CHAPTER 1

1.1

(a) From (1.1) the built-in potential is

$$\psi_o = V_T \ln \frac{N_A N_D}{n_i^2}$$

$$= 26 \ln \frac{8\times10^{15}\times10^{17}}{2.25\times10^{20}} = 751 \text{ mV}$$

From (1.14) the depletion-layer depth in the p-type region is,

$$W_1 = \left[\frac{2\times1.04\times10^{-12}\times5.75}{1.6\times10^{-19}\times8\times10^{15}(1+0.08)}\right]^{1/2}$$

$$= 0.93 \ \mu m$$

In the n-type region using (1.15)

$$W_2 = \left[\frac{2\times1.04\times10^{-12}\times5.75}{1.6\times10^{-19}\times10^{17}\times13.5}\right]^{1/2}$$

$$= 0.074 \ \mu m$$

From (1.7) the maximum field is

$$\mathcal{E}_{MAX} = -\frac{q N_A}{\epsilon} W_1$$

$$= -1.6\times10^{-19}\times\frac{8\times10^{15}\times0.93\times10^{-4}}{1.04\times10^{-12}}$$

$$= -11.4\times10^{4} \text{ V/cm}$$

(b) For zero volts bias,

$$\psi_o + V_R = \psi_o = 0.75$$

$$W_1 = 0.93\sqrt{\frac{0.75}{5.75}} = 0.34 \ \mu m$$

$$W_2 = 0.074\sqrt{\frac{0.75}{5.75}} = 0.027 \ \mu m$$

$$\mathcal{E}_{MAX} = -11.4\times10^{4}\times\frac{0.34}{0.93}$$

$$= -4.17\times10^{4} \text{ V/cm}$$

For 0.3V forward bias,

$$\psi_o + V_R = 0.45 \text{ V}$$

$$W_1 = 0.93\sqrt{\frac{0.45}{5.75}} = 0.26 \ \mu m$$

$$W_2 = 0.074\sqrt{\frac{0.45}{5.75}} = 0.021 \ \mu m$$

$$\mathcal{E}_{MAX} = -11.4\times10^{4}\times\frac{0.26}{0.93}$$

$$= -3.19\times10^{4} \text{ V/cm}$$

1.2

From (1.20), the zero bias junction capacitance is,

$$C_{jo} = A\left[\frac{q\epsilon N_A N_D}{2(N_A+N_D)}\right]^{1/2}\frac{1}{\sqrt{\psi_o}}$$

$$= 2\times10^{-5}\times\left[\frac{1.6\times10^{-19}\times1.04\times10^{-12}\times8\times10^{15}\times10^{17}}{2(8\times10^{15}+10^{17})}\right]^{1/2}\times\frac{1}{\sqrt{0.75}}$$

$$= 0.57 \text{ pF}$$

Using (1.20) with $V_D = -5$V we obtain,

$$C_j = \frac{C_{jo}}{\sqrt{7.7}} = 0.2 \text{ pF}$$

with $V_D = 0.3$ V

$$C_j = \frac{C_{jo}}{\sqrt{0.6}} = 0.74 \text{ pF}$$

1.3

The breakdown voltage can be calculated from (1.24) using

$$|\mathcal{E}_{MAX}| = \mathcal{E}_{CRIT} = 4\times10^5 \text{ V/cm}$$

Thus,

$$4\times10^5 = \left[\frac{2\times1.6\times10^{-19}\times8\times10^{15}\times10^{17}\times V_R}{1.04\times10^{-12}(8\times10^{15}+10^{17})}\right]^{1/2}$$

$$\therefore\ V_R = 70.2 \text{ V}$$

1.4

Junction curvature causes (1.24) to become

$$|\mathcal{E}_{MAX}| = 1.5\left[\frac{2q N_A N_D V_R}{\varepsilon(N_A+N_D)}\right]^{1/2}$$

If $N_A >> N_D$, $|\mathcal{E}_{MAX}| = 3\times10^5$ V/cm and $V_R = 150$ V, then this equation gives

$$9\times10^{10} = 2.25\ \frac{2q N_D V_R}{\varepsilon}$$

$$\therefore\ N_D = \frac{9\times10^{10}\times1.04\times10^{-12}}{3.2\times10^{-19}\times150\times2.25}$$

$$= 8.7\times10^{14} \text{ atoms/cm}^3$$

1.5

The plane breakdown voltage is

$$BV_{CBO} = \frac{\varepsilon(N_A+N_D)}{2q N_A N_D}\mathcal{E}_{CRIT.}^2$$

$$= \frac{1.04\times10^{-12}}{3.2\times10^{-19}\times6\times10^{15}}\times9\times10^{10} = 48.8\text{ V}$$

From (1.81),

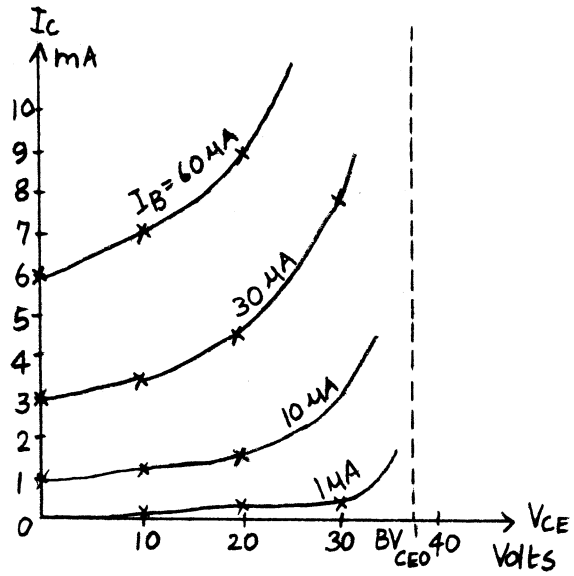
$$BV_{CEO} = \frac{48.8}{\sqrt[4]{200}} = 13\text{ V}$$

