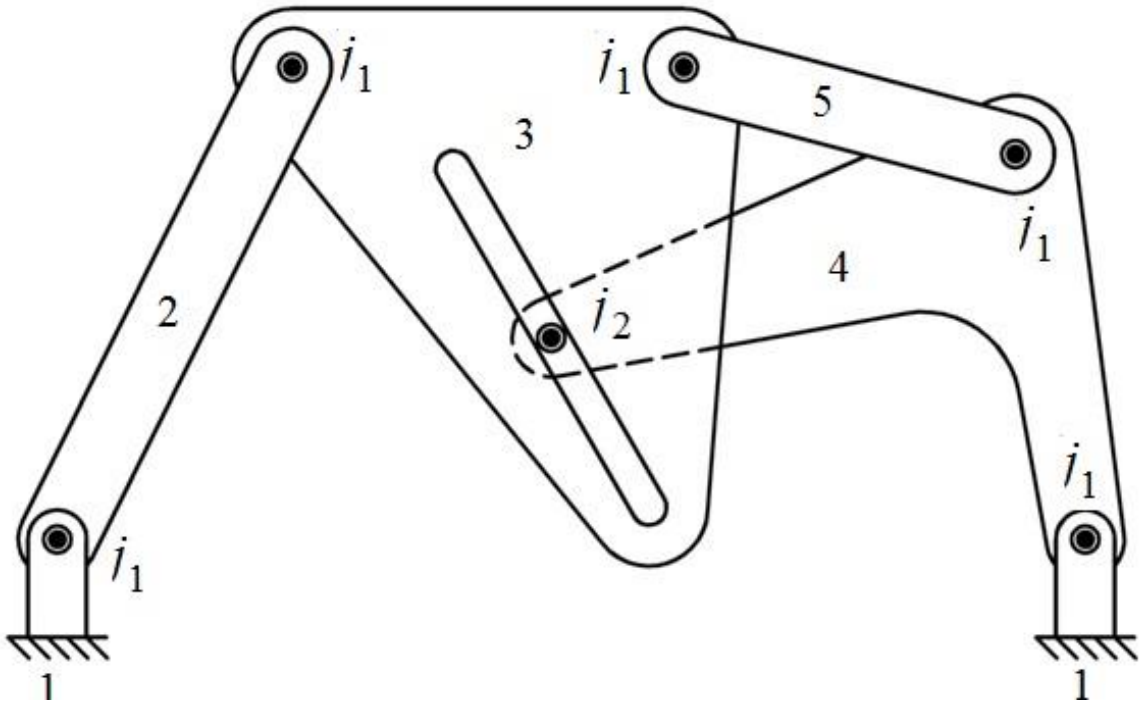
$$m = 3(7-1) - 2(8) - 1(1) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2, link 4, or link 7 would be suitable inputs since they are each pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.29 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 7 links and the joint types are illustrated in the figure below. *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$n = 5$, $j_1 = 5$, and $j_2 = 1$.

Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(5-1) - 2(5) - 1(1) = 1$$ *Ans.*

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

Rotation of either link 2 or link 4 would be suitable inputs since they are each pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.30 The rocker of a crank-rocker four-bar linkage is required to have a length of 6 in and swing through a total angle of 30°. Also, the advance-to-return ratio of the linkage is required to be 1.75. Determine a suitable set of link lengths for the remaining three links.

The advance-to-return-ratio must be

$$Q = \alpha/\beta = 1.75 \tag{1}$$

where

$$\alpha = 180^{\circ} + \phi \qquad \text{and} \qquad \beta = 180^{\circ} - \phi \tag{2}$$

Substituting Eqs. (2) into Eq. (1) gives

$$180° + \phi = 1.75(180° - \phi)$$

Therefore,

$$\phi = 49.09°$$

$$\beta = 130.91^{\circ} \qquad \text{and} \qquad \alpha = 229.09°$$

Note that the dimensions of the synthesized four-bar linkage to satisfy the given design constraints are not unique. However, the graphic procedure follows the steps shown in Example 1.4. (For this problem, refer to the figure below):

(1) Draw the rocker $r_4 = 6$ in of the crank-rocker four-bar linkage to a suitable scale, for example, full scale. Draw the rocker in the two extreme positions, that is, show the swing angle of the rocker of 30 degrees. Label the ground pivot O_4 and label the pin B in the two positions B_1 and B_2.

(2) Through point B_1 draw an arbitrary line (labeled the X-line). Through B_2 draw a line parallel to the X-line.

(3) Lay out the angle $\phi = 49.09^{\circ}$ counterclockwise from the X-line through point B_1. The intersection of this line with the line parallel to the X-line through point B_2 is the input crank pivot O_2.

(4)*i* The length O_2O_4 of the ground link can be measured from the drawing. That is

$$r_1 = O_2O_4 = 8.94 \text{ in}$$ *Ans.*

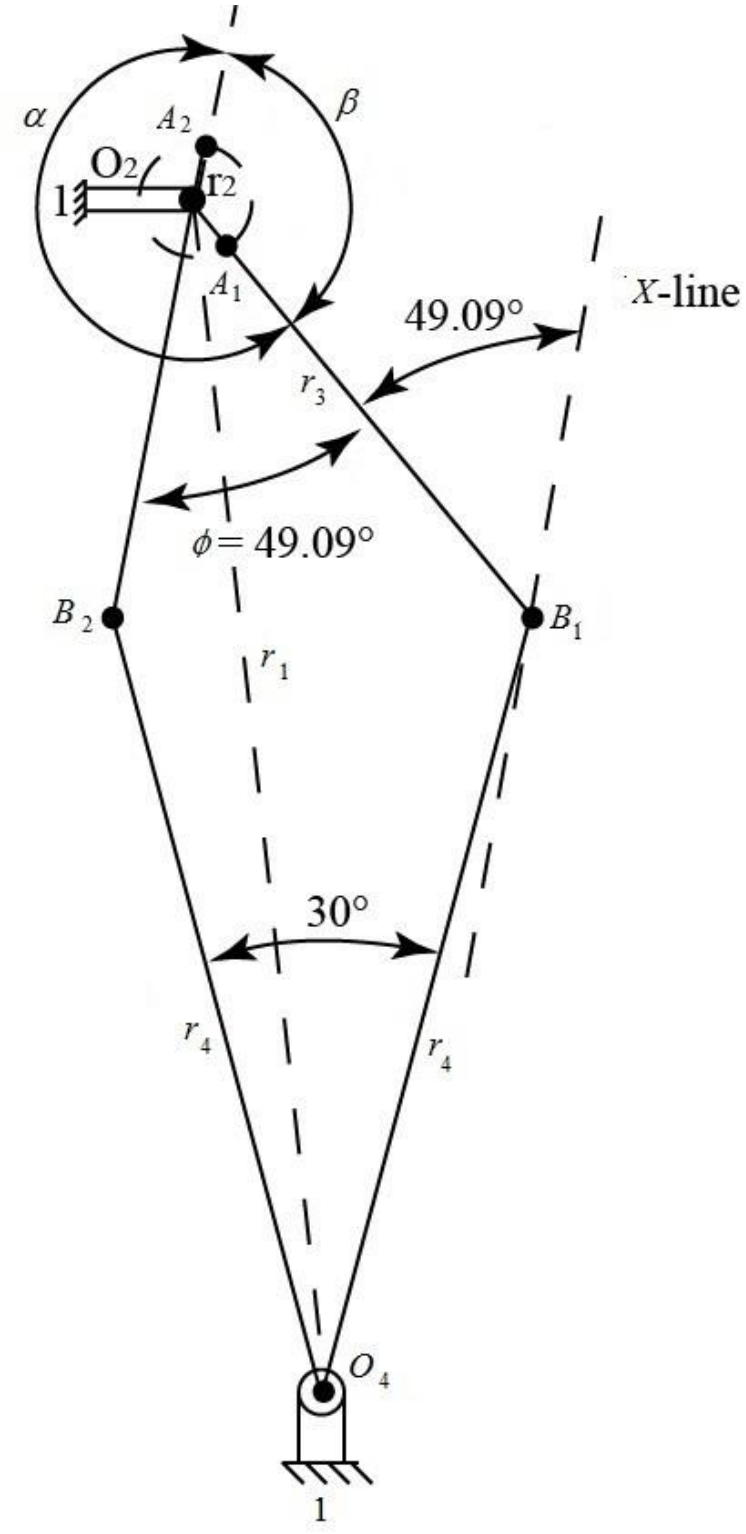
The other measurements are

$O_2B_1 = r_3 + r_2 = 4.02$ in and $O_2B_2 = r_3 - r_2 = 3.18$ in

Therefore, the length of the input link and the length of the coupler link, respectively, must be

$r_2 = O_2A = 0.42$ in and $r_3 = AB = 3.60$ in *Ans.*

This solution of the synthesized four-bar linkage is shown in the figure below.