1.6

$$BV_{CBO} = \frac{1.04\times10^{-12}}{3.2\times10^{-19}\times10^{15}}\times9\times10^{10} = 293\text{ V}$$

$$BV_{CEO} = \frac{293}{\sqrt[4]{400}} = 65.5\text{ V}$$

1.7 (a)



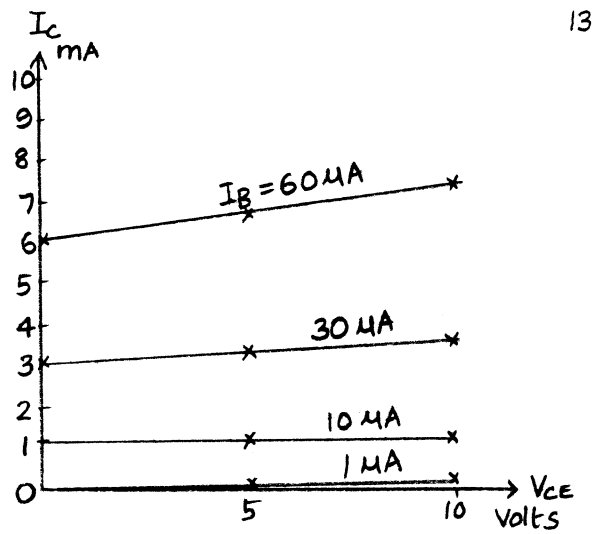
$$I_C = \left(1 + \frac{V_{CE}}{V_A}\right) \frac{M\alpha_F}{1 - M\alpha_F} I_B$$

$$M = \frac{1}{1 - \left(\frac{V_{CB}}{BV_{CBO}}\right)^n}$$

Where $\alpha_F = 0.99$, $V_A = 50\,V$

$BV_{CBO} = 120\,V$, $n = 4$

(b)



1.8

$$g_m = \frac{I_C}{V_T} = \frac{0.2\,mA}{26\,mV} = 7.69\,\frac{mA}{V}$$

$$r_o = \frac{1}{\eta g_m} = \frac{1}{10^{-3}(7.69m)} = 130\,k\Omega$$

$$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{7.69\,m} = 13\,k\Omega$$

$$C_b = \tau_F g_m = (15\,ps)(7.69\,m) = 115\,fF$$

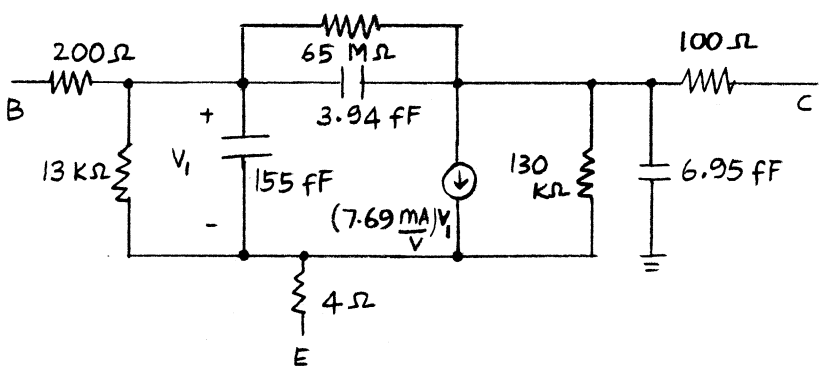
$$r_\mu = 5\beta_o r_o = 5(100)(130\,k) = 65\,M\Omega$$

$$C_{je} \cong 2C_{je0} = 40\,fF$$

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{\psi_o}}} = \frac{10\,fF}{\sqrt{1 + \frac{3}{0.55}}} = 3.94\,fF$$

$$C_{cs} = \frac{C_{cs0}}{\sqrt{1 + \frac{V_{cs}}{\psi_o}}} = \frac{20\,fF}{\sqrt{1 + \frac{4}{0.55}}} = 6.95\,fF$$

$$C_\pi = C_b + C_{je} = 155\,fF$$



1.9

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{26\text{ mV}} = 38.5\ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{1}{\eta\, g_m} = \frac{1}{10^{-3}(38.5\text{m})} = 26\text{ k}\Omega$$

$$r_\pi = \frac{\beta_o}{g_m} = \frac{100}{38.5\text{m}} = 2.6\text{ k}\Omega$$

$$C_b = \tau_F g_m = (15\text{ps})(38.5\text{m}) = 577\text{ fF}$$

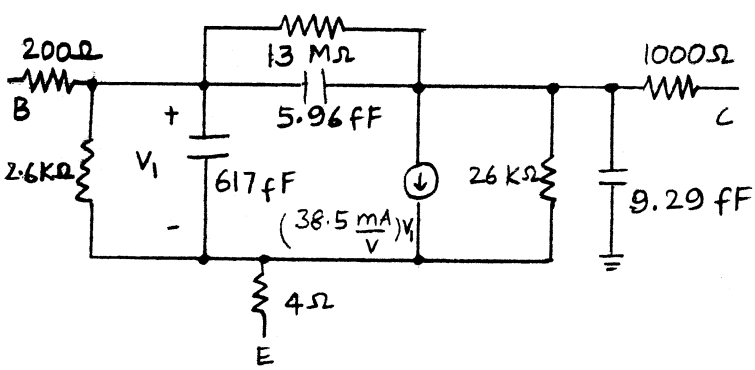
$$r_\mu = 5\beta_o r_o = 5(100)(26\text{ k}) = 13\text{ M}\Omega$$

$$C_{je} \simeq 2C_{je0} = 40\text{ fF}$$

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1+\frac{V_{CB}}{\psi_o}}} = \frac{10\text{ fF}}{\sqrt{1+\frac{1}{0.55}}} = 5.96\text{ fF}$$