The synthesized four-bar linkage.

1.31 Determine a suitable set of link lengths for a slider-crank linkage such that the stroke will be 500 mm and the advance-to-return ratio will be 1.8.

The advance-to-return ratio must be

$$Q = \alpha/\beta = 1.8 \tag{1}$$

where

$$\alpha = 180^{\circ} + \phi \qquad \text{and} \qquad \beta = 180^{\circ} - \phi \tag{2}$$

Substituting Eqs. (2) into Eq. (1) gives

$$180° + \phi = 1.8(180° - \phi)$$

Therefore,

$$\phi = 51.43°$$

$$\beta = 128.57^{\circ} \qquad \text{and} \qquad \alpha = 231.43°$$

Note that the dimensions of the synthesized slider-crank linkage to satisfy the given design constraints are not unique. However, the graphic procedure follows the steps shown in Example 1.5. (For this problem, refer to the figure below):

(1) Draw the stroke of the slider-crank linkage to a suitable scale, for example, 1 in ~ 100 mm. The length of the stroke of this mechanism is specified as $r_4 = 500$ mm. Label the pin *B* in its two extreme positions as B_1 and B_2.

(2) Through point B_2 draw an arbitrary line (labeled the *X*-line). Through point B_1 draw a line parallel to the *X*-line.

(3) Lay out the angle $\phi = 51.43^{\circ}$ clockwise from the *X*-line. The intersection of this line with the line parallel to the *X*-line through point B_1 is the ground pivot O_2.

(4) From the scale drawing below, the ground link; i.e., the offset (the vertical distance), is measured as

$$r_1 = 102.5 \text{ mm} \qquad \underline{\textit{Ans.}}$$

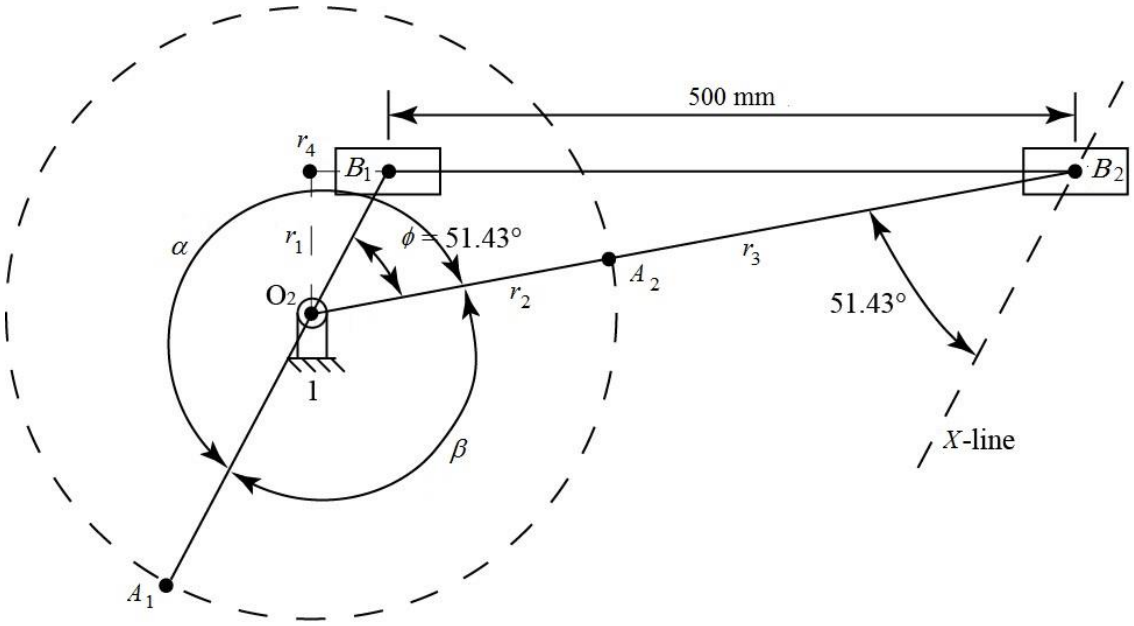
The other measurements are

$O_2B_2 = r_3 + r_2 = 562.5$ mm and $O_2B_1 = r_3 - r_2 = 118.5$ mm

Therefore, the length of the input link and the length of the coupler link, respectively, are

$r_2 = O_2A = 222$ mm and $r_3 = AB = 340.5$ mm $\qquad \underline{\textit{Ans.}}$

The solution of the synthesized slider-crank linkage is shown in the figure below.