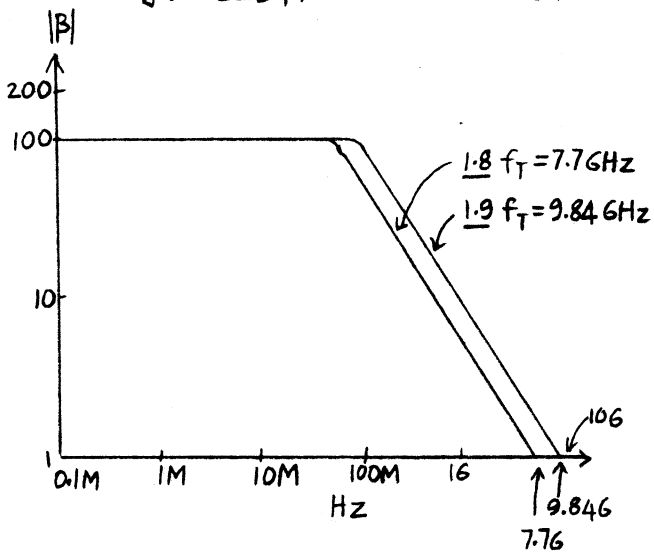
$$C_{cs} = \frac{C_{cs0}}{\sqrt{1+\frac{V_{cs}}{\psi_o}}} = \frac{20\text{ fF}}{\sqrt{1+\frac{2}{0.55}}} = 9.29\text{ fF}$$

$$C_\pi = C_b + C_{je} = 617\text{ fF}$$



For problem (1.9),

$$\beta(j\omega) = \frac{38.5\text{ mA/V}}{j\omega\ 623\text{fF}} = \frac{61.8\text{G}}{j\omega} = \frac{9.84\text{G}}{jf}$$



1.10 At high frequencies,

$$\beta(j\omega) = \frac{\omega_T}{j\omega} = \frac{g_m}{j\omega(C_\mu + C_\pi)}$$

For problem (1.8)

$$\beta(j\omega) = \frac{7.69\text{ mA/V}}{j\omega\ 159\text{ fF}} = \frac{48.4\text{G}}{j\omega} = \frac{7.7\text{G}}{jf}$$

1.11

(a) At $I_C = 1\,mA$, $\tau_{T_1} = \frac{1}{2\pi f_{T_1}} = \frac{1}{2\pi \times 600 \times 10^6}$

$= 0.265\,ns$

At $I_C = 10\,mA$, $\tau_{T_2} = \frac{1}{2\pi f_{T_2}} = \frac{1}{2\pi \times 10^9}$

$= 0.159\,ns$

$\therefore 265 = \tau_F + 26 \times (C_\mu + C_{je})$

$159 = \tau_F + 2.6 \times (C_\mu + C_{je})$

$\therefore 106 = 23.4\,(C_\mu + C_{je})$

$\therefore C_\mu + C_{je} = 4.53\,pF$

$\tau_F = 147\,ps$

Since $C_\mu \simeq 0.15\,pF$, $C_{je} = 4.4\,pF$

$I_C = 0.1\,mA$

$g_m = \frac{0.1 \times 10^{-3}}{26 \times 10^{-3}} = 3.8\,mA/V$

$r_\pi = \frac{100}{3.8}\,K\Omega = 26\,K\Omega$

$r_o = 500\,K\Omega$

$r_\mu = 5 \times 100 \times 500\,K\Omega = 250\,M\Omega$

$C_b = \tau_F g_m = 0.147 \times 10^{-9} \times 3.8 \times 10^{-3}$

$= 0.56\,pF$

$C_\pi = 0.56 + 4.4 = 5.0\,pF$

$C_\mu = 0.15 \sqrt{\frac{1 + \frac{1.0}{0.55}}{1 + \frac{2}{0.55}}} = 0.31\,pF$

$C_{cs} = 1 \sqrt{\frac{1 + \frac{10}{0.55}}{1 + \frac{15}{0.55}}} = 0.82\,pF$

$r_b = 100\,\Omega$, $r_c = 100\,\Omega$

$I_C = 1\,mA$

$g_m = \frac{1}{26} = 38\,mA/V$

$r_\pi = \frac{100}{38}\,K\Omega = 2.6\,K\Omega$

$r_o = 50\,K\Omega$, $r_\mu = 25\,M\Omega$

$C_b = 5.6\,pF$

$C_\pi = 5.6 + 4.4 = 10\,pF$

$C_\mu = 0.31\,pF$, $C_{cs} = 0.82\,pF$

$r_b = 100\,\Omega$, $r_c = 100\,\Omega$

$I_C = 5\,mA$

$g_m = 190\,mA/V$

$r_\pi = \frac{100}{190} = 530\,\Omega$

$r_o = 10\,K\Omega$, $r_\mu = 5\,M\Omega$

$C_b = 28\,pF$

$C_\pi = 28 + 4.4 = 32.4\,pF$

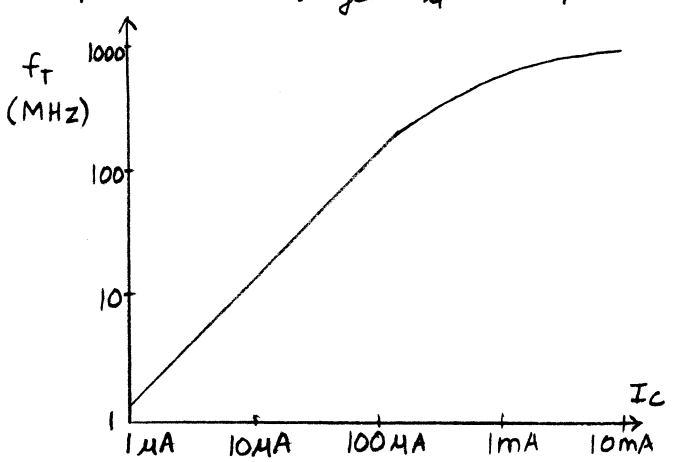
$C_\mu = 0.31\,pF$, $C_{cs} = 0.82\,pF$

$r_b = 100\,\Omega$, $r_b = 100\,\Omega$

(b) $f_T = \frac{1}{2\pi} \frac{g_m}{C_b + C_{je} + C_\mu}$

$= \frac{1}{2\pi} \frac{1}{\tau_F + \frac{V_T}{I_C}(C_{je} + C_\mu)}$

$\tau_F = 0.147\,ns$, $C_{je} + C_\mu = 4.7\,pF$



1.12

(a) $\tau_F = \frac{W_B^2}{2D_P} = \frac{10^{-6}}{26} = 38.5\ ns$

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_b + C_{je}}$$

$$= \frac{1}{2\pi}\frac{1}{\tau_F + \frac{V_T C_{je}}{I_C}}$$

$$= \frac{1}{2\pi}\frac{1}{38.5 + 0.1} \times 10^9 = 4.1\ MHz$$

$$Q = \tau_F I_C = 38.5 \times 10^{-9} \times 0.5 \times 10^{-3}$$

$$= 19.3\ pC$$

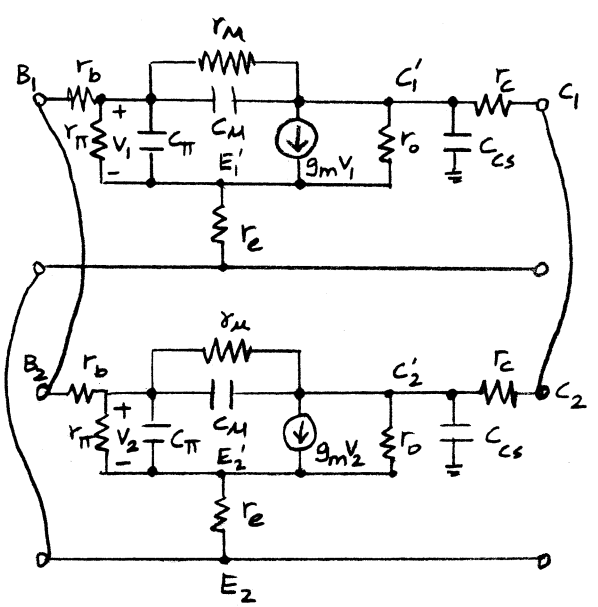
(b) $\frac{\Delta I_C}{I_C} = \frac{\Delta W_B}{W_B} = \frac{0.1}{10}$ per volt of V_{CE}

$\therefore \Delta I_C = 0.5 \times \frac{1}{100}$ mA per volt of V_{CE}

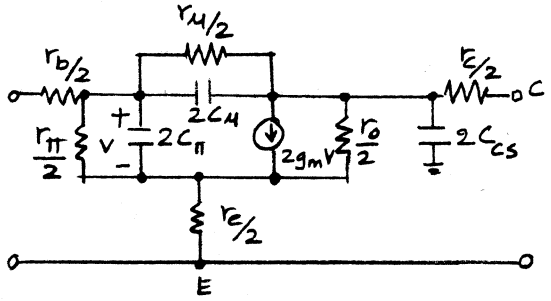
$\therefore r_o = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{100}{0.5}\ K\Omega = 200\ K\Omega$

1.13

Connect two transistors in parallel, each with a collector bias current $\frac{I_C}{2}$

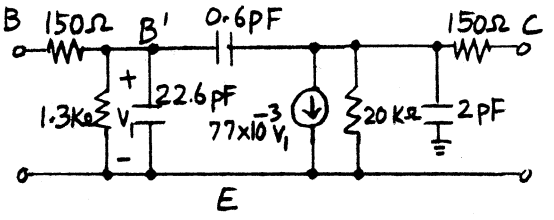


Points B_1' and B_2' will be at the same potential and can be connected. Also C_1' and C_2', E_1' and E_2'. Thus, the composite device becomes as shown with a collector bias current I_C.



Since r_b, r_c and r_e are assumed constant, the values of these resistors are halved in the composite device as compared to the original device at the same total current.

Since C_μ and C_{cs} are independent of bias current, the values of these capacitors are doubled in the composite device. Since r_π, r_o and r_μ are proportional to $1/I_C$, the values of these resistors are unchanged in the composite device as compared to the original device with the same total current. Since g_m is proportional to I_C, its value is unchanged in the composite device.



Finally, $C_\pi = C_{je} + C_b$ where C_{je} is assumed constant and C_b is proportional to I_C. Thus, in the composite device, C_{je} is doubled and C_b is unchanged.

As a consequence, C_π in the composite is larger than in the original device at the same total current, but the increase in C_π is less than two times.

1.14

Since current gain is 9 at 50 MHz,

$f_T = 9 \times 50 = 450$ MHz at $I_C = 1$mA

$$\therefore \tau_T = \frac{1}{2\pi f_T} = 0.354 \text{ ns}$$

From (1.130),

$$\tau_T = \tau_F + \frac{C_{je} + C_\mu}{g_m}$$

$$\therefore 0.354 = 0.25 + (C_{je} + C_\mu) \times 26$$

$$\therefore C_{je} + C_\mu = 4 \text{ pF}$$

Given $C_\mu = 0.6$ pF, $\therefore C_{je} = 3.4$ pF

At $I_C = 2$ mA

$$C_b = \tau_F g_m = 0.25 \times 10^{-9} \times \frac{1}{13} = 19.2 \text{ pF}$$

$$\therefore C_\pi = C_{je} + C_b = 3.4 + 19.2 = 22.6 \text{ pF}$$

$$g_m = \frac{1}{13} = 77 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{40}{2} \text{ kΩ} = 20 \text{ kΩ}$$