The synthesized slider-crank linkage.

1.32 Determine the transmission angle and the mechanical advantage of the four-bar linkage in the posture illustrated. What type of four-bar linkage is this?

Grashof's law for a planar four-bar linkage, Sec. 1.9, Eq. (1.6), states that in order for the four-bar linkage to be a Grashof chain, the dimensions must satisfy

$$s + l \le p + q \tag{1}$$

The lengths of the four links of the four-bar linkage are

$s = 20$ mm, $l = 90$ mm, $p = 60$ mm, and $q = 70$ mm..

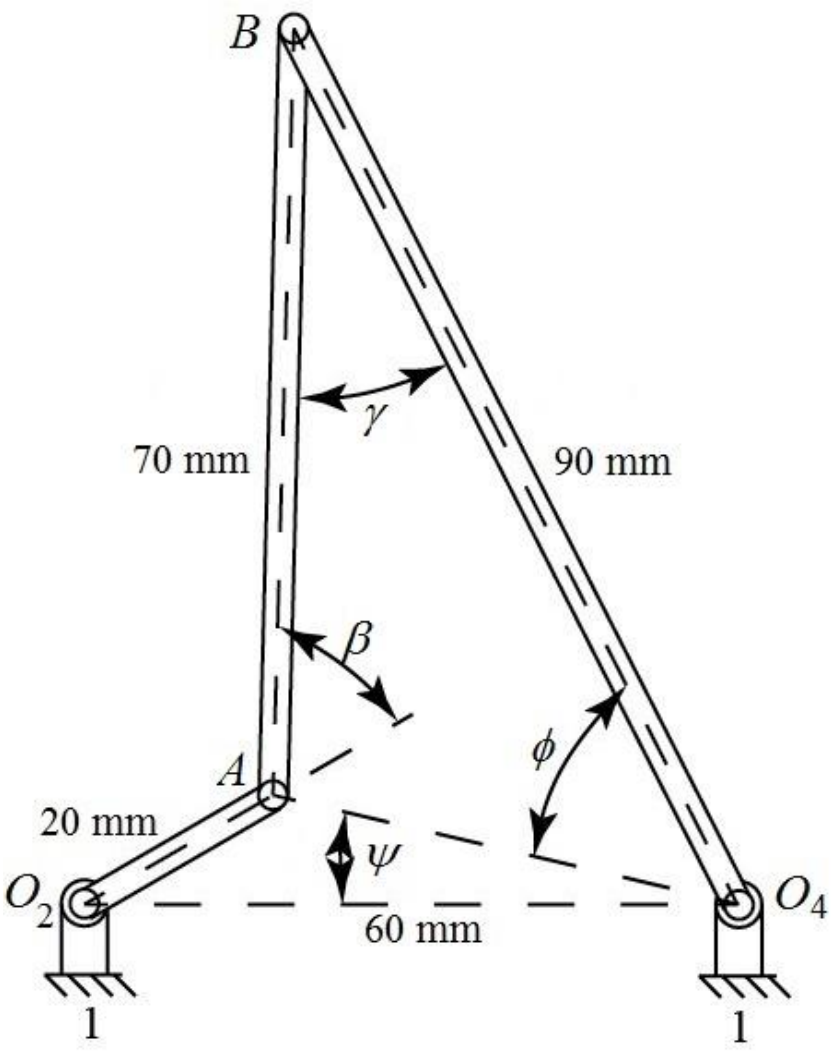
Substituting these dimensions into Eq. (1) gives

$$20 \text{ mm} + 90 \text{ mm} \le 60 \text{ mm} + 70 \text{ mm}$$

or

$$110 \text{ mm} \le 130 \text{ mm}$$

Since this inequality is satisfied, the four-bar linkage is a Grashof chain. Also, since the shortest link *s* is adjacent to the ground link, the shortest link is a crank; see Sec. 1.9. Therefore, this is a crank-rocker four-bar linkage. *Ans.*



$r_2 = 20$ mm, $r_3 = 70$ mm, $r_4 = 90$ mm, and $r_1 = 60$ mm.

In the posture illustrated, the distance between points *A* and O_4 can be found from the triangle AO_2O_4. The law of cosines can be written as

$$\overline{AO_4}^2 = r_2^2 + r_1^2 - 2r_2 r_1 \cos\theta_2$$

Substituting the given dimensions gives

$$\overline{AO_4}^2 = (20\text{ mm})^2 + (60\text{ mm})^2 - 2(20\text{ mm})(60\text{ mm})\cos 30^\circ = 1\,921.54\,(\text{mm})^2$$

Therefore, $\overline{AO_4} = 43.84\text{ mm}$.