$$r_\pi = \frac{\beta_0}{g_m} = 1300 \text{ Ω}$$

1.15

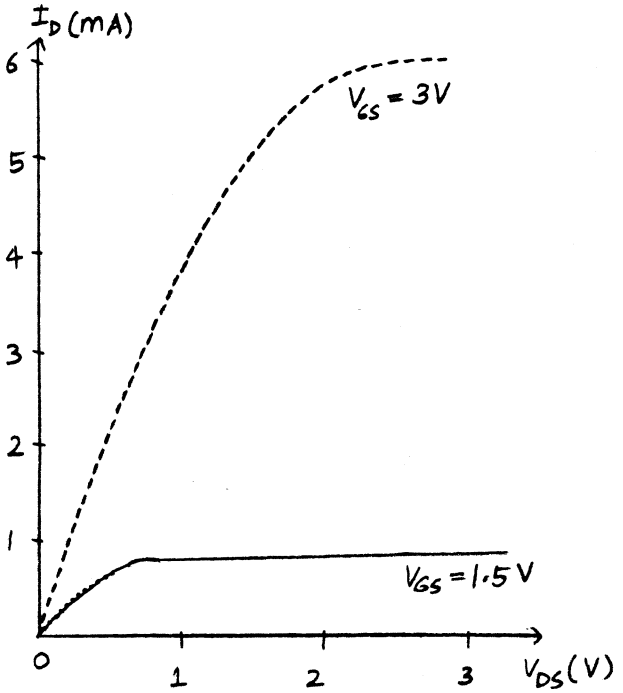
(a) $\frac{K'}{2}\frac{W}{L} = 97 \times 10 = 970\ \mu A/V^2$

so in triode region,

$I_D = 970\left[2(V_{GS}-0.6)V_{DS} - V_{DS}^2\right]\ \mu A$

In active region (saturation),

$I_D = 970\,(V_{GS}-0.6)^2(1+0.024V_{DS})\ \mu A$



(b) From (1.140),

$V_t = V_{to} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

From (1.141),

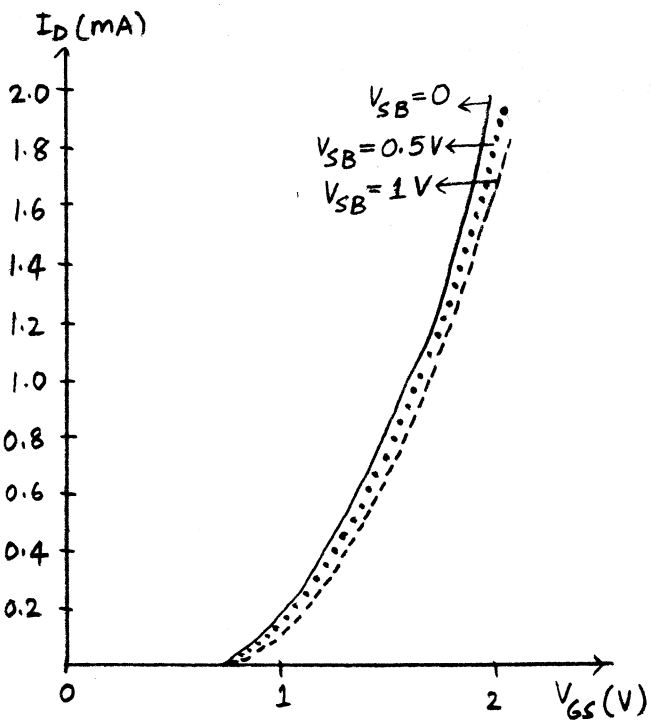
$$\gamma = \frac{\sqrt{2q\varepsilon N_A}}{C_{ox}} = \frac{\sqrt{2\times 1.6\times 10^{-19}\times 1.04\times 10^{-12}\times 5\times 10^{15}}}{4.3\times 10^{-7}}$$

$$= 0.094\sqrt{V}$$

From (1.142),

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.9\times 8.85\times 10^{-14}}{80\times 10^{-8}}\times 10^{-8} = 4.3\ \frac{fF}{\mu m^2}$$

$$= 4.3\times 10^{-7}\ \frac{F}{cm^2}$$



When $V_{SB} = 0$, $V_t = 0.6\,V$

When $V_{SB} = 0.5\,V$, $V_t = 0.63\,V$

When $V_{SB} = 1\,V$, $V_t = 0.65\,V$

1.16

$$V_t = 0.6 + 0.094\left(\sqrt{1.6} - \sqrt{0.6}\right)$$
$$\simeq 0.646 \text{ V}$$

$$I_D = \frac{K'}{2}\frac{W}{L}(V_{GS}-V_t)^2(1+\lambda V_{DS})$$
$$= \frac{194}{2}\frac{10}{1}(1-0.646)^2(1+0.024(2))$$
$$= 127\ \mu A$$

From (1.180),

$$g_m = \sqrt{2K'\frac{W}{L}I_D} = \sqrt{2(194)10(127)}$$
$$= 702\ \mu A/V$$

From (1.199),

$$g_{mb} = \gamma\sqrt{\frac{K'W/L\, I_D}{2(2\phi_F+V_{SB})}} = 0.094\sqrt{\frac{194(10)(127)}{2(0.6+1)}}$$
$$= 26\ \mu A/V$$

From (1.194), $r_o = \frac{1}{\lambda I_D} = \frac{1}{0.024(127\mu)}$

$$\simeq 328\ K\Omega$$

From (1.201), $C_{sb} = \frac{C_{sbo}}{\sqrt{1+\frac{V_{SB}}{\psi_o}}} = \frac{20\,fF}{\sqrt{1+\frac{1}{0.7}}} = 13\,fF$

$V_{DB} = V_{DS} + V_{SB} = 2+1 = 3V$

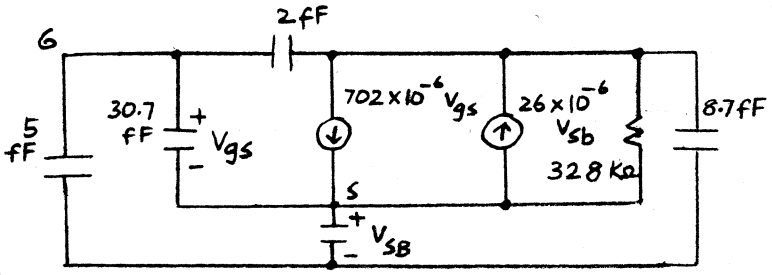
From (1.202), $C_{db} = \frac{C_{dbo}}{\sqrt{1+\frac{V_{DB}}{\psi_o}}} = \frac{20\,fF}{\sqrt{1+\frac{3}{0.7}}} = 8.7\,fF$

$C_{gd} = 2\,fF$

From (1.191),

$C_{gs_i} = \frac{2}{3}WLC_{ox} = \frac{2}{3}(10)(1)(4.3\,f) = 28.7\,fF$

$C_{gs} = C_{gs_i} + C_{gs_{OL}} = 28.7 + 2 = 30.7\,fF$



1.17

From (1.208),

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}+C_{gd}+C_{gb}} = \frac{1}{2\pi}\frac{g_m}{37.7\times10^{-15}}$$

From (1.179)

$$g_m = K'\frac{W}{L}(V_{GS}-V_t)(1+\lambda V_{DS})$$
$$= 194\frac{10}{1}(V_{GS}-0.6)(1+\lambda V_{DS})$$

Here, $1+\lambda V_{DS} = 1+0.024(3) = 1.072$

This term is usually ignored but is included here to demonstrate its use. Ignoring the term would cause a 7.2% error

V_{GS} (V)	$V_{GS}-V_t$ (V)	$g_m\left(\frac{mA}{V}\right)$	f_T (GHz)
1.0	0.4	0.83	3.5
1.5	0.9	1.87	7.9
2.0	1.4	2.91	12.3

1.18

In linear region, from (1.215)

$$I_D = \frac{\mu_n C_{ox}}{2\left(1+\frac{V_{DS}}{\mathcal{E}_c L}\right)} \frac{W}{L}\left[2(V_{GS}-V_t)V_{DS} - V_{DS}^2\right]$$

From (1.220),

$$V_{DS(act.)} = \mathcal{E}_c L\left[\sqrt{1+\frac{2(V_{GS}-V_t)}{\mathcal{E}_c L}} - 1\right]$$

In active region, from (1.223)

$$I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L} V_{DS(act.)}^2$$

$\mathcal{E}_c L = 15\,V$

(a)

V_{GS} (V)	$V_{GS}-V_t$ (V)	$V_{DS(act)}$ (V)
1.0	0.40	0.395
2.0	1.40	1.34
3.0	2.40	2.23

	V_{DS}(V)	No vel. sat. I_D (μA)	vel. sat. I_D (μA)
V_{GS}=1V	0	0	0
	0.2	116.4	114.9
	0.4	155.2	151.2
V_{GS}=2V	V_{DS}(V)	No vel. sat. I_D (μA)	vel. sat. I_D (μA)
	0	0	0
	0.2	504.4	497.8
	0.4	931.2	907.0
	1.4	1901.2	1742.1
V_{GS}=3V	V_{DS} (V)	No vel. sat I_D (μA)	vel. sat. I_D (μA)
	0	0	0
	0.2	892.4	880.7
	0.4	1707.2	1662.9
	1.4	4617.2	4223.0
	2.4	5587.2	4839.7

(b) $\mathcal{E}_c L = 1.5\,V$

V_{GS}(V)	$V_{GS}-V_t$ (V)	$V_{DS(act)}$(V)
1.0	0.40	0.3574
2.0	1.40	1.0397
3.0	2.40	1.5741

	V_{DS}(V)	No vel. sat. I_D(μA)	vel. sat. I_D(μA)
V_{GS}=1V	0	0	0
	0.2	116.4	102.7
	0.4	155.2	123.9
V_{GS}=2V	V_{DS}(V)	No vel. sat. I_D(μA)	vel. sat. I_D(μA)
	0	0	0
	0.2	504.4	445.1
	0.4	931.2	735.2
	1.4	1901.2	1048.5
V_{GS}=3V	V_{DS}(V)	No vel. sat. I_D(μA)	vel. sat. I_D(μA)
	0	0	0
	0.2	892.4	787.4
	0.4	1707.2	1347.8
	1.4	4617.2	2388.2
	2.4	5587.2	2403.4

(c) $\mathcal{E}_{cL} = 0.75\,V$

V_{GS}(V)	$V_{GS}-V_t$ (V)	$V_{DS(act)}$(V)
1.0	0.40	0.3282
2.0	1.40	0.8817
3.0	2.40	1.290

	V_{DS}(V)	No vel. sat I_D(μA)	vel sat. I_D(μA)
V_{GS}=1V	0	0	0
	0.2	116.4	91.89
	0.4	155.2	104.5