To determine the transmission angle γ consider the triangle ABO_4. The law of cosines can be written as

$$\overline{AO_4}^2 = r_3^2 + r_4^2 - 2r_3 r_4 \cos\gamma$$

Substituting the given dimensions gives

$$1921.54\,(\text{mm})^2 = (70\text{ mm})^2 + (90\text{ mm})^2 - 2(70\text{ mm})(90\text{ mm})\cos\gamma$$

This equation gives $\cos\gamma = 0.879\,24$

Therefore, the transmission angle is $\gamma = 28.45^\circ$ <u>*Ans.*</u> (2)

The angle ψ can be found from the triangle O_2O_4A. From the law of sines,

$$\psi = \sin^{-1}\left[\frac{(20\text{ mm})\sin 30^\circ}{43.84\text{ mm}}\right] = 13.19^\circ$$

To determine the angle ϕ, consider the triangle BO_4A. From the law of sines, this angle is

$$\phi = \sin^{-1}\left[\frac{(70\text{ mm})\sin 28.45^\circ}{43.84\text{ mm}}\right] = 49.52^\circ$$

The angle between link 3 (link AB) and the ground link O_2O_4 can be written as

$$\theta_3 = 180^o - \gamma - \phi - \psi = 180^o - 28.45^o - 49.52^o - 13.19^o = 88.84^o$$

The angle β can be written as

$$\beta = \theta_3 - 30^\circ = 88.84^\circ - 30^\circ = 58.84^\circ \quad (3)$$

The mechanical advantage of a four-bar linkage, Sec.1.10, can be written as

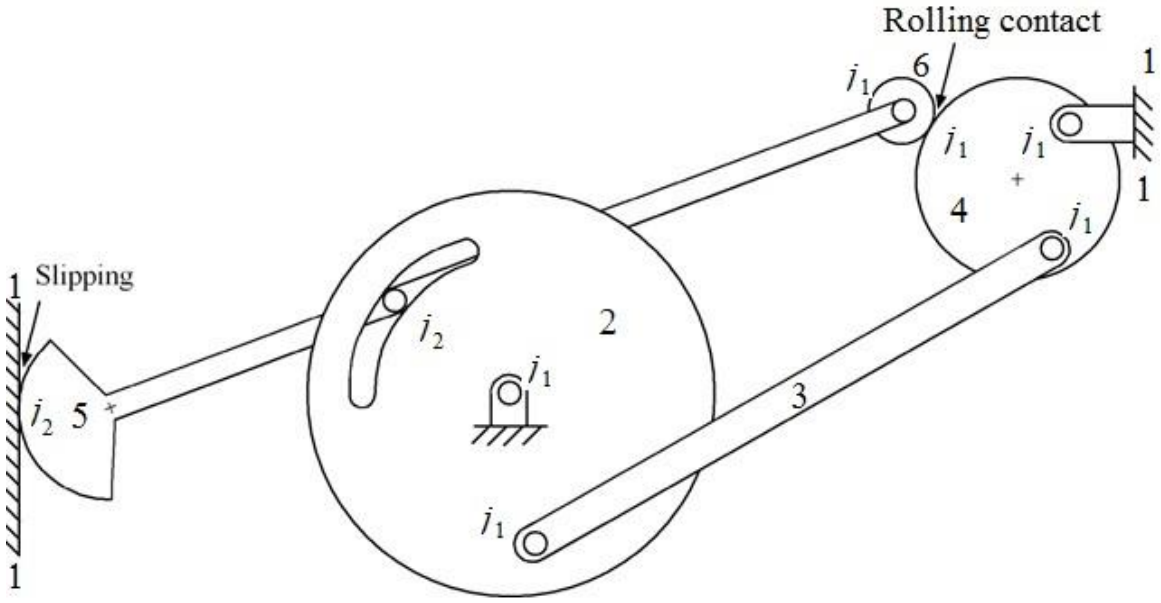
$$MA = \frac{r_4 \sin\gamma}{r_2 \sin\beta}$$

Substituting Eqs. (2) and (3), and the given link lengths, the mechanical advantage of the four-bar linkage (in the given posture) is

$$MA = \frac{(90\text{ mm})\sin 28.45^o}{(20\text{ mm})\sin 58.84^o} = 2.51 \quad \text{<u>*Ans.*</u>}$$

1.33 Determine the mobility of the mechanism. Number each link and label the lower pairs and the higher pairs. Identify a suitable input, or inputs, for the mechanism.

The mechanism has 6 links and the joint types are illustrated in the following figure *Ans.*



The link numbers and joint types of the mechanism. *Ans.*

The number of links, lower pairs, and higher pairs, respectively, are

$$n = 6,\ j_1 = 6, \text{ and } j_2 = 2.$$

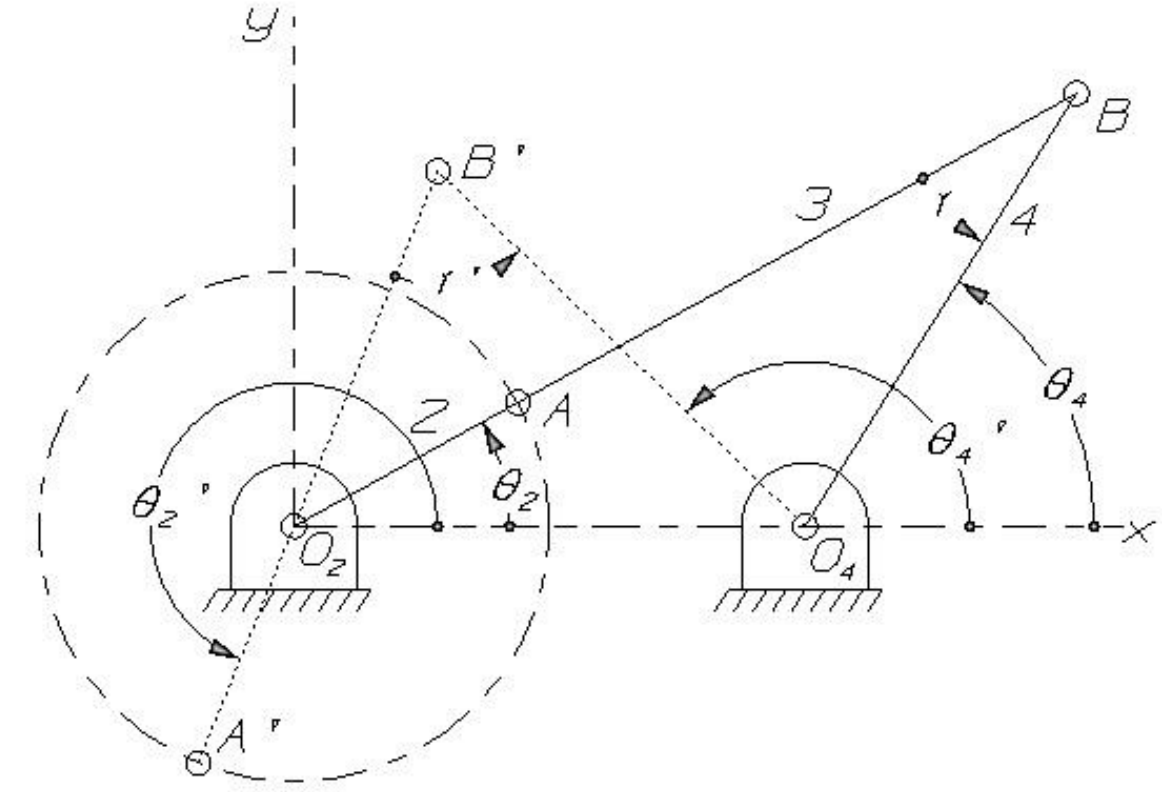
Substituting these values into the Kutzbach mobility criterion, Eq. (1.1), the mobility of the mechanism is

$$m = 3(6-1) - 2(6) - 1(2) = 1 \qquad \textit{Ans.}$$

This is the correct answer for this mechanism; that is, for a single input value there is a unique posture.

The rotation of either link 2 or link 4 would be suitable inputs since they are pinned to the ground link. Other choices for the input are not particularly practical. *Ans.*

1.34 A crank-rocker four-bar linkage is illustrated in one of its two toggle postures. Find θ_2 and θ_4 corresponding to each toggle posture. What is the total rocking angle of link 4? What are the transmission angles at the extremes?



$r_2 = 8$ in, $r_3 = 20$ in, and $r_4 = r_1 = 16$ in.