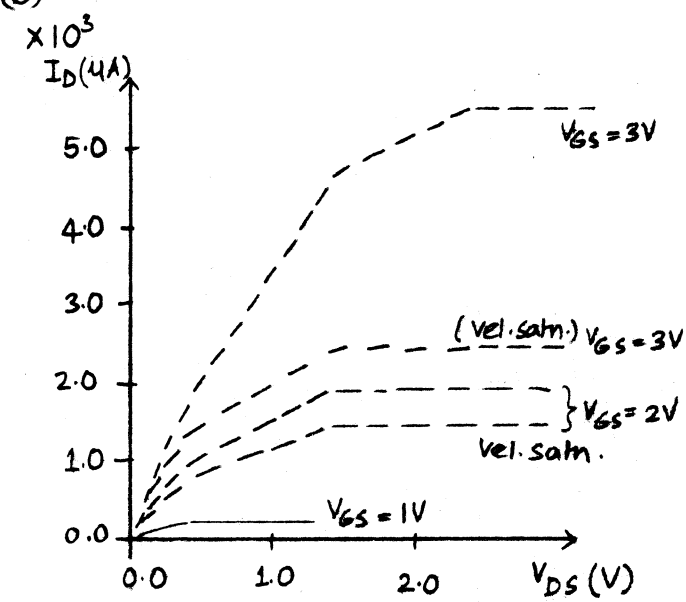
$V_{GS} = 2V$

V_{DS} (V)	No velocity sat. I_D (μA)	vel. satn. I_D (μA)
0	0	0
0.2	504.4	398.2
0.4	931.2	607.3
1.4	1901.2	754.1

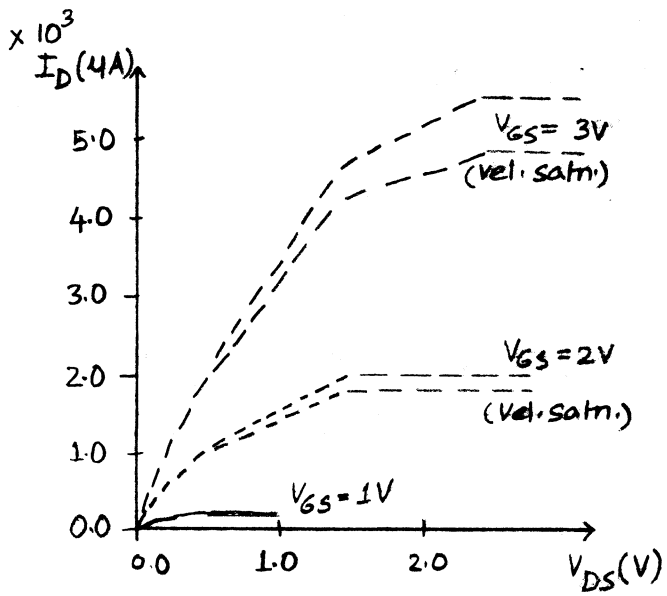
$V_{GS} = 3V$

V_{DS} (V)	No velocity sat. I_D (μA)	vel. satn. I_D (μA)
0	0	0
0.2	892.4	704.4
0.4	1707.2	1113.4
1.4	4617.2	1610.7
2.4	5587.2	1614.7

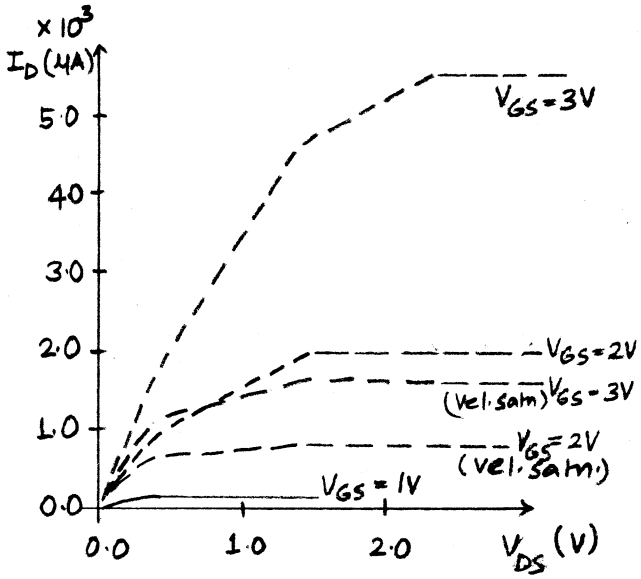
(b)



(a)



(c)



1.19

Direct calculation :

$$I_D = \frac{k'}{2}\frac{W}{L} V_{DS_{act}}^2$$

$$V_{DS_{act}} = \varepsilon_c L \left[\sqrt{1 + \frac{2(V_{GS}-V_t)}{\varepsilon_c L}} - 1 \right]$$

$L = 0.5\ \mu m$

$\varepsilon_c L = 0.75$

$$\frac{k'}{2}\frac{W}{L} = \frac{194}{2}\frac{2}{0.5} = 388\ \frac{\mu A}{V^2}$$

$$V_{DS(act)} = 0.75 \left[\sqrt{1 + \frac{2(V_{GS}-0.6)}{0.75}} - 1 \right]$$

V_{GS} (V)	$V_{GS}-V_t$ (V)	$V_{DS\,act.}$ (V)	I_D (μA)
0.6	0.0	0	0
0.8	0.2	0.179	12.43
1.0	0.4	0.328	41.7
1.2	0.6	0.459	81.7
1.6	1.0	0.686	182.59
1.8	1.2	0.787	240.31
1.9	1.3	0.835	270.52
2.0	1.4	0.882	301.83

Now use (1.230),

$$\frac{k'}{2}\frac{W}{L} = \frac{194}{2}\frac{2}{0.5} = 388\ \frac{\mu A}{V^2}$$

$$R_{Sx} = \frac{1}{\varepsilon_c k' W} = \frac{1}{1.5\times10^6 (194) 2\mu} = 1718.2$$

$$k'\frac{W}{L} R_{Sx} = 194 \times 4 \times 1718.2 \times 10^{-6} = 1.333$$

V_{GS} (V)	$V_{GS}-V_t$ (V)	$\frac{1}{1+k'\frac{W}{L}R_{Sx}(V_{GS}-V_t)}$	I (μA)
0.6	0.0	1.0	0
0.8	0.2	0.790	12.4
1.0	0.4	0.652	40.9
1.2	1.6	0.556	78.4
1.6	1.0	0.429	168
1.8	1.2	0.385	217