(*a*) From isosceles triangle O_4O_2B we can calculate

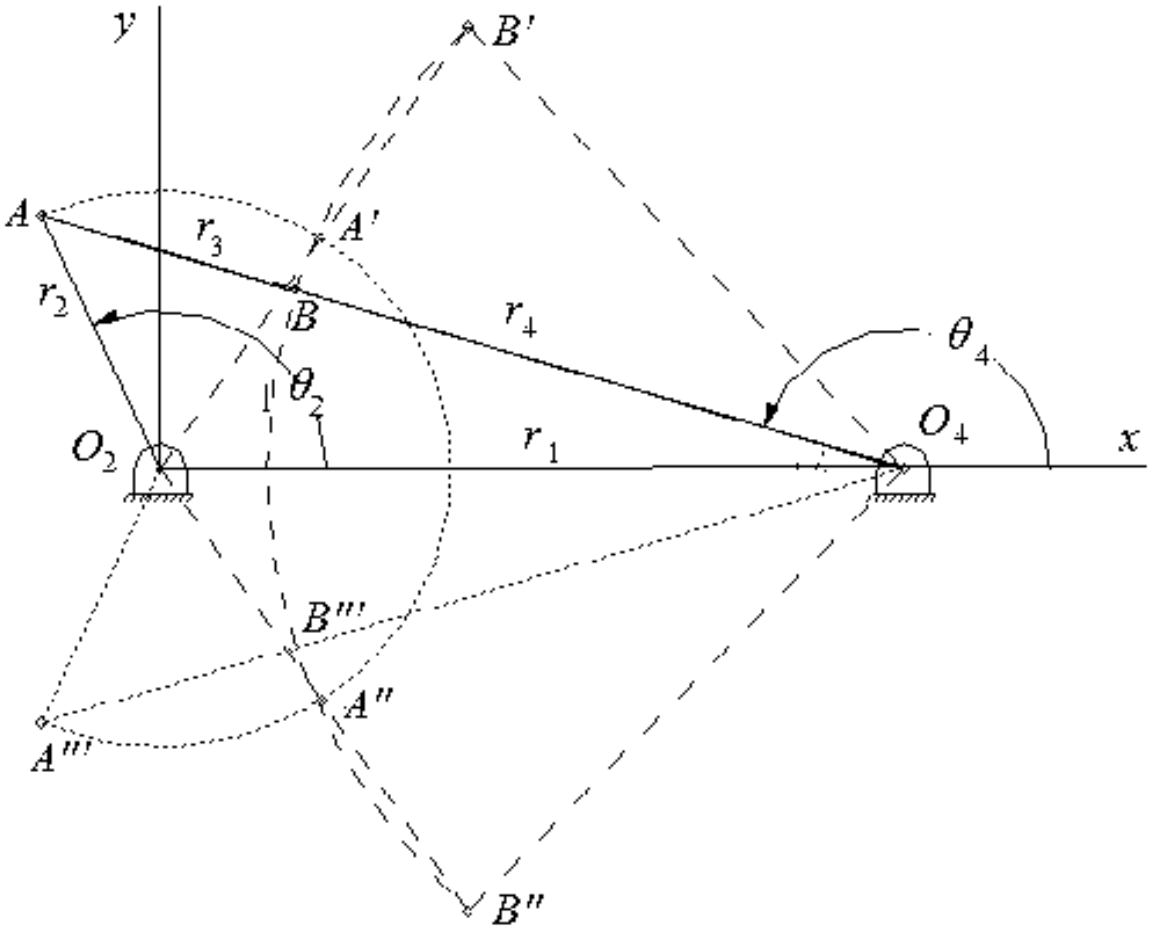
$$\theta_2 = \cos^{-1}\frac{(r_2 + r_3)/2}{r_1} = \cos^{-1}\frac{8\text{ in} + 20\text{ in}}{2(16\text{ in})} = 28.955^\circ,\ \theta_4 = 57.910^\circ, \qquad \underline{\textit{Ans.}}$$

$$\theta_2' = \cos^{-1}\frac{r_3 - r_2}{2r_1} = \cos^{-1}\frac{20\text{ in} - 8\text{ in}}{2(16\text{ in})} = 247.976^\circ,\ \theta_4' = 135.951^\circ\ . \qquad \underline{\textit{Ans.}}$$

(*b*) Then $\Delta\theta_4 = \theta_4' - \theta_4 = 78.041^\circ$ <u>*Ans.*</u>

(*c*) Finally, from isosceles triangle O_2BO_4, $\gamma = 28.955^\circ$ and $\gamma' = 67.976^\circ$. <u>*Ans.*</u>