10% error occurs when $V_{GS} = 1.8$ V

Reasonable since,

$$(V_{GS}-V_t)^2 = (1.2)^2 = 1.44$$

$$(I_D R_{Sx})^2 = \left[242.8 \times 10^{-6} (1718.2)\right]^2$$

$$= 0.1740$$

0.17 is 12% of 1.44

1.20 simple calculation:

$$g_m = K' \frac{W}{L}(V_{GS}-V_t)$$

$$= \mu_n \frac{\epsilon_{ox}}{t_{ox}} \frac{W}{L}(V_{GS}-V_t)$$

$$= \frac{\mu_n \epsilon_{ox}}{L}(50)\frac{W}{L}(V_{GS}-V_t)$$

$$= 450 \frac{cm^2}{Vs}(3.9)(8.854\times10^{-14}\frac{F}{cm}) \times \frac{50(10\mu m)}{L^2} \times \frac{10^4 \mu m}{1\,cm}(0.1)$$

$$= \frac{7.769\times10^{-4}}{L^2}(0.1)$$

$$= \frac{1.1654\times10^{-3}}{L} R \ \frac{\mu A}{V^2}$$

L (μm)	R	$g_m = \frac{1.1654\times10^{-3}R}{L}$ μA/V
10	6.6×10^{-3}	0.77
2	3.175×10^{-2}	18.5
1	6.066×10^{-2}	70.7
0.5	0.11148	260
0.4	0.134	390

L (μm)	g_m (μA/V²)	
0.4	485	(24% error)
0.5	310	(20% error)
1.0	77.7	(10% error)
2.0	19.4	
10.0	0.77	

Now with velocity saturation,

$g_m = WC_{ox}\mu_n \mathcal{E}_c R$ where,

$$R = \frac{\sqrt{1+\frac{2(V_{GS}-V_t)}{\mathcal{E}_c L}} - 1}{\sqrt{1+\frac{2(V_{GS}-V_t)}{\mathcal{E}_c L}}}$$

$$g_m = (10\mu m)(3.9)\frac{(8.854\times10^{-14}\frac{F}{cm})}{L/50} 450 \frac{cm^2}{V\,sec} \times 1.5\,R \times \frac{10^4\mu m}{1\,cm}$$

$$= \frac{7.77\times10^{-5}}{L} R(\mathcal{E}_c W)\ \mu A/V^2$$

1.21

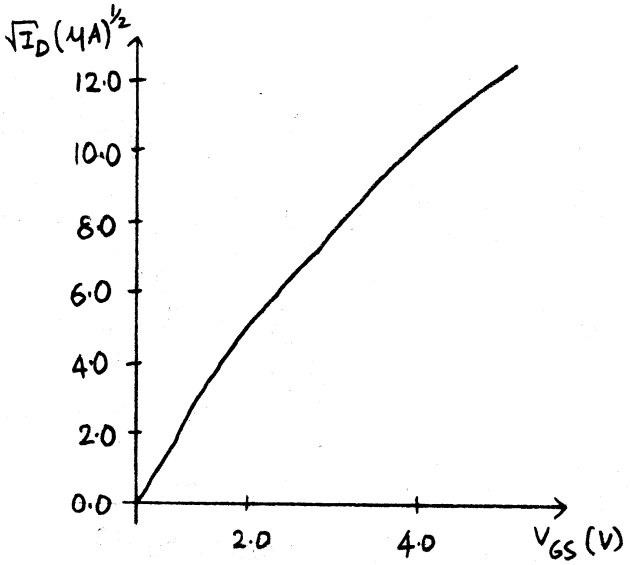
From (1.223), $I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [V_{DS(act)}]^2$

From (1.220), $V_{DS(act.)} = \varepsilon_c L \left[\sqrt{1 + \frac{2(V_{GS} - V_t)}{\varepsilon_c L}} - 1 \right]$

$\varepsilon_c L = 1.5$ $\quad V_t = 0.7$ V

V_{GS} (V)	$V_{DS(act.)}$ (V)	I_D (μA)	$\sqrt{I_D}$ $(\mu A)^{1/2}$	$\frac{K'}{2}\frac{W}{L}(V_{GS}-V_t)^2$ (μA)	$\sqrt{\frac{K'}{2}\frac{W}{L}(V_{GS}-V_t)^2}$ $(\mu A)^{1/2}$	$V_{GS}-V_t$ (V)
0.7	0.0	0.0	0.0	0.0	0.0	0.0
0.8	0.097	0.253	0.503	0.27	0.520	0.1
0.9	0.188	0.956	0.978	1.08	1.04	0.2
1.0	0.275	2.04	1.43	2.43	1.56	0.3
1.5	0.656	11.6	3.41	17.3	4.16	0.8
2.0	0.980	25.9	5.09	45.6	6.75	1.3
3.0	1.52	62.4	7.89	143	12.0	2.3
4.0	1.99	106.9	10.3	294	17.1	3.3
5.0	2.39	154.2	12.4	499	22.3	4.3

For large V_{GS}, current is limited by velocity saturation.



1.22

$$I_D = K_x \frac{W}{L} e^{V_{GS}/nV_T}\left(1 - e^{-V_{DS}/V_T}\right)$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{nV_T} K_x \frac{W}{L} e^{V_{GS}/nV_T}\left(1 - e^{-V_{DS}/V_T}\right)$$

$$= \frac{1}{nV_T} I_D = \frac{10 \times 10^{-9}}{1.5 \times 26 \times 10^{-3}} = 2.56 \times 10^{-7}$$

$$= 0.256\ \mu A/V$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd} + C_{gb})} = \frac{0.256\ \mu A/V}{2\pi(10fF)}$$

$$= 4.076\ MHz$$