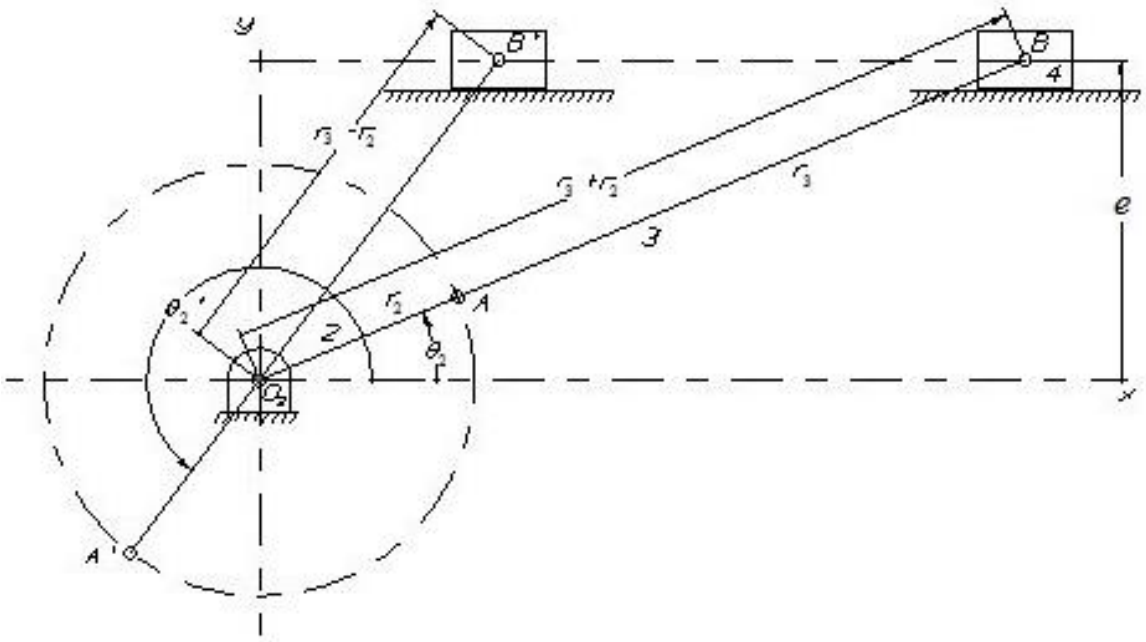
1.35 Find θ_2 and θ_4 corresponding to a dead-center posture. Is there a toggle posture?



$r_2 = 110$ mm, $r_3 = 100$ mm, $r_4 = 240$ mm, and $r_1 = 280$ mm.

For the given dimensions, there are two dead-center postures, and they correspond to the two extreme travel postures of crank O_2A. From ΔO_4AO_2 using the law of cosines, we can find $\theta_2 = 114.05°$, $\theta_4 = 162.82°$ and, symmetrically, $\theta_2''' = -114.05°$, $\theta_4''' = -162.82°$. There are also two toggle postures; these occur at $\theta_2' = 56.50°$, $\theta_4' = 133.14°$ and, symmetrically, at $\theta_2'' = -56.50°$, $\theta_4'' = -133.14°$. *Ans.*

1.36 Determine the advance-to-return ratio for the slider-crank linkage with the offset *e*. Also, determine in which direction the crank should rotate to provide quick return.



An offset slider-crank linkage in the two dead-center postures.

From the figure we can see that $e = (r_3 + r_2)\sin\theta_2 = (r_3 - r_2)\sin(\theta_2' - 180°)$ or

$$\theta_2 = \sin^{-1}\left(\frac{e}{r_3 + r_2}\right), \qquad \theta_2' = 180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)$$

$$\Delta\theta_{drive} = \theta_2' - \theta_2 = 180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 + r_2}\right)$$

$$\Delta\theta_{return} = \theta_2 + 360° - \theta_2' = 180° + \sin^{-1}\left(\frac{e}{r_3 + r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)$$

The advance-to-return ratio is

$$Q = \frac{180° + \sin^{-1}\left(\frac{e}{r_3 - r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 + r_2}\right)}{180° + \sin^{-1}\left(\frac{e}{r_3 + r_2}\right) - \sin^{-1}\left(\frac{e}{r_3 - r_2}\right)} \qquad \underline{Ans.}$$

Assuming driving is when *B* is sliding to the right, the crank should rotate clockwise. <u>*Ans.*</u>